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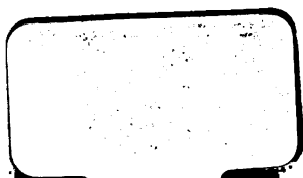
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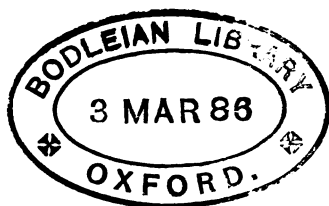
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# CONTENTS.

## SECOND SECTION.

### LENGTHS.

| CHAPTER                                   | PAGE |
|-------------------------------------------|------|
| V. Right-angled Triangle . . . . .        | 1    |
| VI. Similar Figures . . . . .             | 4    |
| VII. Chords of a Circle . . . . .         | 6    |
| VIII. Circumference of a Circle . . . . . | 8    |
| IX. Arc of a Circle . . . . .             | 10   |

## THIRD SECTION.

### AREAS.

|                                                            |    |
|------------------------------------------------------------|----|
| XI. Rectangle . . . . .                                    | 11 |
| XII. Parallelogram . . . . .                               | 18 |
| XIII. Triangle . . . . .                                   | 19 |
| XIV. Quadrilaterals . . . . .                              | 24 |
| XV. Rectilineal Figure . . . . .                           | 28 |
| XVI. Circle . . . . .                                      | 31 |
| XVII. Sector of a Circle and Segment of a Circle . . . . . | 36 |
| XVIII. Simpson's Rule . . . . .                            | 40 |
| XIX. Similar Figures . . . . .                             | 41 |

## FOURTH SECTION.

### VOLUMES.

|                                                             |    |
|-------------------------------------------------------------|----|
| XXII. Rectangular Parallelepiped . . . . .                  | 45 |
| XXIII. Parallelepiped, Prism, Cylinder . . . . .            | 49 |
| XXIV. Segments of a Right Circular Cylinder. Ring . . . . . | 53 |
| XXV. Pyramid. Cone . . . . .                                | 55 |
| XXVI. Frustum of a Pyramid or Cone . . . . .                | 58 |
| XXVII. Wedge . . . . .                                      | 62 |

| CHAPTER                                     | PAGE |
|---------------------------------------------|------|
| XXVIII. Prismoid . . . . .                  | 64   |
| XXIX. Sphere . . . . .                      | 66   |
| XXX. Zone and Segment of a Sphere . . . . . | 70   |
| XXXI. Irregular Solids . . . . .            | 73   |
| XXXII. Similar Solids . . . . .             | 73   |

## FIFTH SECTION.

## AREAS OF THE SURFACES OF SOLIDS.

|                                                             |    |
|-------------------------------------------------------------|----|
| XXXIII. Plane Surfaces . . . . .                            | 76 |
| XXXIV. Right Circular Cylinder . . . . .                    | 79 |
| XXXV. Segments of a Right Circular Cylinder. Ring . . . . . | 82 |
| XXXVI. Right Circular Cone . . . . .                        | 83 |
| XXXVII. Frustum of a Right Circular Cone . . . . .          | 87 |
| XXXVIII. Sphere . . . . .                                   | 88 |
| XXXIX. Zone of a Sphere. Segment of a Sphere . . . . .      | 90 |

## SIXTH SECTION.

## PRACTICAL APPLICATIONS.

|                                 |    |
|---------------------------------|----|
| XL. Introduction . . . . .      | 92 |
| XLI. Artificers' Work . . . . . | 94 |
| XLII. Timber Measure . . . . .  | 96 |
| XLIII. Gauging . . . . .        | 97 |

## SEVENTH SECTION.

## LAND SURVEYING.

|                                  |     |
|----------------------------------|-----|
| XLVI. The Field-Book . . . . .   | 98  |
| XLVIII. Duodecimals . . . . .    | 104 |
| XLIX. Metrical System . . . . .  | 104 |
| Miscellaneous Examples . . . . . | 106 |

# MENSURATION.

## CHAPTER V.

$$\begin{aligned}
 1. \quad & (\text{By Euc. I. 47}) \text{Hypoth.}^2 = \text{Base}^2 + \text{Perpen.}^2 = 532^2 \text{ sq. ft.} + 165^2 \text{ sq. ft.} \\
 & = (500 + 32)^2 \text{ sq. ft.} + (200 - 35)^2 \text{ sq. ft.} \\
 & = (250000 + 32000 + 32^2) \text{ sq. ft.} + (40000 - 14000 + 35^2) \text{ sq. ft.} \\
 & = \{250000 + 32000 + (30 + 2)^2\} \text{ sq. ft.} + \{40000 - 14000 + (40 - 5)^2\} \text{ sq. ft.} \\
 & = (250000 + 32000 + 900 + 120 + 4) \text{ sq. ft.} + (40000 - 14000 + 1600 - 400 + 25) \text{ sq. ft.} \\
 & = (283024 + 27225) \text{ sq. ft.} = 310249 \text{ sq. ft.} \\
 & \therefore \text{Hypoth.} = \sqrt{(310249 \text{ sq. ft.})} = 557 \text{ ft.}
 \end{aligned}$$

$$\begin{aligned}
 3. \quad & \text{Hypoth.}^2 = (278 \text{ ft. } 8 \text{ in.})^2 + (262 \text{ ft. } 6 \text{ in.})^2 = (278\frac{2}{3} \text{ ft.})^2 + (262\frac{1}{2} \text{ ft.})^2 \\
 & = \left(\frac{836}{3} \text{ ft.}\right)^2 + \left(\frac{525}{2} \text{ ft.}\right)^2 = \left(\frac{800 + 36}{3} \text{ ft.}\right)^2 + \left(\frac{500 + 25}{2} \text{ ft.}\right)^2 \\
 & = \frac{640000 + 57600 + 1296}{9} \text{ sq. ft.} + \frac{250000 + 25000 + 625}{4} \text{ sq. ft.} \\
 & = \left(\frac{698896}{9} + \frac{275625}{4}\right) \text{ sq. ft.} = \frac{5276209}{36} \text{ sq. ft.} \\
 & \therefore \text{Hypoth.} = \sqrt{\left(\frac{5276209}{36} \text{ sq. ft.}\right)} = \frac{2297}{6} \text{ ft.} = 382 \text{ ft. } 10 \text{ in.}
 \end{aligned}$$

$$\begin{aligned}
 4. \quad & \text{Hypoth.}^2 = \left(\frac{1}{2} \text{ mile}\right)^2 + (345\frac{1}{2} \text{ yds.})^2 = (880 \text{ yds.})^2 + \left(\frac{1036}{3} \text{ yds.}\right)^2 \\
 & = (800 + 80)^2 \text{ sq. yds.} + \left\{\frac{1}{3}(1000 + 36)\right\}^2 \text{ sq. yds.} = (640000 + 128000 + 6400) \text{ sq. yds.} \\
 & + \left\{\frac{1}{3}(1000000 + 72000 + 1296)\right\} \text{ sq. yds.} = \left(774400 + \frac{1073296}{9}\right) \text{ sq. yds.} \\
 & = \frac{8042896}{9} \text{ sq. yds.}
 \end{aligned}$$

$$\therefore \text{Hypoth.} = \sqrt{\left(\frac{8042896}{9} \text{ sq. yds.}\right)} = \frac{2836}{3} \text{ yds.} = 945 \text{ yds. } 1 \text{ ft.}$$

T. M. K.

$$9. \text{ Hypoth.}^2 = \text{Base}^2 + \text{Perpen.}^2;$$

$$\therefore \text{Base}^2 = \text{Hypoth.}^2 - \text{Perpen.}^2 = (\text{Hypoth.} + \text{Perpen.})(\text{Hypoth.} - \text{Perpen.})$$

$$= (725 + 644) \text{ ft.} \times (725 - 644) \text{ ft.} = 1369 \times 81 \text{ sq. ft.}$$

$$= 37^2 \times 9^2 \text{ sq. ft.} = 333^2 \text{ sq. ft.}$$

$$\therefore \text{base} = \sqrt{(333^2 \text{ sq. ft.})} = 333 \text{ ft.}$$

$$17. \text{ Here } AB = 22620 \text{ ft., } AC = 12815 \text{ ft.,}$$

$$AD = 11484 \text{ ft. Required } BC. \text{ (By Euc. I. 47)}$$

$$BD^2 = BA^2 - AD^2 = (BA + AD)(BA - AD)$$

$$= (22620 + 11484) \text{ ft.} \times (22620 - 11484) \text{ ft.}$$

$$= 34104 \times 11136 \text{ sq. ft.} = (29 \times 24 \times 49)$$

$$\times (29 \times 24 \times 16) \text{ sq. ft.} = 29^2 \times 24^2 \times 7^2 \times 4^2 \text{ sq. ft.,}$$

$$\therefore BD = \sqrt{(29^2 \times 24^2 \times 7^2 \times 4^2 \text{ sq. ft.})} = 29 \times 24 \times 7 \times 4 \text{ ft.} = 19488 \text{ ft.}$$

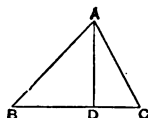
$$\text{Again, } DC^2 = AC^2 - AD^2 = (AC + AD)(AC - AD)$$

$$= (12815 + 11484) \text{ ft.} \times (12815 - 11484) \text{ ft.} = 24299 \times 1331 \text{ sq. ft.}$$

$$= (11 \times 47 \times 47) \times (11 \times 11 \times 11) \text{ sq. ft.} = 11^2 \times 11^2 \times 47^2 \text{ sq. ft.;}$$

$$\therefore DC = \sqrt{(11^2 \times 11^2 \times 47^2 \text{ sq. ft.})} = 11^2 \times 47 \text{ ft.} = 121 \times 47 \text{ ft.} = 5687 \text{ ft.};$$

$$\therefore BC = BD + DC = (19488 + 5687) \text{ ft.} = 25175 \text{ ft.}$$



$$18. \text{ Let } a = \text{base, } b = \text{height, and } c = \text{hypotenuse. Then } a = 3925 \text{ ft., } c - b = 625 \text{ ft. Required } b \text{ and } c.$$

$$\text{From Euc. I. 47, or Art. 60 of the Mensuration,}$$

$$a^2 = c^2 - b^2 = (c + b)(c - b) = (c + b) \times 625;$$

$$\text{Thus } c + b = 3925^2 \div 625 = 15405625 \div 625 = 24649.$$

$$\text{Add } c - b \text{ or } 625 \text{ to this, and divide by 2, we get } c = 12637;$$

$$\text{Also subtract } c - b \text{ or } 625, \text{ and divide by 2, we get } b = 12012.$$

$$19. \text{ Here } AB = CD = 25 \text{ ft., } AC = 1 \text{ ft. Required } DB.$$

$$\therefore BC = AB - AC = (25 - 1) \text{ ft.} = 24 \text{ ft.}$$

$$\text{Now (Euc. I. 47) } DB^2 = DC^2 - BC^2 = (DC + BC)(DC - BC)$$

$$= (25 + 24) \text{ ft.} \times (25 - 24) \text{ ft.} = 49 \text{ sq. ft.}$$

$$\therefore DB = \sqrt{(49 \text{ sq. ft.})} = 7 \text{ ft.}$$

$$20. \text{ Here } DC = AC = 40 \text{ ft., } AB = 24 \text{ ft., } DE = 32 \text{ ft.}$$

$$\text{Required } BE.$$

$$\text{(Euc. I. 47) } BC^2 = AC^2 - AB^2$$

$$= (AC + AB)(AC - AB) = (40 + 24) \text{ ft.} \times (40 - 24) \text{ ft.}$$

$$= 64 \times 16 \text{ sq. ft.} = 8^2 \times 4^2 \text{ sq. ft.};$$

$$\therefore BC = \sqrt{(8^2 \times 4^2 \text{ sq. ft.})} = 8 \times 4 \text{ ft.} = 32 \text{ ft.,}$$

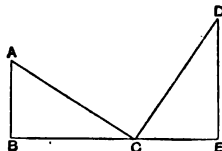
$$\text{and } CE^2 = CD^2 - DE^2 = (CD + DE)(CD - DE)$$

$$= (40 + 32) \text{ ft.} \times (40 - 32) \text{ ft.} = 72 \times 8 \text{ sq. ft.}$$

$$= 9 \times 8 \times 8 \text{ sq. ft.} = 3^2 \times 8^2 \text{ sq. ft.};$$

$$\therefore CE = \sqrt{(3^2 \times 8^2 \text{ sq. ft.})} = 3 \times 8 \text{ ft.} = 24 \text{ ft.};$$

$$\therefore BE = BC + CE = (32 + 24) \text{ ft.} = 56 \text{ ft.}$$



21. (Vide diagram to preceding.) Here  $CE = 14$  ft.,  $ED = 48$  ft.,  $AB = 40$  ft. Required  $BE$ .

(By Euc. I. 47)

$$DC = \sqrt{(DE^2 + EC^2)} = \sqrt{(48^2 + 14^2)} \text{ sq. ft.} = \sqrt{(2304 + 196)} \text{ sq. ft.} \\ = \sqrt{(2500 \text{ sq. ft.})} = 50 \text{ ft.}; \therefore AC \text{ also} = 50 \text{ ft.};$$

$$\text{and } BC = \sqrt{(AC^2 - AB^2)} = \sqrt{\{(AC + AB)(AC - AB)\}} \\ = \sqrt{\{(50 + 40) \text{ ft.} \times (50 - 40) \text{ ft.}\}} = \sqrt{(90 \times 10 \text{ sq. ft.})} \\ = \sqrt{(900 \text{ sq. ft.})} = 30 \text{ ft.};$$

$$\therefore BE = BC + CE = 44 \text{ ft.}$$

22.  $\text{Diag.}^2 = \text{side}^2 + \text{side}^2 = 2 \text{ side}^2 = 2 \times 1 \text{ sq. in.} = 2 \text{ sq. in.};$

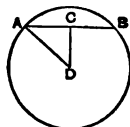
$$\therefore \text{diag.} = \sqrt{(2 \text{ sq. in.})} = 1.4142135624 \text{ in.}$$

24. Here  $AD = 82.66$  ft.,  $CD = 71.1$  ft. Required  $AB$ .

$$AC = \sqrt{(AD^2 - DC^2)} = \sqrt{\{(AD + DC)(AD - DC)\}} \\ = \sqrt{(153.76 \text{ ft.} \times 11.56 \text{ ft.})} = \sqrt{(1777.4656 \text{ sq. ft.})} \\ = 42.16 \text{ ft.};$$

$$\therefore (\text{by Euc. III. 8}) AC = CB;$$

$$\therefore AB = AC + CB = 2AC = 84.32 \text{ ft.}$$



25. Let  $a$  = length, and  $b$  = breadth, and  $d$  = diagonal of rectangle. Here  $a = 147$  yds.,  $b = 196$  yds. Required  $(a + b) - d$ .

(By Euc. I. 47)

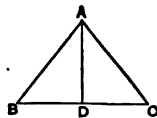
$$d = \sqrt{(a^2 + b^2)} = \sqrt{(147^2 + 196^2)} \text{ sq. yds.} \\ = \sqrt{\{(3^2 \times 7^2 \times 7^2 + 7^2 \times 2^2 \times 7^2 \times 2^2)\} \text{ sq. yds.}} \\ = \sqrt{\{7^2 \times 7^2 (3^2 + 4^2)\} \text{ sq. yds.}} = 7 \times 7 \times 5 \text{ yds.} = 245 \text{ yds.}; \\ \therefore a + b - d = (147 + 196 - 245) \text{ yds.} = 98 \text{ yds.}$$

26. Here  $AB = AC = 17$  ft.,  $BC = 28$  ft. Required  $AD$ .

$$(\text{By Euc. I. 5 and 26}) BD = DC = \frac{BC}{2} = 14 \text{ ft.};$$

$\therefore$  (by Euc. I. 47)

$$AD = \sqrt{(AB^2 - BD^2)} = \sqrt{\{(AB + BD)(AB - BD)\}} \\ = \sqrt{\{(17 + 14) \text{ ft.} \times (17 - 14) \text{ ft.}\}} = \sqrt{(93 \text{ sq. ft.})} = 9.64 \text{ ft.}$$



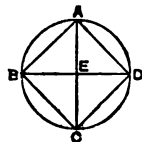
27. Here  $AB = BC = CD = DA = 8$  ft. Required  $AE$ .

(By Euc. I. 47)

$$AB^2 = AE^2 + EB^2 = 2AE^2 \text{ (Euc. I. 15 def.)};$$

$$\therefore AE^2 = \frac{AB^2}{2} = \frac{8^2}{2} \text{ sq. ft.} = 32 \text{ sq. ft.};$$

$$\therefore AE = \sqrt{(32 \text{ sq. ft.})} = \sqrt{(16 \times 2)} \text{ sq. ft.} = 4\sqrt{2} \text{ ft.} \\ = 4 \times 1.4142 \text{ ft.} = 5.6568 \text{ ft.} = 5.66 \text{ ft. nearly.}$$



28. (Vide preceding diagram.) Here  $AE = 6$  ft. Required  $AB$ .

Now (as in preceding)  $AB^2 = 2AE^2 = 2 \times 6^2 \text{ sq. ft.};$

$$\therefore AB = \sqrt{(2 \times 6^2 \text{ sq. ft.})} = 6\sqrt{2} \text{ ft.} = 6 \times 1.4142 \text{ ft.} = 8.4852 \text{ ft.}$$

29. (Vide diag. to 24.) Here  $AD=7$  ft.,  $AB=8$  ft. Required  $CD$ .

(By Euc. III. 8)  $AC=CB=\frac{1}{2}AB=4$  ft.;  $\therefore$  (Euc. I. 47),  
 $CD=\sqrt{(AD^2-AC^2)}=\sqrt{\{(7+4)\text{ ft.} \times (7-4)\text{ ft.}\}}=\sqrt{(11 \times 3\text{ sq. ft.})}$   
 $=\sqrt{(33\text{ sq. ft.})}=5.74\text{ ft.}$

30. (Vide diag. to 24.) Here  $AD=17$  in.,  $CD=18$  in. Required  $AB$ .

$AC=\sqrt{(AD^2-CD^2)}=\sqrt{\{(AD+CD)(AD-CD)\}}=\sqrt{\{(17+18)\text{ in.} \times (17-18)\text{ in.}\}}$   
 $=\sqrt{(30 \times 4\text{ sq. in.})}=\sqrt{(120\text{ sq. in.})}=10.954\text{ in.};$   
 $\therefore$  (by Euc. III. 8)  $AC=CB$ ;  $\therefore AB=AC+CB=2AC=21.908\text{ in.}$

31. Here  $AB=BC=CD=DE=EF=FK=\frac{1}{4}AK=\frac{1}{4} \times 1\text{ ft.}=2\text{ in.}$ ; and  
 $KL=KM=KN=KO=KP=12\text{ in.}$

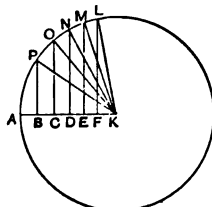
$LF=\sqrt{(LK^2-KF^2)}=\sqrt{(144-4)\text{ sq. in.}}$   
 $=\sqrt{(140\text{ sq. in.})}=11.832\text{ in.};$

$EM=\sqrt{(MK^2-KE^2)}=\sqrt{\{(144-16)\text{ sq. in.}\}}$   
 $=\sqrt{(128\text{ sq. in.})}=11.314\text{ in.};$

$DN=\sqrt{(KN^2-KD^2)}=\sqrt{\{(144-36)\text{ sq. in.}\}}$   
 $=\sqrt{(108\text{ sq. in.})}=10.392\text{ in.};$

$OC=\sqrt{(KO^2-KC^2)}=\sqrt{\{(144-64)\text{ sq. in.}\}}$   
 $=\sqrt{(80\text{ sq. in.})}=8.944\text{ in.};$

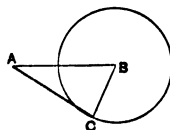
$PB=\sqrt{(KP^2-KB^2)}=\sqrt{\{(144-100)\text{ sq. in.}\}}=\sqrt{(44\text{ sq. in.})}=6.633\text{ in.}$



32. Here  $AB=12$  ft.,  $BC=7$  ft.

$\therefore$  (by Euc. III. 18)  $\angle ACB$  is a rt.  $\angle$ ;

$\therefore$  (Euc. I. 47)  $AC=\sqrt{(AB^2-BC^2)}$   
 $=\sqrt{\{(144-49)\text{ sq. ft.}\}}$   
 $=\sqrt{(95\text{ sq. ft.})}=9.75\text{ ft.}$



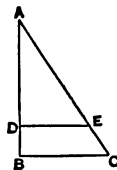
## CHAPTER VI.

1. Here  $AD=5$  in.,  $DE=4$  in.,  $AB=7$  in. Required  $BC$ .

In similar triangles  $ABC$  and  $ADE$  (by Euc. VI. 4),

$$BC : BA :: DE : AD;$$

$$\therefore BC = \frac{BA \times DE}{AD} = \frac{7 \times 4}{5}\text{ in.} = 5.6\text{ in.}$$



2. Here  $AB=BC=CA=80$  in. Required  $AD$ .

(By Euc. I. 5 and 26)

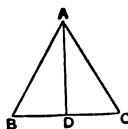
$$BD=DC=\frac{1}{2}BC=\frac{1}{2} \times 80\text{ in.}=40\text{ in.};$$

$$\therefore AD=\sqrt{(AB^2-BD^2)}=\sqrt{\{(AB+BD)(AB-BD)\}}$$

$$=\sqrt{\{(80+40)\text{ in.} \times (80-40)\text{ in.}\}}=\sqrt{(40 \times 120)\text{ sq. in.}}$$

$$=\sqrt{\{(15 \times 8)\text{ sq. in.}\}}=15\sqrt{3}\text{ in.}=15 \times 1.732\text{ in.}$$

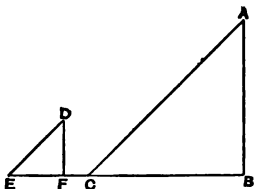
$$=25.98\text{ in.}$$



3. Here  $DF=6$  ft.,  $FE=8\frac{1}{2}$  ft.,  $BC=56\frac{1}{2}$  ft. Required  $AB$ .

(By Euc. VI. 4)  $AB : BC :: DF : FE$ ;

$$\therefore AB = BC \times DF \div FE = 56\frac{1}{2} \times 6 \div 8\frac{1}{2} = 40 \text{ ft.}$$



4. (Vide diag. to preceding.) Here  $DF=3$  ft.,  $EF=4\frac{1}{2}$  ft.,  $AB=45$  ft. Required  $BC$ .

(By Euc. VI. 4)  $BC : BA :: EF : FD$ ;

$$\therefore BC = BA \times EF \div FD = 45 \times 4\frac{1}{2} \div 3 = 67\frac{1}{2} \text{ ft.}$$

5. Now since 1 mile of country =  $\frac{1}{2}$  of an inch on map;  $\therefore$  500 miles on country =  $250$  in., i.e.  $2\frac{1}{2}$  in. on map.

6. Now since 31 miles of country =  $\frac{1}{2}$  in. on map;  $\therefore$  1 mile of country =  $\frac{1}{62}$  in., i.e.  $\frac{1}{62}$  in. on map.

7. Distance required is found from the following proportion

$$6\frac{1}{2} \text{ in.} : 8\frac{1}{2} \text{ in.} :: 54 \text{ miles} : 68 \text{ miles.}$$

8. (Vide diag. to 1.) Here  $BC=20$  in.,  $DE=16$  in.,  $BD=6$  in. Required  $BA$ .

Now  $AD=BA-BD=BA-6$  in.; and (Euc. VI. 4)  $BA : BC :: AD : DE$ ;

i.e.  $BA : BC :: BA - 6 \text{ in.} : DE$ ; i.e.  $BA : 20 \text{ in.} :: BA - 6 \text{ in.} : 16 \text{ in.}$ ;

$\therefore 20 \text{ in.} \times BA - 120 \text{ sq. in.} = 16 \text{ in.} \times BA$ ;  $\therefore 4 \text{ in.} \times BA = 120 \text{ sq. in.}$ ;

$$\therefore BA = 30 \text{ in.}$$

9. (Vide diag. to 1.) Here  $AD=8$  in.,  $DE=7$  in.,  $BD=3$  in. Required  $BC$ .

Now  $AB=AD+DB=(8+3) \text{ in.} = 11 \text{ in.}$ ; and (Euc. VI. 4)

$$BC : BA :: DE : DA$$
;

$$\therefore BC = \frac{BA \times DE}{DA} = \frac{11 \times 7}{8} \text{ in.} = 9\frac{7}{8} \text{ in.}$$

10. (Vide diag. to 1.)  $DE=7$  in.,  $BC=10$  in.,  $BD=2$  in. Required  $DA$ .

Now  $AB=AD+DB=AD+2$  in.; and (Euc. VI. 4)  $AD : DE :: AB : BC$ ;

i.e.  $AD : 7 \text{ in.} :: AD + 2 \text{ in.} : 10 \text{ in.}$ ;  $\therefore 10 \text{ in.} \times AD = 7 \text{ in.} \times AD + 14 \text{ sq. in.}$ ;

$$\therefore 3 \text{ in.} \times AD = 14 \text{ sq. in.}; \therefore AD = 4\frac{2}{3} \text{ in.}$$

11. Here  $DE=16$  ft.,  $BC=20$  ft.,  $GF=5$  ft.;

$$\therefore AF=AG+GF=AG+5 \text{ ft.}$$

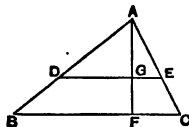
Now from similar triangles  $ABC$  and  $ADE$   
(by Euc. VI. 4)  $BC : AB :: DE : DA$ ;

$\therefore BC : DE :: BA : DA$ ; again from similar  
triangles  $ABF$  and  $ADG$ ,  $AF : AB :: AG : AD$ ;

$\therefore AF : AG :: AB : AD$ ; but, as we have shewn  
 $BC : DE :: BA : DA$ ;

$\therefore AF : AG :: BC : DE$ ;  $\therefore AF : AG :: 20 \text{ ft.} : 16 \text{ ft.}$ , i.e. as  $5 \text{ ft.} : 4 \text{ ft.}$ ;  
i.e.  $AG+5 \text{ ft.} : AG :: 5 \text{ ft.} : 4 \text{ ft.}$ ;  $\therefore 4 \text{ ft.} \times AG + 20 \text{ sq. ft.} = 5 \text{ ft.} \times AG$ ;

$$\therefore AG=20 \text{ ft.}; \therefore AF=(20+5) \text{ ft.}=25 \text{ ft.}$$



12. Here  $AD=8$  ft.,  $BC=14$  ft.,

$$DM=MN=NF;$$

$\therefore DF=3DM$ , and  $DN=\frac{2}{3}DF$ .

Draw  $DE$  parallel to  $AB$  (Euc. I. 31);

$$\therefore (\text{by Euc. I. 34}) BE=KO=GP=AD=8 \text{ ft.}$$

$$\therefore EC=BC-BE=(14-8) \text{ ft.}$$

Now (as in preceding)  $PH : EC :: DM : DF$ ;

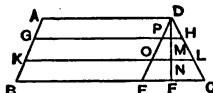
$$\text{i.e. } PH=EC \times DM \div DF=6 \text{ ft.} \times DM \div 3DM=2 \text{ ft.}$$

Again

$$OL : EC :: DN : DF;$$

$$\therefore OL=EC \times DN \div DF=6 \text{ ft.} \times \frac{2}{3}DF \div DF=4 \text{ ft.};$$

$$\therefore GH=GP+PH=(8+2) \text{ ft.}=10 \text{ ft.}; \text{ and } KL=KO+OL=(8+4) \text{ ft.}=12 \text{ ft.}$$



## CHAPTER VII.

[The diagram in Example 1 is used for all the Examples.]

1. Here  $CD=15$  in.;  $AC=4$  ft. 6 in. Required  $CF$ .

Join  $A, F$ .

(By Euc. VI. 8 Cor.)  $CF : CA :: CA : CD$ ;

$$\therefore CF=CA^2 \div CD=(\frac{3}{2} \text{ ft.})^2 \div \frac{1}{2} \text{ ft.}$$

$$= \frac{9}{4} \text{ sq. ft.} \times \frac{2}{1} \text{ ft.} = \frac{9}{2} \text{ ft.} = 4 \frac{1}{2} \text{ ft.}$$

2. Here  $CD=2 \cdot 28$  ft.,  $AC=7 \cdot 15$  ft. Required  $CF$ .

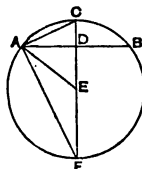
(As in preceding)

$$CF=CA^2 \div CD=(7 \cdot 15 \text{ ft.})^2 \div 2 \cdot 28 \text{ ft.} = \frac{51 \cdot 1225}{2 \cdot 28} \text{ ft.} = 22 \cdot 42 \text{ ft. nearly.}$$

3. Here  $AC=3$  ft. 4 in.=40 in.,  $CF=25$  ft.=300 in. Required  $CD$ .

(By Euc. VI. 8 Cor.)  $CD : CA :: CA : CF$ ;

$$\therefore CD=CA^2 \div CF=40^2 \text{ sq. in.} \div 300 \text{ in.} = \frac{1600}{3} \text{ in.} = 5 \frac{1}{3} \text{ in.}$$



4. Here  $AC = 6.43$  ft.,  $CF = 23.65$  ft. Required  $CD$ .

(As in preceding)  $CD = CA^2 \div CF$

$$= (6.43 \text{ ft.})^2 \div 23.65 \text{ ft.} = 41.3449 \text{ sq. ft.} \div 23.65 \text{ ft.} = 1.74 \text{ ft. nearly.}$$

5. Here  $CD = 1$  ft. 3 in.  $= \frac{5}{4}$  ft.,  $CF = 11$  ft. 8 in.  $= \frac{47}{4}$  ft. Required  $AC$ .

(By Euc. VI. 8 Cor.)  $CD : AC :: AC : CF$ ;

$$\therefore AC^2 = CD \times CF = \frac{5}{4} \times \frac{47}{4} \text{ sq. ft.} = \frac{5 \times 5 \times 9}{4 \times 4} \text{ sq. ft.} = \frac{5^2 \times 3^2}{4^2} \text{ sq. ft.}$$

$$\therefore AC = \sqrt{\left(\frac{5^2 \times 3^2}{4^2} \text{ sq. ft.}\right)} = \frac{5 \times 3}{4} \text{ ft.} = 3 \text{ ft. 9 in.}$$

6. Here  $CD = 3.24$  ft.,  $CF = 28.76$  ft. Required  $AC$ .

(As in preceding)  $AC^2 = CD \times CF = 3.24 \times 28.76 \text{ sq. ft.} = 93.1824 \text{ sq. ft.}$ ;

$$\therefore AC = \sqrt{(93.1824 \text{ sq. ft.})} = 9.65 \text{ ft. nearly.}$$

7. Here  $AB = 20$  ft.,  $CD = 4$  ft. Required  $CF$ .

(By Euc. III. 3)

$AD = DB = \frac{1}{2}AB = 10$  ft., and (by Euc. VI. 8 Cor.)  $DF : DA :: DA : DC$ ;

$$\therefore DF = DA^2 \div DC = (10 \text{ ft.})^2 \div 4 \text{ ft.} = 100 \text{ sq. ft.} \div 4 \text{ ft.} = 25 \text{ ft.};$$

$$\therefore CF = CD + DF = 29 \text{ ft.}$$

8. Here  $AB = 15.78$  ft.,  $CD = 2.8$  ft. Required  $CF$ .

(As in preceding)

$AD = \frac{1}{2}AB = 7.89$  ft.; and  $DF = DA^2 \div DC = (7.89 \text{ ft.})^2 \div 2.8 \text{ ft.}$

$$= 62.2521 \text{ sq. ft.} \div 2.8 \text{ ft.} = 22.23 \text{ ft. nearly};$$

$$\therefore CF = CD + DF = (2.8 + 22.23) \text{ ft.} = 25.03 \text{ ft.}$$

9. Here  $AB = 15$  in.,  $CF = 20$  in. Required  $AC$ .

(As in 7)

$AD = \frac{1}{2}AB = \frac{15}{2}$  in.; and (Euc. I. def. 15)  $AE = \frac{1}{2}$  diameter  $= \frac{1}{2}CF = 10$  in.

Now (Euc. I. 47)  $DE = \sqrt{(AE^2 - AD^2)} = \sqrt{\{(10 \text{ in.})^2 - (\frac{15}{2} \text{ in.})^2\}}$

$$= \sqrt{\{(10 + \frac{15}{2})(10 - \frac{15}{2}) \text{ sq. in.}\}} = \sqrt{\{\frac{15}{2} \times \frac{5}{2} \text{ sq. in.}\}} = \sqrt{\left(\frac{7 \times 5^2}{2^2} \text{ sq. in.}\right)}$$

$$= \frac{5}{2} \sqrt{7} \text{ in.} = \frac{5}{2} \times 2.6457 \text{ in.} = 6.61 \text{ in. nearly};$$

$\therefore CD = CE - ED = (10 - 6.61) \text{ in.} = 3.39 \text{ in.}$  Lastly (by Euc. VI. 8 Cor.)

$CD : CA :: CA : CF$ ;  $\therefore CA^2 = CD \times CF = (3.39 \times 20) \text{ sq. in.} = 67.8 \text{ sq. in.}$ ;

$$\therefore CA = \sqrt{(67.8 \text{ sq. in.})} = 8.23 \text{ in. nearly.}$$

10. Here  $AB = 80$  in.,  $CF = 100$  in. Required  $AC$ .

(As in preceding)  $AD = \frac{1}{2}AB = 40$  in., and  $AE = \frac{CF}{2} = 50$  in. Now (Euc. I. 47)

$$DE = \sqrt{(AE^2 - AD^2)} = \sqrt{\{(AE + AD)(AE - AD)\}} = \sqrt{\{(50 + 40)(50 - 40) \text{ sq. in.}\}}$$

$$= \sqrt{(90 \times 10 \text{ sq. in.})} = \sqrt{(900 \text{ sq. in.})} = 30 \text{ in.};$$

$$\therefore CD = CE - ED = (50 - 30) \text{ in.} = 20 \text{ in.};$$

$\therefore$  (as in preceding)  $AC^2 = CD \times CF = 20 \times 100 \text{ sq. in.} = 4 \times 5 \times 10^2 \text{ sq. in.}$ ;

$$\therefore AC = \sqrt{\{(4 \times 5 \times 10^2) \text{ sq. in.}\}} = 2 \times 10 \sqrt{5} \text{ in.} = 20 \times 2.236 \text{ in.} = 44.72 \text{ in.}$$

11. Here  $AC = 2$  ft. 6 in.  $= \frac{5}{2}$  ft.,  $CF = 4$  ft. 2 in.  $= \frac{5}{2}$  ft. Required  $AB$ .  
 (By Euc. VI. 8 Cor.)  $CD : CA :: CA : CF$ ;  
 $\therefore CD = CA^2 \div CF = (\frac{5}{2} \text{ ft.})^2 \div \frac{5}{2} \text{ ft.} = \frac{25}{4} \text{ sq. ft.} \times \frac{2}{5} \text{ ft.} = \frac{5}{2} \text{ ft.}$  Again (Euc. I. 47)  
 $AD = \sqrt{(AC^2 - CD^2)} = \sqrt{\{(AC + CD)(AC - CD)\}} = \sqrt{\{(\frac{5}{2} + \frac{5}{2})(\frac{5}{2} - \frac{5}{2}) \text{ sq. ft.}\}}$   
 $= \sqrt{(4 \times 1 \text{ sq. ft.})} = 2 \text{ ft.}$ ; and (Euc. III. 3)  $AD = DB$ ;  
 $\therefore AB = 2 \times AD = 2 \times 2 \text{ ft.} = 4 \text{ ft.}$

12. Here  $AC = 2$  ft. 4 in.,  $CF = 16$  ft. Required  $AB$ .  
 (As in preceding)  $CD = AC^2 \div CF = (2 \cdot 4 \text{ ft.})^2 \div 16 \text{ ft.} = \cdot 6 \times \cdot 6 \text{ ft.} = \cdot 36 \text{ ft.}$ ;  
 and  $AD = \sqrt{(AC^2 - CD^2)} = \sqrt{\{(AC + CD)(AC - CD)\}}$   
 $= \sqrt{\{(2 \cdot 4 + \cdot 36) \text{ ft.} \times (2 \cdot 4 - \cdot 36) \text{ ft.}\}} = \sqrt{\{2 \cdot 76 \times 2 \cdot 04 \text{ sq. ft.}\}} = \sqrt{5 \cdot 6304 \text{ sq. ft.}}$   
 $= 2 \cdot 373 \text{ ft. nearly.} \therefore AB = 2AD = 4 \cdot 746 \text{ ft.} = 4 \cdot 75 \text{ ft. nearly.}$

13. Here  $AB = 12$  yds.  $= 36$  ft.,  $AC = 19$  ft. 6 in.  $= 19 \cdot 5$  ft. Required  $CF$ .  
 (By Euc. III. 3)  $AD = \frac{1}{2} AB = 18$  ft.  $\therefore$  (Euc. I. 47)  $CD = \sqrt{(AC^2 - AD^2)}$   
 $= \sqrt{\{(AC + AD)(AC - AD)\}} = \sqrt{\{(19 \cdot 5 + 18)(19 \cdot 5 - 18) \text{ sq. ft.}\}}$   
 $= \sqrt{37 \cdot 5 \times 1 \cdot 5 \text{ sq. ft.}} = \sqrt{25 \times 1 \cdot 5 \times 1 \cdot 5 \text{ sq. ft.}} = \sqrt{5^2 \times (1 \cdot 5)^2 \text{ sq. ft.}}$   
 $= 5 \times 1 \cdot 5 \text{ ft.} = 7 \cdot 5 \text{ ft.}$

Now (Euc. VI. 8 Cor.)  $CD : AC :: AC : CF$ ;

$$\therefore CF = AC^2 \div CD = (19 \cdot 5 \text{ ft.})^2 \div 7 \cdot 5 \text{ ft.}$$

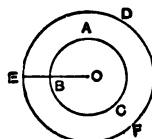
$$= \frac{19 \cdot 5 \times 19 \cdot 5}{7 \cdot 5} \text{ ft.} = \frac{3 \cdot 9 \times 19 \cdot 5}{1 \cdot 5} \text{ ft.} = \frac{3 \cdot 9 \times 3 \cdot 9}{\cdot 3} \text{ ft.} = 13 \times 3 \cdot 9 \text{ ft.} = 50 \cdot 7 \text{ ft.}$$

14. Here  $AB = 49$  ft.,  $AC = 25$  ft. Required  $CF$ .  
 (Euc. III. 3)  $AD = \frac{1}{2} AB = 24 \cdot 5$  ft.; and (Euc. I. 47)  $CD = \sqrt{(AC^2 - AD^2)}$   
 $= \sqrt{\{(AC + CD)(AC - CD)\}} = \sqrt{\{(25 + 24 \cdot 5) \text{ ft.} \times (25 - 24 \cdot 5) \text{ ft.}\}}$   
 $= \sqrt{\{49 \cdot 5 \times \cdot 5 \text{ sq. ft.}\}} = \sqrt{24 \cdot 75 \text{ sq. ft.}} = 4 \cdot 9749 \dots \text{ft.}$ ; Now (as in preceding)  
 $CF = AC^2 \div CD = (25 \text{ ft.})^2 \div 4 \cdot 9749 \text{ ft.} = (625 \div 4 \cdot 9749) \text{ ft.} = 125 \cdot 63 \text{ ft. nearly.}$

## CHAPTER VIII.

1. Circum.  $= \frac{2}{3} \times 14 \text{ ft.} = 44 \text{ ft.}$
2. Circum.  $= \frac{2}{3} \times 86 \text{ yds. 1 ft.} = \frac{2}{3} \times 86 \frac{1}{3} \text{ yds.} = \frac{2}{3} \times \frac{259}{3} \text{ yds.}$   
 $= \frac{22 \times 37}{3} \text{ yds.} = \frac{814}{3} \text{ yds.} = 271 \text{ yds. 1 ft.}$
3. Circum.  $= \frac{2}{3} \times 213 \text{ yds. 2 ft. 8 in.} = 22 \times 30 \text{ yds. 1 ft. 8 in.} = 672 \text{ yds. 8 in.}$
4. Circum.  $= \frac{2}{3} \times 1 \text{ fur. 60 yds.} = \frac{2}{3} \times 1 \frac{1}{3} \text{ fur.} = \frac{2}{3} \times \frac{4}{3} \text{ fur.} = 4 \text{ fur.}$
5. Circum.  $= 3 \cdot 1416 \times 27 \text{ ft.} = 84 \cdot 8232 \text{ ft.}$
6. Circum.  $= 3 \cdot 1416 \times 61 \text{ yds. 2 ft.} = 3 \cdot 1416 \times 185 \text{ ft.} = 581 \cdot 196 \text{ ft.}$

7. Circum. =  $3 \cdot 1416 \times 555$  yds. 1 ft. 6 in. =  $3 \cdot 1416 \times 1666\frac{1}{2}$  ft.  
 =  $3 \cdot 1416 \times 1666\frac{1}{2}$  ft. = 5285'4764 ft.
8. Circum. =  $3 \cdot 1416 \times 1$  fur. 80 yds. =  $3 \cdot 1416 \times 300$  yds. = 942'48 yds.
9. Diam. = circum.  $\div 2\frac{1}{2}$  = 66 yds.  $\times \frac{1}{2}$  = 21 yds.
10. Diam. = circum.  $\div 2\frac{1}{2}$  = 10 ch.  $\div 2\frac{1}{2}$  = 220 yds.  $\times \frac{1}{2}$  = 70 yds.
11. Diam. = circum.  $\div 2\frac{1}{2}$  = 3 fur. 4 ch.  $\div 2\frac{1}{2}$  = 34 ch.  $\times \frac{1}{2}$   
 = 34  $\times$  22 yds.  $\times \frac{1}{2}$  = 238 yds.
12. Diam. = circum.  $\div 2\frac{1}{2}$  = 1 mile  $\div 2\frac{1}{2}$  = 1760  $\times \frac{1}{2}$  yds. = 560 yds.
13. Diam. = circum.  $\div 3 \cdot 1416$  = 1 ft.  $\div 3 \cdot 1416$  = '8183... of a foot.
14. Diam. = circum.  $\div 3 \cdot 1416$  = 25 ft.  $\div 3 \cdot 1416$  = 7'9577... ft.
15. Diam. = circum.  $\div 3 \cdot 1416$  = 108 yds. 1 ft.  $\div 3 \cdot 1416$  = 325 ft.  $\div 3 \cdot 1416$   
 = 103'45047... ft. = 103'4505 ft. nearly.
16. Diam. = circum.  $\div 3 \cdot 1416$  = 1 fur.  $\div 3 \cdot 1416$  = 220 yds.  $\div 3 \cdot 1416$   
 = 5 yds.  $\div \cdot 0714$  = 70'028... yds.
17. Circum. of circle described by Mercury round the sun  
 =  $2 \times 2\frac{1}{2} \times 37000000$  miles; Now since the planet describes this circum. in  
 88 days, i.e. in  $88 \times 24 \times 60 \times 60$  seconds;  $\therefore$  No. of miles it describes in  
 1 second =  $\frac{2 \times 22 \times 37000000}{7 \times 88 \times 24 \times 60 \times 60} = \frac{28125}{756} = 30 \cdot 58 = 30 \cdot 6$  nearly.
18. Circum. of carriage wheel =  $2\frac{1}{2} \times 28$  in. = 88 in.; now since in 1 revolution, a wheel passes over a distance equal to its own circum.;  $\therefore$  the carriage wheel makes 1 revolution in passing over 88 in.;  $\therefore$  no. of revolutions it will make in passing over  $\frac{1}{2}$  a mile =  $880 \times 3 \times 12$  in.  $\div$  88 in. = 360.
19. Let the circle  $ABC$  represent the shrubbery, and let the space between the two circles  $ABC$  and  $DEF$  represent the road. Now  $EB$  (i.e. the breadth of the road,)
- =  $OE - OB = \frac{\text{circum. of } DEF}{2 \times 3 \cdot 1416} - \frac{\text{circum. of } ABC}{2 \times 3 \cdot 1416}$
- =  $\frac{1}{2 \times 3 \cdot 1416}$  (circum. of  $DEF$  - circum. of  $ABC$ )
- =  $\frac{600 \text{ ft.} - 480 \text{ ft.}}{2 \times 3 \cdot 1416} = \frac{120}{2 \times 3 \cdot 1416} \text{ ft.} = \frac{60}{8 \cdot 1416} \text{ ft.} = 19 \cdot 0985 \dots \text{ft.}$
20. Circum. - Diam. = 10 ft., i.e. 10 ft. =  $3 \cdot 1416 \times$  Diam. - Diam.  
 = Diam.  $\times (3 \cdot 1416 - 1)$  = Diam.  $\times 2 \cdot 1416$ ;  
 $\therefore$  Diam. = 10 ft.  $\div 2 \cdot 1416$  = 4'669... ft. = 4'67 ft. nearly.



## CHAPTER IX.

(Note. In the following examples we shall take  $L$  to represent the length of the arc,  $A$  the angle subtended by the arc at the centre of the circle, and  $\odot^\circ$  the circumference of the circle.)

1. (By Euc. VI. 33)  $L : \odot^\circ :: A : 360^\circ$ ;

$$\therefore L = \frac{\odot^\circ \times A}{360^\circ} = \frac{2 \times 3.1416 \times 10 \text{ in.} \times 72^\circ}{360^\circ} = 12.5664 \text{ in.}$$

2. (as in preceding)

$$L = \frac{2 \times 3.1416 \times 19 \text{ ft.} \times 10^\circ 24'}{360^\circ} = \frac{2 \times 3.1416 \times 235 \text{ in.} \times 10^\circ \frac{2}{3}}{360^\circ} \\ = \frac{639.8392}{15} \text{ in.} = 42.6559 \dots \text{ in.} = 42.656 \text{ in. nearly.}$$

3. (By Euc. VI. 33)  $A : 360^\circ :: L : \odot^\circ$ ;

$$\therefore A = \frac{360^\circ \times 15 \text{ in.}}{2 \times 3.1416 \times 2 \text{ ft.}} = \frac{360^\circ \times 15 \text{ in.}}{2 \times 3.1416 \times 24 \text{ in.}} = \frac{75^\circ}{2.0944} = 35^\circ.809 \dots = 35^\circ.81 \text{ nearly.}$$

4. (as in preceding)

$$A = \frac{360^\circ \times 1 \text{ ft.}}{2 \times 3.1416 \times 1 \text{ ft.}} = \frac{15^\circ}{2.618} = 57^\circ.29 \dots = 57^\circ.3 \text{ nearly.}$$

5.  $L = \frac{8 \times \text{chord of half the arc} - \text{chord of whole arc}}{3}$

$$= \frac{1}{3} (8 \times 19 \text{ in.} - 36 \text{ in.}) = 11\frac{2}{3} \text{ in.} = 38\frac{2}{3} \text{ in.}$$

6. (Vide diag. to Chap. VII. 1.) Here  $AB = 56 \text{ in.}$ ,  $AE = EC = 197 \text{ in.}$  Required arc  $BAC$ . (By Euc. III. 3)  $AD = DB = \frac{1}{2} AB = 28 \text{ in.}$ ; and (Euc. I. 47)  $DE = \sqrt{(AE^2 - AD^2)} = \sqrt{\{(AE + AD)(AE - AD)\}} = \sqrt{\{(197 + 28)(197 - 28) \text{ sq. in.}\}} \\ = \sqrt{(225 \times 169 \text{ sq. in.})} = \sqrt{\{(15^2 \times 13^2) \text{ sq. in.}\}} = 15 \times 13 \text{ in.} = 195 \text{ in.};$   
 $\therefore CD = CE - ED = (197 - 195) \text{ in.} = 2 \text{ in.}$ ; and (Euc. I. 47)  $AC = \sqrt{(AD^2 + DC^2)} = \sqrt{\{(28^2 + 2^2) \text{ sq. in.}\}} = \sqrt{\{2^2 (14^2 + 1) \text{ sq. in.}\}} \\ = 2\sqrt{197} \text{ in.} = 2 \times 14.035 \dots \text{ in.} = 28.07 \text{ in.}$   $\therefore$  Length of arc  $ACB = \frac{1}{3} (8AC - AB) = \frac{1}{3} (224.56 - 56) \text{ in.} = 56.186 \text{ in.} = 56.19 \text{ in. nearly.}$

7. (Vide diag. to Chap. VII. 1.) Here  $AB = 6 \text{ in.}$ ,  $AE = 9 \text{ in.}$ ; Required arc  $ACB$ .

(As in preceding)  $AD = \frac{1}{2} AB = 3 \text{ in.}$ , and

$$DE = \sqrt{\{(9^2 - 3^2) \text{ sq. in.}\}} = \sqrt{(12 \times 6 \text{ sq. in.})} = \sqrt{(6^2 \times 2 \text{ sq. in.})} \\ = 6 \times \sqrt{2} \text{ in.} = 6 \times 1.4142135 \text{ in.} = 8.485281 \text{ in.};$$

$$\therefore DC = CE - DE = (9 - 8.485281) \text{ in.} = .514719 \text{ in.};$$

and  $AC = \sqrt{(9 \text{ sq. in.} + .264936 \text{ sq. in. nearly})}$

$$= \sqrt{(9.264936 \text{ sq. in.})} = 3.044 \text{ in. nearly};$$

$\therefore$  Length of arc  $ACB = \frac{1}{3} (8 \times 3.044 - 6) \text{ in.} = \frac{1}{3} \times 18.352 \text{ in.} = 6.117 \text{ in. nearly.}$

(Note.  $AC$  might also have been found by Euc. VI. 8. Cor., as in Chap. VII.)

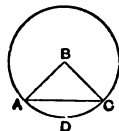
8. Here Radius = 5 in., angle of sector =  $90^\circ$ ; Required perimeter of sector. (By Euc. VI. 33) Arc of sector : circum. of circle ::  $90^\circ$  :  $360^\circ$ ;

$$\therefore \text{arc of sector} = \frac{1}{4} \text{ circum. of circle} = \frac{1}{4} \times 2 \times 3.1416 \times 5 \text{ in.}$$

$$\therefore \text{perimeter of sector} = \text{arc} + 2 \times \text{radius} = \frac{1}{4} \times 3.1416 \times 10 \text{ in.} + 10 \text{ in.} \\ = 10 \text{ in.} (.7854 + 1) = 10 \times 1.7854 \text{ in.} = 17.854 \text{ in.}$$

9. Here  $AB = BC = 16 \text{ in.}$ ,  $\angle ABC = 90^\circ$ ; Required perimeter of segment  $ADC$ . (By Euc. I. 47)

$$AC = \sqrt{(AB^2 + BC^2)} = \sqrt{2AB^2} = AB\sqrt{2} \\ = 16 \times 1.4142135 \text{ in.} = 22.6274 \text{ in. nearly.}$$



Again (by Euc. VI. 33)

$$\text{arc } ADC : \text{circum. of circle} :: 90^\circ : 360^\circ;$$

$$\therefore \text{arc } ADC = \frac{1}{4} \text{ circum. of circle} = \frac{1}{4} \times 2 \times 3.1416 \times 16 \text{ in.} = 25.1328 \text{ in.};$$

$$\therefore \text{perimeter of segment } ADC = \text{arc } ADC + \text{chord } AC = (25.1328 + 22.6274) \text{ in.} \\ = 47.7602 \text{ in.}$$

$$10. \text{ Arc of semicircle} = \frac{1}{2} \text{ circum. of circle} = \frac{1}{2} \times 2 \times 3.1416 \times 1 \text{ in.} = 3.1416 \text{ in.};$$

$$\therefore \text{perimeter of semicircle} = \text{arc of semicircle} + \text{diam.} = (3.1416 + 2) \text{ in.} \\ = 5.1416 \text{ in.}$$

11. Perimeter of semicircle (as in preceding)

$$= \frac{1}{2} \text{ circum. of circle} + \text{diam.} = \frac{1}{2} \times 2 \times 3.1416 \times \text{radius} + 2 \text{ radius} \\ = \text{radius } (3.1416 + 2) = \text{radius} \times 5.1416 = (\text{by hyp.}) 100 \text{ ft.};$$

$$\therefore \text{radius} = 100 \text{ ft.} \div 5.1416 = 19.45 \text{ ft. nearly.}$$

12. Radius = 25 in.,  $A = 32^\circ 31' 12'' .4 = 32^\circ \frac{1872.4}{8600} = \frac{117072^\circ .4}{8600}$ ; Required length of arc. (As in 1)

$$L = 2 \times 3.1416 \times 25 \text{ in.} \times \frac{117072^\circ .4}{8600} \div 360^\circ = \frac{3831.19429 \text{ in.}}{270} = 14.1896 \text{ in. nearly.}$$

## CHAPTER XI.

$$1. \text{ Area of square} = \text{side}^2 = 14^2 \text{ sq. yds.} = 196 \text{ sq. yds.}$$

$$5. \text{ Area of square} = \text{side}^2 = (10 \text{ yds. } 2 \text{ ft.})^2 = (10\frac{1}{3} \text{ yds.})^2 = (\frac{31}{3} \text{ yds.})^2 \\ = 12\frac{1}{9} \text{ sq. yds.} = 113 \text{ sq. yds. } 7 \text{ ft.}$$

$$9. \text{ Area of square} = \text{side}^2 = (3 \text{ yds. } 2 \text{ ft. } 4 \text{ in.})^2 = (3\frac{1}{3} \text{ yds.})^2 = (\frac{10}{3} \text{ yds.})^2 \\ = 11\frac{1}{9} \text{ sq. yds.} = 14 \text{ sq. yds. } 2 \text{ ft. } 64 \text{ in.}$$

$$13. \text{ Area of square} = \text{side}^2 = (4 \text{ ch. } 50 \text{ lks.})^2 = (4\frac{1}{2} \text{ ch.})^2 = \frac{25}{4} \text{ sq. ch.} \\ = \frac{81}{4 \times 10} \text{ ac.} = 2\frac{1}{10} \text{ ac.} = 2 \text{ ac. } 4 \text{ po.}$$

80. Let  $ABCD$  be the given square; then  $AB = 85$  yds.  $= 255$  ft. Let  $EF$ ,  $FG$ ,  $GH$ ,  $HE$ , be the boundaries of the path; then  $KA = BL = 10$  yds.  $= 30$  ft.  $\therefore$  (by Eucl. I. 34)

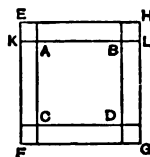
$$EH = KL = AB + KA + BL = (255 + 30 + 30) \text{ ft.} = 315 \text{ ft.}$$

Now area of path

$$= (\text{area of square } EFGH - \text{area of square } ABCD)$$

$$= (315 \text{ ft.})^2 - (255 \text{ ft.})^2 = (315 + 255) \text{ ft.} \times (315 - 255) \text{ ft.} = 570 \times 60 \text{ sq. ft.};$$

and area of 1 stone  $= 1 \text{ ft. } 4 \text{ in.} \times 10 \text{ in.} = \frac{1}{3} \text{ ft.} \times \frac{5}{6} \text{ ft.} = \frac{5}{18} \text{ sq. ft.}$  Now since 1 stone covers an area of  $\frac{5}{18} \text{ sq. ft.}$ , the no. of stones required to cover an area of  $570 \times 60 \text{ sq. ft.} = 570 \times 60 \text{ sq. ft.} \div \frac{5}{18} \text{ sq. ft.} = 80780$ .



81. (As in preceding) Length of outer rectangle  $= (63 + 4\frac{1}{2} + 4\frac{1}{2}) \text{ ft.} = 72 \text{ ft.}$ , and its breadth  $= (36 + 4\frac{1}{2} + 4\frac{1}{2}) \text{ ft.} = 45 \text{ ft.}$   $\therefore$  (as in preceding) area of path  $= (72 \times 45 - 63 \times 36) \text{ sq. ft.} = \{(36 \times 2 \times 9 \times 5) - (9 \times 7 \times 36)\} \text{ sq. ft.}$

$$= 36 \times 9 (10 - 7) \text{ sq. ft.} = 36 \times 9 \times 3 \text{ sq. ft.};$$

and area of 1 brick  $= 9 \text{ in.} \times 4\frac{1}{2} \text{ in.} = \frac{3}{4} \text{ ft.} \times \frac{3}{8} \text{ ft.} = \frac{9}{32} \text{ sq. ft.}$ ;  $\therefore$  (as in preceding) no. of bricks required  $= 36 \times 9 \times 3 \text{ sq. ft.} \div \frac{9}{32} \text{ sq. ft.} = 3456$ .

82. Area of court yard  $= 1296 \times 9 \times \frac{1}{2} \text{ sq. in.} = 648 \times 81 \text{ sq. in.}$ ; area of pavement  $= \frac{1}{2} \times 648 \times 81 \text{ sq. in.} = 648 \times 9 \text{ sq. in.}$ ; and area of 1 tile

$$= 6 \times 6 \text{ sq. in.} = 36 \text{ sq. in.};$$

$$\therefore \text{no. of tiles required} = 648 \times 9 \text{ sq. in.} \div \text{by } 36 \text{ sq. in.} = 162.$$

83. Area of one rectangle  $= 9 \times 16 = 3^2 \times 4^2$ ;  $\therefore$  side of square equal in area to this rectangle  $= \sqrt{(3^2 \times 4^2)} = 3 \times 4 = 12$ ; and area of the other rectangle  $= 36 \times 25 = 6^2 \times 5^2$ ;  $\therefore$  side of square equal in area to this rectangle

$$= \sqrt{(6^2 \times 5^2)} = 6 \times 5 = 30; \therefore \frac{\text{side of 1st square}}{\text{side of 2nd square}} = \frac{12}{30} = \frac{2}{5};$$

$\therefore$  side of 1st square : side of 2nd square  $:: 2 : 5$ .

84. Area of 4 walls of room  $=$  area of the 2 long walls  $+$  area of the 2 short walls  $= 2$  area of 1 long wall  $+$  2 area of 1 short wall  $= 2 \times \text{length} \times \text{height of room} + 2 \times \text{breadth} \times \text{height} = 2 \times \text{height} (\text{length} + \text{breadth})$

$$= 2 \times 10\frac{1}{2} \text{ ft.} (18 + 12) \text{ ft.} = 2 \times 1\frac{1}{2} \times 30 \text{ sq. ft.} = 21 \times 30 \text{ sq. ft.}$$

Now since the paper must cover these walls, its area must equal their area; i.e. length of paper  $\times$  its breadth  $= 21 \times 30 \text{ sq. ft.}$ ;

$$\therefore \text{length of paper} \times 27 \text{ in. (i.e. } \frac{3}{4} \text{ ft.)} = 21 \times 30 \text{ sq. ft.}$$

$$\therefore \text{length of paper} = 21 \times 30 \text{ sq. ft.} \div \frac{3}{4} \text{ ft.} = 280 \text{ ft.}$$

85. Area of 4 walls of room (as in preceding)

$$= 2 \times 18 \text{ ft. } 6 \text{ in.} (24 \text{ ft. } 10 \text{ in.} + 16 \text{ ft.}) = 2 \times 18\frac{1}{2} \text{ ft.} \times 40\frac{1}{2} \text{ ft.}$$

$$= 2 \times 1\frac{1}{2} \times 16\frac{1}{2} \text{ sq. ft.} = 2 \times 24\frac{1}{2} \text{ sq. ft.} = 49\frac{1}{2} \text{ sq. ft.}$$

$=$  no. of sq. ft. of paper required (vide preceding).

86. Perimeter of rectangle = sum of its four sides = 2 length + 2 breadth = 96 ft. + 56 ft. = 152 ft. And perimeter of square = sum of its four sides = 4 × one side; but, by hyp., the perimeter of the square = perimeter of the rectangle; ∴ 4 × side of square = 152 ft.; ∴ side of square = 38 ft.; ∴ area of square = (38 ft.)<sup>2</sup> = 1444 sq. ft.

87. Here length = 3 × breadth; ∴ area of rectangle = length × breadth = 3 × breadth × breadth = 3 breadth<sup>2</sup> = (by hyp.) 1828 sq. ft.;  
∴ breadth<sup>2</sup> = 441 sq. ft. = 21<sup>2</sup> sq. ft.; ∴ breadth = √21<sup>2</sup> sq. ft.) = 21 ft.;  
and length = 3 × breadth = 3 × 21 ft. = 63 ft.

88. Area of the seven sheets

$$= 7 \times 9 \text{ in.} \times 6\frac{7}{8} \text{ in.} = 7 \times 9 \times \frac{55}{8} \text{ sq. in.} = \frac{63 \times 55}{8} \text{ sq. in.};$$

and area of the one sheet = 18 $\frac{3}{4}$  in. × 11 in. =  $\frac{55}{4}$  × 11 sq. in. Now since 7 sheets, the area of which is  $\frac{63 \times 55}{8}$  sq. in., weigh 1 oz.; the sheet, the area of which is  $\frac{55}{4}$  × 11 sq. in., will weigh  $\frac{55}{4} \times 11 \div \frac{63 \times 55}{8}$  oz., i.e.  $\frac{4}{11}$  oz.

89. Let  $x$  = longer side of rectangle, and  $y$  = the shorter side; therefore  $2(x+y)$  = perimeter of rectangle (vide 86); and  $4 \times$  side of square = perimeter of square (ibid.); but, by hyp., perimeter of square = perimeter of rectangle; ∴  $4 \times$  side of square =  $2(x+y)$ ; ∴ side of square =  $\frac{2(x+y)}{4} = \frac{x+y}{2}$ ; ∴ area of square =  $\left(\frac{x+y}{2}\right)^2 = \frac{x^2 + 2xy + y^2}{4}$ , and area of rectangle =  $xy$ . Now area of square is  $\geq$  area of rectangle, according as  $\frac{x^2 + 2xy + y^2}{4} \geq xy$ ; i.e. according as  $x^2 + 2xy + y^2 \geq 4xy$ ; i.e. according as  $x^2 - 2xy + y^2 \geq 0$ ; i.e. according as  $(x-y)^2 \geq 0$ ; i.e. according as  $(x-y) \geq 0$ ; but  $x-y$  is  $> 0$ , because  $x$  is larger than  $y$ , by hypothesis; ∴ area of square  $>$  area of rectangle. Now since  $x$  and  $y$  are general quantities, the truth we have demonstrated will hold for all values of  $x$  and  $y$ , provided that  $x > y$ .

90. Take  $x$  and  $y$  as in the preceding. ∴ (as in preceding) area of square =  $\frac{x^2 + 2xy + y^2}{4}$ , and area of rectangle =  $xy$ ; ∴ area of square - area of rectangle =  $\frac{x^2 + 2xy + y^2}{4} - xy = \frac{x^2 + 2xy + y^2 - 4xy}{4} = \frac{x^2 - 2xy + y^2}{4} = \left(\frac{x-y}{2}\right)^2$  = the area of a square, each of whose sides is half the difference of the two sides of the given rectangle.

91. Area of field

$$= 1 \text{ fur. } 20 \text{ po.} \times 10 \text{ po. } 1 \text{ yd.} = 60 \text{ po.} \times \frac{11}{4} \text{ po.} = \frac{60 \times 11}{4} \text{ ac.} = 1\frac{1}{2} \text{ ac.};$$

$$\therefore \text{Rent} = 1\frac{1}{2} \times \text{£}1. 18s. = 1\frac{1}{2} \times \text{£}2\frac{1}{2} = \text{£}2\frac{1}{2} = \text{£}2. 6s.$$

$$92. \text{ Area of land} = 4235 \times 280 \text{ sq. yds.} = \frac{4235 \times 280}{4840} \text{ ac.};$$

$$\therefore \text{Rent} = \frac{4235 \times 280}{4840} \times £4. 10s. = \frac{4235 \times 280}{4840} \times £\frac{2}{3} = £1102. 10s.$$

$$93. \text{ Area of court} = 18 \text{ ft. } 6 \text{ in.} \times 12 \text{ ft. } 3 \text{ in.} = \frac{3}{4} \text{ ft.} \times \frac{5}{2} \text{ ft.}; \therefore \text{expense of paving} = \frac{3}{4} \times \frac{5}{2} \times 4d. = \frac{3}{4} \times \frac{5}{2} \times \frac{4}{100} = £\frac{3}{10} = £3. 15s. 6\frac{1}{2}d.$$

$$94. (\text{Vide diagram to 17.}) \text{ Here } BD = 30 \text{ yds. (By Euc. I. 47) } AB^2 + BD^2, \text{ i.e. } 2AB^2 = BD^2 = 900 \text{ sq. yds.}, \therefore AB^2 = 450 \text{ sq. yds.} = \text{area of square } ABCD. \text{ Now since cost of gravelling } 9 \text{ sq. yds.} = 1s. = \frac{1}{12}d., \text{ the cost of gravelling } 450 \text{ sq. yds.} = 450 \div 9 \times \frac{1}{12}d. = £2. 10s.$$

$$95. \text{ Area} = 32 \text{ ft. } 3 \text{ in.} \times 16 \text{ ft. } 6 \text{ in.} = \frac{4}{3} \text{ yds.} \times \frac{1}{3} \text{ yds.}; \therefore \text{expense of paving} = \frac{4}{3} \times \frac{1}{3} \times 6s. 4d. = \frac{4}{3} \times \frac{1}{3} \times \frac{4}{100} = £\frac{2}{15} = £18. 14s. 5\frac{1}{2}d.$$

$$96. \text{ Area of street} = 1 \text{ fur. } 92 \text{ yds. } 1 \text{ ft. } 6 \text{ in.} \times 22 \text{ yds. } 8 \text{ in.} = 312\frac{1}{2} \text{ yds.} \times 22\frac{1}{2} \text{ yds.} = \frac{3}{4} \times \frac{1}{2} \text{ sq. yds.} = \frac{3}{4} \times \frac{1}{2} \text{ sq. yds.}; \therefore \text{cost of paving} = \frac{3}{4} \times \frac{1}{2} \times 8\frac{1}{2}d. = \frac{3}{4} \times \frac{1}{2} \times \frac{8}{100} = £245. 18s. 11\frac{1}{2}d.$$

$$97. \text{ Area of court} = 96 \text{ ft.} \times 84 \text{ ft.} = 32 \text{ yds.} \times 28 \text{ yds.} = 896 \text{ sq. yds.}; \text{ area of } 4 \text{ grass plots} = 4 \times 22\frac{1}{2} \text{ ft.} \times 18 \text{ ft.} = 4 \times \frac{1}{2} \text{ yds.} \times 6 \text{ yds.} = 180 \text{ sq. yds.}; \therefore \text{portion of court remaining} = (896 - 180) \text{ sq. yds.} = 716 \text{ sq. yds.}; \therefore \text{cost of paving} = 716 \times 8\frac{1}{2}d. = 716 \times \frac{1}{4} \times \frac{1}{100} = £25. 7s. 2d.$$

98. Let  $ABCD$  represent the rectangular piece of ground. Then  $AD = 85$  yds. and  $AB = 56$  yds. Let the space enclosed between the boundaries of the two rectangles  $ABCD$  and  $EFGH$  represent the path; then the breadth of the path

$$= EK = HL = EM = FN = 4 \text{ yds.}$$

(By Euc. I. 34)  $KL = AD = 85$  yds.;

$$\text{i.e. } KE + EH + HL = 85 \text{ yds.};$$

$$\therefore EH + 8 \text{ yds.} = 85 \text{ yds.}; \therefore EH = (85 - 8) \text{ yds.} = 77 \text{ yds.}$$

In the same manner we find that

$$EF = AB - 8 \text{ yds.} = (56 - 8) \text{ yds.} = 48 \text{ yds.};$$

$$\therefore \text{area of path} = \text{area of } ABCD - \text{area of } EFGH = 85 \times 56 \text{ sq. yds.} - 77 \times 48 \text{ sq. yds.} = (4760 - 3696) \text{ sq. yds.} = 1064 \text{ sq. yds.};$$

$$\therefore \text{cost of paving} = 1064 \times 1s. 2d. = 1064 \times \frac{2}{100} = £21. 2s. 4d.$$

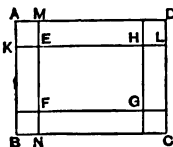
99. Since 1 sq. yd. costs 3s. 9d., i.e.  $\frac{3}{4}d.$ ;  $\therefore$  no. of sq. yds. that cost £38. 10s. 5d., i.e.  $\frac{1}{4} \times \frac{1}{100} = \frac{1}{4} \times \frac{1}{100} = \frac{1}{400}$ , and this, by hyp., is the area of the square court yard;  $\therefore$  side of this court yard

$$= \sqrt{(\frac{1}{400} \text{ sq. yds.})} = \frac{1}{20} \text{ yds.} = 43 \text{ ft.}$$

101. (As in 99.) No. of acres in field

$$= £27. 5s. \div £2. 14s. 6d. = 10.$$

$\therefore$  side of the field  $= \sqrt{(10 \text{ ac.})} = \sqrt{(48400 \text{ sq. yds.})} = 220 \text{ yds.}$  Now since the paling goes round the field,  $\therefore$  the 4 sides are paled;  $\therefore$  no. of yds. of paling  $= 4 \times 220 = 880$ ;  $\therefore$  cost  $= 880 \times 9d. = 880 \times \frac{3}{4}d. = £33.$



102. Area of floor = 18 ft.  $\times$  16 ft. = 6 yds.  $\times$   $\frac{1}{3}$  yds. = 82 sq. yds.;  $\therefore$  length of carpet required =  $\frac{82 \text{ sq. yds.}}{1 \text{ yd.}}$  (vide 84) = 82 yds.

108. Area of floor, or quantity of carpet required

$$= 12 \text{ ft. } 4 \text{ in. } \times 16 \text{ ft. } 8 \text{ in.} = 17 \times \frac{1}{4} \text{ sq. ft.};$$

$$\therefore \text{expense} = 17 \times \frac{1}{4} \times 1s. 6d. = 17 \times \frac{1}{4} \times 2 \frac{1}{2} = 21 \frac{1}{4} = £15. 0s. 7 \frac{1}{2}d.$$

111. Area of floor = 84 ft.  $\times$  16 ft. 6 in. = 84  $\times$   $\frac{1}{2}$  sq. ft. = 17  $\times$  87 sq. ft.; and length of carpet required =  $\frac{17 \times 87 \text{ sq. ft.}}{2 \text{ ft.}}$  (vide 84) =  $\frac{17 \times 87}{2 \times 8}$  yds.;  $\therefore$  expense

$$= \frac{17 \times 87}{2 \times 8} \times 4s. 6d. = \frac{17 \times 87}{2 \times 8} \times 2 \frac{1}{4} = 21 \frac{1}{4} \times 2 = £28. 11s. 9d.$$

118. Since 1 sq. yd. cost 5s., i.e.  $2 \frac{1}{2}$ , the no. of sq. yds. that will cost £6. 5s., i.e.  $2 \frac{1}{4}$ , =  $\frac{1}{4} \div \frac{1}{2} = 25$  sq. yds., which is therefore the area of the carpet. Now since the length of the carpet = length of the room, and that the latter = 25 ft.,  $\therefore$  also length of carpet = 25 ft.;  $\therefore$  breadth of carpet

$$= \frac{\text{area of carpet}}{\text{length of carpet}} = \frac{25 \text{ sq. yds.}}{25 \text{ ft.}} = \frac{25 \times 9 \text{ sq. ft.}}{25 \text{ ft.}}$$

= 9 ft. = likewise, breadth of room.

119. Quantity of carpet required

$$= 18 \text{ ft. } 6 \text{ in. } \times 18 \text{ ft. } 9 \text{ in.} = 17 \text{ ft. } \times 17 \text{ ft.} = 3 \text{ yds. } \times \frac{1}{4} \text{ yds.} = 2 \frac{1}{4} \text{ sq. yds.};$$

$$\therefore \text{length of carpet} = 2 \frac{1}{4} \text{ sq. yds.} + 27 \text{ in.} = 2 \frac{1}{4} \text{ sq. yds.} + \frac{2}{3} \text{ yd.} = 2 \frac{1}{4} \text{ yds.};$$

$$\therefore \text{cost} = 2 \frac{1}{4} \times 4s. 6d. = 2 \frac{1}{4} \times 2 \frac{1}{2} = £28. 8s. 9d.$$

Again length of room

$$= 18 \text{ ft. } 9 \text{ in. } + 2 \frac{1}{2} \text{ ft. } + 2 \frac{1}{2} \text{ ft.} = 28 \text{ ft. } 9 \text{ in.} = \frac{1}{4} \text{ ft. (vide 80),}$$

$$\text{and breadth of room} = 18 \text{ ft. } 6 \text{ in. } + 2 \frac{1}{2} \text{ ft. } + 2 \frac{1}{2} \text{ ft.} = 18 \text{ ft. } 6 \text{ in.} = \frac{1}{4} \text{ ft.};$$

$$\therefore \text{area of portion uncovered} = \text{area of whole room} - \text{area of carpeted portion}$$

$$= \frac{1}{4} \times \frac{1}{4} \text{ sq. ft.} - 2 \frac{1}{4} \text{ sq. yds.} = (\frac{1}{16} - 2 \frac{1}{4}) \text{ sq. ft.} = \frac{1}{16} \text{ sq. ft.} = \frac{1}{16} \text{ sq. yds.} = 20 \frac{1}{4} \text{ sq. yds.}$$

124. Area of 4 walls = 2  $\times$  11 ft.  $\times$  (24 + 15) ft. = 22  $\times$  89 sq. ft. = 858 sq. ft.; and area of fireplace, door, and 2 windows, together

$$= 4 \frac{1}{2} \text{ ft. } \times 8 \text{ ft. } + 7 \text{ ft. } \times 4 \text{ ft. } + 2 \times 6 \frac{1}{2} \text{ ft. } \times 5 \text{ ft.} = (17 + 28 + 65) \text{ sq. ft.} = 106 \frac{1}{2} \text{ sq. ft.};$$

$\therefore$  area of portion to be painted

$$= (858 - 106 \frac{1}{2}) \text{ sq. ft.} = 751 \frac{1}{2} \text{ sq. ft.} = 1 \frac{1}{4} \text{ sq. ft.};$$

$$\therefore \text{cost} = 1 \frac{1}{4} \times 8d. = 1 \frac{1}{4} \times 2 \frac{1}{2} = £29. 7s. 10 \frac{1}{2}d.$$

## CHAPTER XII.

1. Area = base  $\times$  height = 14 yds.  $\times$  5 yds. = 70 sq. yds.

5. Base  $\times$  height = area;

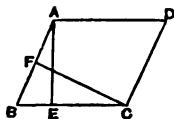
$\therefore$  height = area  $\div$  base = 1125 sq. ft.  $\div$  15 yds. = 1125 sq. ft.  $\div$  45 ft. = 25 ft.

9. Here  $BC = 4$  ft. 6 in. =  $\frac{5}{2}$  ft.,  $AB = 8$  ft.,

$AE = 2$  ft. 8 in. =  $\frac{5}{4}$  ft.

Draw  $CF$  perpendicular to  $AB$  (Euc. I. 12). Now as (by Euc. I. 34)  $CF$  is equal to the perpendicular drawn from any other point in  $CD$  on  $AB$ , we are required to find  $CF$ . Triangles  $AEB$  and  $BFC$  being equiangular, we have (by Euc. VI. 4)  $CF : BC :: AE : AB$ ;

$\therefore CF = BC \times AE \div AB = \frac{5}{2}$  ft.  $\times \frac{5}{4}$  ft.  $\div 8$  ft. = 4 ft.



10. (Vide diag. to preceding.) Here  $AB = 8$  ft.,  $BC = 16$  ft.;  $\therefore$  perimeter of  $ABCD = 2(AB + BC) = 2(8 + 16)$  ft. = 48 ft. = (by hyp.), perimeter of square =  $4 \times$  side of square;  $\therefore$  side of square = 12 ft.;

$\therefore$  area of square =  $(12 \text{ ft.})^2 = 144$  sq. ft.;

$\therefore$  (by hyp.) area of  $ABCD = \frac{1}{2} \times 144$  sq. ft. = 72 sq. ft.

But  $BC \times AE$ , i.e. 16 ft.  $\times AE$  = area of  $ABCD$ ;  $\therefore 16 \text{ ft.} \times AE = 72$  sq. ft.;

$\therefore AE = \frac{72 \text{ sq. ft.}}{16 \text{ ft.}} = \frac{9}{2}$  ft. = 4 ft. 6 in.

Again, (as in preceding)  $CF = BC \times AE \div AB = 16 \text{ ft.} \times \frac{9}{2} \text{ ft.} \div 8 \text{ ft.} = 9 \text{ ft.}$

11. Here  $AB = BC = CD = DA = AC = 24$  ft.;

$\therefore$  (by Euc. I. 6)  $\triangle ABC$  and  $ADC$  are equilateral. Draw  $AE$  perpendicular to  $BC$ . Then (by Euc. I. 5 and 26)  $BE = EC = \frac{1}{2} BC = 12$  ft.; and (by Euc. I. 47)

$AE = \sqrt{(AB^2 - BE^2)} = \sqrt{(AB + BE)(AB - BE)}$

$= \sqrt{\{(24 + 12) \text{ ft.} \times (24 - 12) \text{ ft.}\}}$

$= \sqrt{\{(36 \times 12) \text{ sq. ft.}\}} = \sqrt{(12^2 \times 3 \text{ sq. ft.})} = 12\sqrt{3} \text{ ft.}$

Now area of  $ABCD = BC \times AE = 24 \text{ ft.} \times 12\sqrt{3} \text{ ft.} = 288 \times 1.732 \text{ sq. ft.}$

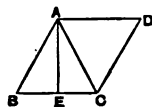
$= 498.816 \text{ sq. ft.} = 498.8 \text{ sq. ft. nearly.}$

Or, applying the rule of Art. 147 Example (3) of the Mensuration,

Area =  $24^2 \times .866 = 498.816 \text{ sq. ft.}$

12. (Vide diag. to preceding.) Here  $AB = BC = CD = DA = 32$  ft., and  $\angle BAD = \angle BCD = 2 \angle ABC = 2 \angle ADC$ . Draw the diagonal  $AC$ . Then (by Euc. I. 5, 34, and Ax. 1),  $\angle BAC = \angle DAC = \angle ACB = \angle DCA$ ; and each of these angles is half of  $\angle BAD$  or  $\angle BCD$ ; and each of them, therefore =  $\angle ABC$  or  $\angle ADC$ . Wherefore, (by Euc. I. 6),  $\triangle ABC$  and  $ADC$  are equilateral. Draw  $AE$  perpendicular to  $BC$ . Then, (as in preceding),  $AE = 16\sqrt{3} \text{ ft.}$ ;  $\therefore$  area of  $ABCD = BC \times AE = 32 \text{ ft.} \times 16\sqrt{3} \text{ ft.}$

$= 512 \times 1.732 \text{ sq. ft.} = 886.784 \text{ sq. ft.} = 886.8 \text{ sq. ft. nearly.}$



CHAPTER XIII.

$$1. \text{ Area} = \frac{\text{base} \times \text{height}}{2} = \frac{18 \text{ ft.} \times 8 \text{ ft.}}{2} = 72 \text{ sq. ft.}$$

$$\begin{aligned} 5. \text{ Here } AB &= 421, BC = 29. \text{ (Euc. I. 47),} \\ AC &= \sqrt{(AB^2 - BC^2)} = \sqrt{\{(AB + BC)(AB - BC)\}} \\ &= \sqrt{\{(421 + 29)(421 - 29)\}} = \sqrt{(450 \times 392)} = \sqrt{\{25 \times 9 \times 2\} \times \{2 \times 196\}} \\ &= \sqrt{(5^2 \times 3^2 \times 2^2 \times 14^2)} = 5 \times 3 \times 2 \times 14 = 420. \\ \therefore \text{ Area of } \triangle ABC &= \frac{1}{2}(BC \times AC) = \frac{1}{2} \times 29 \times 420 = 6090. \end{aligned}$$



7. (Vide diag. to preceding.) (Euc. I. 47), and (as in preceding)

$$\begin{aligned} AC &= \sqrt{\{(10 + 7)(10 - 7)\}} = \sqrt{17 \times 3} = \sqrt{51} = 7.1414; \\ \therefore \text{ area of } \triangle ABC &= \frac{1}{2}(BC \times AC) = \frac{1}{2} \times 7 \times 7.1414 = 24.9949 = 24.995 \text{ nearly.} \end{aligned}$$

9. (Note. When the 3 sides of a triangle are given, to find the area, the formula made use of in Art. 152 of the Mensuration is  $\sqrt{s(s-a)(s-b)(s-c)}$ , where  $a$ ,  $b$ , and  $c$ , are the sides of the triangle, and  $s = \frac{a+b+c}{2}$ . In practice, however, it is sometimes more convenient to adopt another formula, viz.

$$\frac{1}{4} \sqrt{\{(b+c)^2 - a^2\} \{a^2 - (b-c)^2\}},$$

which is deduced from the former. In order to apply the latter formula correctly, take  $b$  and  $c$  such that  $b$  is greater than  $c$ , and the difference between them the least possible, not 0, however. Then of course  $a$  will be the remaining side.)

Here  $a=5$ ,  $b=6$ ,  $c=5$ ;

$$\begin{aligned} \therefore \text{ area of } \triangle &= \frac{1}{4} \sqrt{\{(6+5)^2 - 5^2\} \{5^2 - (6-5)^2\}} = \frac{1}{4} \sqrt{\{(11^2 - 5^2)(5^2 - 1)\}} \\ &= \frac{1}{4} \sqrt{(16 \times 6 \times 6 \times 4)} = \frac{1}{4} \times 4 \times 6 \times 2 = 12. \end{aligned}$$

$$\begin{aligned} 13. \text{ Here } a &= 68, b = 77, c = 75; \therefore \text{ area of } \triangle \\ &= \frac{1}{4} \sqrt{\{(77+75)^2 - 68^2\} \{68^2 - (77-75)^2\}} = \frac{1}{4} \sqrt{\{(152^2 - 68^2)(68^2 - 2^2)\}} \\ &= \frac{1}{4} \sqrt{(220 \times 84 \times 70 \times 66)} = \frac{1}{4} \sqrt{\{(22 \times 10) \times (7 \times 4 \times 3) \times (7 \times 10) \times (22 \times 3)\}} \\ &= \frac{1}{4} \sqrt{(22^2 \times 10^2 \times 7^2 \times 2^2 \times 3^2)} = \frac{1}{4} \times 22 \times 10 \times 7 \times 2 \times 3 = 2310. \end{aligned}$$

$$\begin{aligned} 21. \text{ Area of } \triangle &= \frac{1}{4} \sqrt{\{(4+3)^2 - 2^2\} \{2^2 - (4-3)^2\}} = \frac{1}{4} \sqrt{\{(7^2 - 2^2)(2^2 - 1)\}} \\ &= \frac{1}{4} \sqrt{(9 \times 5 \times 3 \times 1)} = \frac{3}{4} \sqrt{15} = \frac{3}{4} \times 3.8729 = \frac{11.6187}{4} = 2.9046 = 2.905 \text{ nearly.} \end{aligned}$$

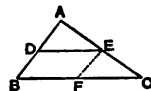
$$\begin{aligned} 27. \text{ Area of } \triangle &= \frac{1}{4} \sqrt{\{(31+24)^2 - 11^2\} \{11^2 - (31-24)^2\}} \\ &= \frac{1}{4} \sqrt{\{(55^2 - 11^2)(11^2 - 7^2)\}} = \frac{1}{4} \sqrt{(66 \times 44 \times 18 \times 4)} \\ &= \frac{1}{4} \sqrt{(6 \times 11 \times 11 \times 4 \times 6 \times 3 \times 4)} = \frac{1}{4} \sqrt{(6^2 \times 11^2 \times 4^2 \times 3)} = \frac{1}{4} \times 6 \times 11 \times 4 \sqrt{3} = 66\sqrt{3}. \end{aligned}$$

$$\begin{aligned} 28. \text{ Area of } \triangle &= \frac{1}{4} \sqrt{\{(63+62)^2 - 61^2\} \{61^2 - (63-62)^2\}} \\ &= \frac{1}{4} \sqrt{\{(125^2 - 61^2)(61^2 - 1)\}} = \frac{1}{4} \sqrt{(186 \times 64 \times 62 \times 60)} \\ &= \frac{1}{4} \sqrt{(3 \times 62 \times 64 \times 62 \times 3 \times 4 \times 5)} = \frac{1}{4} \sqrt{(3^2 \times 62^2 \times 8^2 \times 2^2 \times 5)} \\ &= \frac{1}{4} \times 3 \times 62 \times 8 \times 2 \sqrt{5} = 744\sqrt{5}. \end{aligned}$$

29. Here  $AB=68$ ,  $AC=75$ ,  $BC=77$ ,  $AD=DB$ , and  $AE=EC$ ; from  $E$  draw  $EF$  parallel to  $AB$ , (Euc. I. 31). Then (Euc. I. 29 and 26)  $\triangle ADE=\triangle EFC$ , and  $DE=FC$ ; but  $DE=BF$  (Euc. I. 34). The parallelogram  $DF=2\triangle EFC$  (Euc. I. 38 and 41)  $=2\triangle ADE$ ;  $\therefore DBCE=3\triangle ADE$ .

$$\begin{aligned}\text{Now area of } \triangle ADE &= \frac{1}{2}\sqrt{\left\{\left(\frac{1}{2}77 + \frac{1}{2}75\right)^2 - 34^2\right\} \{34^2 - \left(\frac{1}{2}77 - \frac{1}{2}75\right)^2\}} \\ &= \frac{1}{2}\sqrt{\{(76^2 - 34^2)(34^2 - 1)\}} = \frac{1}{2}\sqrt{(110 \times 42 \times 35 \times 33)} \\ &= \frac{1}{2}\sqrt{(11 \times 5 \times 2 \times 7 \times 3 \times 2 \times 5 \times 7 \times 11 \times 3)} \\ &= \frac{1}{2}\sqrt{(11^2 \times 5^2 \times 2^2 \times 7^2 \times 3^2)} \\ &= \frac{1}{2} \times 11 \times 5 \times 2 \times 7 \times 3 = 1135 = 577.5.\end{aligned}$$

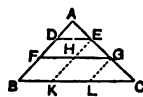
$$\therefore \text{area of } DBCE = 3 \times 577.5 = 1732.5.$$



30. Here  $AB=111$ ,  $AC=175$ ,  $BC=176$ ;  
 $AD=DF=FB=\frac{1}{3}AB=37$ ;  $AE=EG=GC=\frac{1}{3}AC=11\frac{2}{3}$ .

From similar  $\triangle ADE$  and  $\triangle ABC$  (by Euc. VI. 4),

$$DE:AD::BC:BA; \therefore DE=AD \times BC \div 3AD = \frac{BC}{3} = \frac{176}{3}.$$



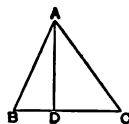
$$\begin{aligned}\therefore \text{area of } \triangle ADE &= \frac{1}{2}\sqrt{\left\{\left(\frac{1}{3}176 + \frac{1}{3}11\frac{2}{3}\right)^2 - 37^2\right\} \{37^2 - \left(\frac{1}{3}176 - \frac{1}{3}11\frac{2}{3}\right)^2\}} \\ &= \frac{1}{2}\sqrt{\left\{(117^2 - 37^2) \left(37^2 - \frac{1}{9}\right)\right\}} = \frac{1}{2}\sqrt{(154 \times 80 \times \frac{1}{9} \times \frac{1}{9})} \\ &= \frac{1}{2}\sqrt{(2 \times 7 \times 11 \times 16 \times 5 \times 7 \times 16 \times 5 \times 2 \times 11 \times \frac{1}{9})} \\ &= \frac{1}{2}\sqrt{\left(2^2 \times 7^2 \times 11^2 \times 16^2 \times 5^2 \times \frac{1}{81}\right)} = \frac{1}{2} \times 2 \times 7 \times 11 \times 16 \times 5 \times \frac{1}{3} = 1026\frac{2}{3}.\end{aligned}$$

Now (by Euc. I. 38, 41),  $DFHE=2\triangle EHG=2\triangle ADE$  (as in preceding),  
 $\therefore$  area of  $DFGE=3 \times$  area of  $\triangle ADE=3 \times 1026\frac{2}{3}=3080$ . In the same manner it may be shewn that

$$FBCG=5 \text{ area of } \triangle ADE=5 \times 1026\frac{2}{3}=5133\frac{1}{3}.$$

31. Here  $AB=13$  ft.,  $BC=14$  ft.,  $AC=15$  ft. Draw  $AD$  perpendicular to  $BC$ . We are to find  $AD$ .

$$\begin{aligned}\text{Area of } \triangle ABC &= \frac{1}{2}\sqrt{\{(15 \text{ ft.} + 14 \text{ ft.})^2 - (13 \text{ ft.})^2\} \\ &\quad \{(13 \text{ ft.})^2 - (15 \text{ ft.} - 14 \text{ ft.})^2\}} \\ &= \frac{1}{2}\sqrt{\{(29^2 - 13^2) \text{ sq. ft.} \times (13^2 - 1) \text{ sq. ft.}\}} \\ &= \frac{1}{2}\sqrt{\{(42 \times 16) \text{ sq. ft.} \times (14 \times 12) \text{ sq. ft.}\}} \\ &= \frac{1}{2}\sqrt{\{7 \times 6 \times 16 \text{ sq. ft.} \times 7 \times 2 \times 6 \times 2 \text{ sq. ft.}\}} = \frac{1}{2}\sqrt{\{7^2 \times 6^2 \text{ sq. ft.} \times 4^2 \times 2^2 \text{ sq. ft.}\}} \\ &= \frac{1}{2} \times 7 \times 6 \times 4 \times 2 \text{ sq. ft.} = 84 \text{ sq. ft.}\end{aligned}$$



But area of  $\triangle ABC$  also  $=\frac{1}{2}BC \times AD=7 \text{ ft.} \times AD$ ;

$$\therefore 7 \text{ ft.} \times AD=84 \text{ sq. ft.}; \therefore AD=12 \text{ ft.}$$

32. (Vide diag. to preceding.) Here  $AB=51$  ft.,  $BC=52$  ft.,  $AC=53$  ft.;  
 $\therefore$  area of  $\triangle ABC=\frac{1}{2}\sqrt{\{(53 \text{ ft.} + 52 \text{ ft.})^2 - (51 \text{ ft.})^2\} \{(51 \text{ ft.})^2 - (53 \text{ ft.} - 52 \text{ ft.})^2\}}$   
 $=\frac{1}{2}\sqrt{(156 \times 54 \text{ sq. ft.} \times 52 \times 50 \text{ sq. ft.})}$   
 $=\frac{1}{2}\sqrt{(3 \times 52 \times 3 \times 9 \times 2 \text{ sq. ft.} \times 52 \times 2 \times 25 \text{ sq. ft.})}$   
 $=\frac{1}{2}\sqrt{(3^2 \times 52^2 \times 3^2 \text{ sq. ft.} \times 2^2 \times 5^2 \text{ sq. ft.})} = \frac{1}{2} \times 3 \times 52 \times 3 \times 2 \times 5 \text{ sq. ft.} = 1170 \text{ sq. ft.};$   
 $\therefore$  (as in preceding)  $AD=1170 \text{ sq. ft.} \div \frac{1}{2}BC=1170 \text{ sq. ft.} \div 26 \text{ ft.} = 45 \text{ ft.}$

Now (by *Eucl. I. 47*)  $BD = \sqrt{(AB^2 - AD^2)} = \sqrt{\{(AB + AD)(AB - AD)\}}$   
 $= \sqrt{\{(51 + 45) \text{ ft.} \times (51 - 45) \text{ ft.}\}} = \sqrt{(96 \times 6 \text{ sq. ft.})} = \sqrt{(16 \times 6 \times 6 \text{ sq. ft.})}$   
 $= \sqrt{(4^2 \times 6^2 \text{ sq. ft.})} = 4 \times 6 \text{ ft.} = 24 \text{ ft.};$

$\therefore$  area of  $\triangle ADB = \frac{1}{2} BD \times AD = \frac{1}{2} \times 24 \times 45 \text{ sq. ft.} = 540 \text{ sq. ft.}$ ; and area of  $\triangle ADC = \text{area of } \triangle ABC - \text{area of } \triangle ADB = (1170 - 540) \text{ sq. ft.} = 630 \text{ sq. ft.}$

38. Here  $AB = BC = CD = DA = 100 \text{ ft.}$ ;  $AE = 60 \text{ ft.}$ ,  
 $BE = 80 \text{ ft.}$  Now  $80^2 + 60^2 = 100^2$ ;  $\therefore AB^2 = AE^2 + EB^2$ ;  
 $\therefore \angle AEB$  is a right angle (*Eucl. I. 48*);  $\therefore$  area of  
 $\triangle AEB = \frac{1}{2} AE \times EB = \frac{1}{2} \times 80 \times 60 \text{ sq. ft.} = 2400 \text{ sq. ft.}$

But area of  $\triangle AEB = \frac{1}{2} EF \times AB = 50 \text{ ft.} \times EF$ ;

$\therefore 50 \text{ ft.} \times EF = 2400 \text{ sq. ft.}$ ;

$\therefore EF = 2400 \text{ sq. ft.} \div 50 \text{ ft.} = 48 \text{ ft.}$ ;

$\therefore EG = FG - FE = AD - FE$  (*Eucl. I. 84*)  $= (100 - 48) \text{ ft.} = 52 \text{ ft.}$ ;

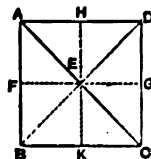
$\therefore$  area of  $\triangle DEC = \frac{1}{2} \times DC \times EG = \frac{1}{2} \times 100 \times 52 \text{ sq. ft.} = 2600 \text{ sq. ft.}$

Again (by *Eucl. I. 47*),  $AF = \sqrt{(AE^2 - EF^2)} = \sqrt{\{(AE + EF)(AE - EF)\}}$   
 $= \sqrt{(108 \times 12 \text{ sq. ft.})} = \sqrt{(9 \times 12 \times 12 \text{ sq. ft.})} = 3 \times 12 \text{ ft.} = 36 \text{ ft.} = EH$  (by *Eucl. I. 84*),

$\therefore$  area of  $\triangle AED = \frac{1}{2} \times AD \times EH = \frac{1}{2} \times 100 \times 36 \text{ sq. ft.} = 1800 \text{ sq. ft.}$

Lastly,  $EK = HK - HE = AB - HE$  (*Eucl. I. 84*)  $= (100 - 36) \text{ ft.} = 64 \text{ ft.}$ ;

$\therefore$  area of  $\triangle BEC = \frac{1}{2} \times BC \times EK = \frac{1}{2} \times 100 \times 64 \text{ sq. ft.} = 3200 \text{ sq. ft.}$



34. Here  $AD = 18 \text{ ft.}$ ,  $DE = 5 \text{ ft.}$ ,  $DF = 10 \cdot 4 \text{ ft.}$

(By *Eucl. I. 47*)  $AE = \sqrt{(AD^2 - DE^2)} = \sqrt{\{(AD + DE)(AD - DE)\}}$

$= \sqrt{(18 \text{ ft.} \times 8 \text{ ft.})} = \sqrt{(144 \text{ sq. ft.})} = 12 \text{ ft.}$  Now from similar

$\triangle ADB$  and  $\triangle ADE$ , (by *Eucl. VI. 8 Cor.*)  $AB : AD :: AD : AE$ ;

$\therefore AB = AD^2 \div AE = 169 \text{ sq. ft.} \div 12 \text{ ft.} = 14 \frac{1}{3} \text{ ft.}$  Again

$DB : DA :: DE : AE$ ;

$\therefore DB = DA \times DE \div AE = 18 \times 5 \text{ sq. ft.} \div 12 \text{ ft.} = 7 \frac{1}{2} \text{ ft.}$

(By *Eucl. I. 47*)  $AF = \sqrt{(AD^2 - DF^2)} = \sqrt{\{(AD + DF)(AD - DF)\}}$

$= \sqrt{(23 \cdot 4 \text{ ft.} \times 2 \cdot 6 \text{ ft.})} = \sqrt{(60 \cdot 84 \text{ sq. ft.})} = 7 \cdot 8 \text{ ft.}$  Now from similar

$\triangle ADC$  and  $\triangle ADF$ , (by *Eucl. VI. 8 Cor.*)  $AC : AD :: AD : AF$ ;

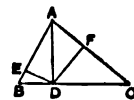
$\therefore AC = AD^2 \div AF = 169 \text{ sq. ft.} \div 7 \cdot 8 \text{ ft.} = 21 \cdot 6 \text{ sq. ft.} \div 2 \cdot 6 \text{ ft.} = 8 \frac{1}{3} \text{ ft.}$ ;

and  $DC : DA :: DF : AF$ ;  $\therefore DC = DA \times DF \div AF = 18 \times 10 \cdot 4 \text{ sq. ft.} \div 7 \cdot 8 \text{ ft.}$

$= 18 \times 4 \times 2 \cdot 6 \text{ sq. ft.} \div 8 \times 2 \cdot 6 \text{ ft.} = 12 \text{ ft.}$ ;

$\therefore BC = BD + DC = (7 \frac{1}{2} + 12) \text{ ft.} = 19 \frac{1}{2} \text{ ft.} = 21 \frac{1}{2} \text{ ft.}$  Gathering up these results,  
we have  $AB = 14 \frac{1}{3} \text{ ft.}$ ,  $AC = 8 \frac{1}{3} \text{ ft.}$ ,  $BD = 7 \frac{1}{2} \text{ ft.}$ ,  $DC = 12 \text{ ft.}$  Lastly,

area of  $\triangle ABC = \frac{1}{2} BC \times AD = \frac{1}{2} \times 21 \frac{1}{2} \times 18 \text{ sq. ft.} = 191 \frac{1}{4} \text{ sq. ft.} = 147 \frac{1}{2} \text{ sq. ft.}$



35. Area of field  $= \frac{1}{2} \times 1166 \times 788 \text{ sq. lks.} = \frac{1166 \times 788}{2 \times 100,000} \text{ ac.}$  Now since  
rent of this no. of acres is £24, the rent of one acre

$$= £24 \div \frac{1166 \times 788}{200,000} = £ \frac{4,800,000}{860,508},$$

which equals £5. 11s. 6½d. nearly.

## 36. Area of field

$$\begin{aligned}
 &= \frac{1}{2} \sqrt{\{(440+350)^2 \text{ sq. yds.} - 750^2 \text{ sq. yds.}\} \{750^2 \text{ sq. yds.} - (440-350)^2 \text{ sq. yds.}\}} \\
 &= \frac{1}{2} \sqrt{\{(790^2 - 750^2) \text{ sq. yds.} \times (750^2 - 90^2) \text{ sq. yds.}\}} \\
 &= \frac{1}{2} \sqrt{(1540 \times 40 \text{ sq. yds.} \times 840 \times 660 \text{ sq. yds.})} \\
 &= \frac{1}{2} \sqrt{\{(10 \times 22 \times 7) \times (10 \times 4) \text{ sq. yds.} \times (10 \times 4 \times 7 \times 3) \times (10 \times 22 \times 3) \text{ sq. yds.}\}} \\
 &= \frac{1}{2} \sqrt{(10^2 \times 10^2 \times 22^2 \text{ sq. yds.} \times 7^2 \times 4^2 \times 3^2 \text{ sq. yds.})} \\
 &= \frac{1}{2} \times 10 \times 10 \times 22 \times 7 \times 4 \times 3 \text{ sq. yds.} = 46200 \text{ sq. yds.} = \frac{46200}{4840} \text{ ac.} = \frac{11}{11} \text{ ac.}
 \end{aligned}$$

Now since rent of  $\frac{11}{11}$  ac. is £28. 5s., i.e. £1 $\frac{5}{8}$ ,  
the rent of 1 ac. = £1 $\frac{5}{8}$  ×  $\frac{11}{11}$  = £1 $\frac{5}{8}$  = £2. 15s.

37. Area of  $\Delta$ 

$$\begin{aligned}
 &= \frac{1}{2} \sqrt{\{(84+72)^2 \text{ sq. in.} - 60^2 \text{ sq. in.}\} \{60^2 \text{ sq. in.} - (84-72)^2 \text{ sq. in.}\}} \\
 &= \frac{1}{2} \sqrt{\{(156^2 - 60^2) \text{ sq. in.} \times (60^2 - 12^2) \text{ sq. in.}\}} \\
 &= \frac{1}{2} \sqrt{(216 \times 96 \text{ sq. in.} \times 72 \times 48 \text{ sq. in.})} \\
 &= \frac{1}{2} \sqrt{\{(36 \times 6) \times (16 \times 6) \text{ sq. in.} \times (12 \times 6) \times (12 \times 4) \text{ sq. in.}\}} \\
 &= \frac{1}{2} \sqrt{(6^2 \times 6^2 \times 4^2 \text{ sq. in.} \times 12^2 \times 2^2 \times 6 \text{ sq. in.})} \\
 &= \frac{1}{2} \times 6 \times 6 \times 4 \times 12 \times 2 \times \sqrt{6} \text{ sq. in.} = 864 \times 2.449 \text{ sq. in. nearly,} \\
 &= 2115.9 \text{ sq. in.} = 2116 \text{ sq. in. to the nearest inch.}
 \end{aligned}$$

38. Here  $AB=200$  yds.,  $BC=100$  yds.;  $\therefore$  area of  $\Delta ABC = \frac{1}{2} BC \times AB = \frac{1}{2} \times 100 \times 200$  sq. yds. = 10,000 sq. yds.  
Now (by Euc. I. 47)  $AC = \sqrt{(AB^2 + BC^2)} = \sqrt{\{(200^2 + 100^2) \text{ sq. yds.}\}} = \sqrt{\{100^2 (2^2 + 1) \text{ sq. yds.}\}} = 100\sqrt{5}$  yds.

Now area of  $\Delta ABC$  also =  $\frac{1}{2} AC \times DB = 50\sqrt{5}$  yds.  $\times DB$ ;  
 $\therefore 50\sqrt{5}$  yds.  $\times DB = 10,000$  sq. yds.;

$$\begin{aligned}
 \therefore DB &= 10,000 \text{ sq. yds.} \div 50\sqrt{5} \text{ yds.} = \frac{10,000}{50\sqrt{5}} \text{ yds.} = \frac{200}{\sqrt{5}} \text{ yds.} \\
 &= \frac{200\sqrt{5}}{\sqrt{5} \times \sqrt{5}} \text{ yds.} = \frac{200\sqrt{5}}{5} \text{ yds.} = 40\sqrt{5} \text{ yds.}
 \end{aligned}$$

and (by Euc. VI. 8 Cor.)  $DA : AB :: AB : AC$ ;

$$\therefore DA = AB^2 \div AC = \frac{40,000}{100\sqrt{5}} \text{ yds.} = \frac{400}{\sqrt{5}} \text{ yds.} = 80\sqrt{5} \text{ yds.}$$

$$\begin{aligned}
 \therefore \text{area of } \Delta ADB &= \frac{1}{2} AD \times DB = \frac{1}{2} \times 80\sqrt{5} \times 40\sqrt{5} \text{ sq. yds.} \\
 &= 40 \times 40 \times 5 \text{ sq. yds.} = 8000 \text{ sq. yds.}
 \end{aligned}$$

$$\begin{aligned}
 \therefore \text{area of } \Delta BDC &= \text{area of } \Delta ABC - \text{area of } \Delta ADB \\
 &= (10,000 - 8000) \text{ sq. yds.} = 2000 \text{ sq. yds.}
 \end{aligned}$$

39. Let  $a, c, b$ , be the sides of the triangle. Then, by hyp.,

$$a : b :: 5 : 12; \therefore b = \frac{12}{5}a; \text{ and } a : c :: 5 : 12; \therefore c = \frac{12}{5}a;$$

$\therefore a^2 + c^2 = a^2 + (\frac{12}{5}a)^2 = \frac{169}{25}a^2$ ; but  $b^2$  also =  $\frac{169}{25}a^2$ ; wherefore the triangle is right angled (Euc. I. 48). Now, perimeter of triangle

$$= a + b + c = a + \frac{12}{5}a + \frac{12}{5}a = 6a = (\text{by hyp.}) 50 \text{ yds.} = 150 \text{ ft.}; \therefore a = 25 \text{ ft.};$$

$$\therefore c = \frac{12}{5}a = \frac{12}{5} \times 25 \text{ ft.} = 60 \text{ ft.};$$

$$\therefore \text{area of triangle} = \frac{1}{2} \times a \times c = \frac{1}{2} \times 25 \text{ ft.} \times 60 \text{ ft.} = 25 \times 30 \text{ sq. ft.} = 750 \text{ sq. ft.}$$



40. Let  $a, c, b$ , be the sides of the triangle. Then, by hyp.,

$$a : b :: 18 : 15; \therefore b = \frac{5}{6}a; \text{ and } a : c :: 18 : 14; \therefore c = \frac{7}{9}a.$$

Now perimeter of triangle

$$= a + b + c = a + \frac{5}{6}a + \frac{7}{9}a = \frac{47}{18}a = (\text{by hyp.}) 70 \text{ yds.} = 210 \text{ ft.};$$

$$\therefore a = 210 \text{ ft.} \times \frac{18}{47} = 80 \text{ ft.}; \therefore b = \frac{5}{6} \times 80 \text{ ft.} = 66 \frac{2}{3} \text{ ft.}, \text{ and } c = \frac{7}{9} \times 80 \text{ ft.} = 62 \frac{2}{3} \text{ ft.};$$

$\therefore$  area of triangle

$$= \frac{1}{2} \sqrt{\{(75+70)^2 \text{ sq. ft.} - 65^2 \text{ sq. ft.}\} \{65^2 \text{ sq. ft.} - (75-70)^2 \text{ sq. ft.}\}}$$

$$= \frac{1}{2} \sqrt{\{(145^2 - 65^2) \text{ sq. ft.} \times (65^2 - 5^2) \text{ sq. ft.}\}}$$

$$= \frac{1}{2} \sqrt{\{210 \times 80 \text{ sq. ft.} \times 70 \times 60 \text{ sq. ft.}\}}$$

$$= \frac{1}{2} \sqrt{\{(70 \times 3) \times (4 \times 20) \text{ sq. ft.} \times 70 \times 20 \times 3 \text{ sq. ft.}\}}$$

$$= \frac{1}{2} \sqrt{\{70^2 \times 3^2 \text{ sq. ft.} \times 2^2 \times 20^2 \text{ sq. ft.}\}} = \frac{1}{2} \times 70 \times 3 \times 2 \times 20 \text{ sq. ft.} = 2100 \text{ sq. ft.}$$

41. Let  $ABDEC$  represent the gable end. Then

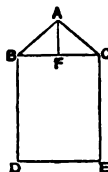
$$AF = 12 \text{ ft.}, BC = 27 \text{ ft.}, BD = 38 \text{ ft.};$$

$\therefore$  area of  $ABDEC$  = area of  $\triangle ABC$  + area of rectangle  $BE$

$$= \frac{1}{2} \times BC \times AF + BD \times BC = BC \left( \frac{AF}{2} + BD \right) = 27 \text{ ft.} (6 + 38) \text{ ft.}$$

$$= 27 \times 44 \text{ sq. ft.} = 9 \times 13 \text{ sq. yds.} = 117 \text{ sq. yds.};$$

$$\therefore \text{cost} = 117 \times 1s. 9d. = 117 \times \frac{9}{4} = \frac{1053}{4} = \text{£}10. 4s. 9d.$$



42. Here  $293^2 = 285^2 + 68^2$ ; wherefore (Euc. I. 48), the triangle is right-angled. Hence (by Euc. III. 31), the hypotenuse of this triangle is the diameter of the circle circumscribing the triangle;  $\therefore$  Diam. of circumscribing circle = 293.

43. Here  $AB = 99$ ,  $AC = 125$ ,  $BC = 136$ . Describe the circle  $ABEC$  about the triangle  $ABC$  (by Euc. IV. 5). Draw the diameter  $AE$ , which we are required to find. Draw  $AD$  perpendicular to  $BC$  (Euc. I. 12), and join  $CE$ .

$$\text{Area of } \triangle ABC = \frac{1}{2} \sqrt{\{(136+125)^2 - 99^2\} \{99^2 - (136-125)^2\}}$$

$$= \frac{1}{2} \sqrt{\{(261^2 - 99^2) (99^2 - 11^2)\}} = \frac{1}{2} \sqrt{\{360 \times 162 \times 110 \times 88\}}$$

$$= \frac{1}{2} \sqrt{\{10 \times 36 \times 2 \times 81 \times 10 \times 11 \times 11 \times 4 \times 2\}}$$

$$= \frac{1}{2} \sqrt{\{10^2 \times 6^2 \times 2^2 \times 9^2 \times 11^2 \times 2^2\}}$$

$$= \frac{1}{2} \times 10 \times 6 \times 2 \times 9 \times 11 \times 2 = 60 \times 99.$$

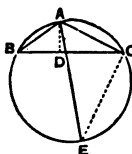
Now, from similar  $\triangle ABD$  and  $\triangle AEC$ , (which are similar because by Euc. III. 81,  $\angle ACE$  is a right angle and is  $\therefore \angle ADB$ , and  $\angle ABD = \angle AEC$ , by Euc. III. 21;  $\therefore$  by Euc. I. 32, and Ax. 8,  $\angle BAD = \angle CAE$ ), we have (by Euc. VI. 4)  $AE : AC :: AB : AD$ ;  $\therefore AE = \frac{AB \times AC}{AD}$ ; and multiplying the

numerator and denominator of this fraction by  $BC$ , we have

$$AE = \frac{AB \times AC \times BC}{AD \times BC};$$

$$\text{but } AD \times BC = 2 \text{ area of } \triangle ABC = 2 \times 60 \times 99;$$

$$\therefore AE = \frac{99 \times 125 \times 136}{2 \times 60 \times 99} = \frac{425}{2} = 212 \frac{1}{2}.$$





$$\begin{aligned}\therefore \text{area of } AKMD &= DO \times \frac{AD+KM}{2} = 5 \text{ ft.} \times \frac{8 \text{ ft.} + 4 \text{ ft.}}{2} \\ &= 5 \times \frac{1}{2} \text{ sq. ft.} = 2\frac{1}{2} \text{ sq. ft.} = 17\frac{1}{2} \text{ sq. ft.};\end{aligned}$$

and area of

$$KBCM = OG \times \frac{KM+BC}{2} = 5 \text{ ft.} \times \frac{4 \text{ ft.} + 5 \text{ ft.}}{2} = 5 \text{ ft.} \times \frac{9}{2} \text{ ft.} = 4\frac{1}{2} \text{ sq. ft.} = 22\frac{1}{2} \text{ sq. ft.}$$

(Note. Hence the line joining the bisections of the oblique sides of a trapezoid is equal to half the sum of the parallel sides of the trapezoid.)

16. Here  $AD=14$  yds.,  $BC=20$  yds.,  
 $AM=12$  yds.,  $AK=KL=LM=\frac{1}{3}AM=4$  yds.

Now (as was shewn in 15)  $GH=\frac{EF+BC}{2}$ , and

$$EF=\frac{AD+GH}{2}=\frac{1}{2}\left(AD+\frac{EF+BC}{2}\right);$$

$$\therefore 2EF=AD+\frac{EF+BC}{2}; 4EF=2AD+EF+BC; 3EF=2AD+BC;$$

$$\therefore EF=\frac{2AD+BC}{3}=\frac{28 \text{ yds.}+20 \text{ yds.}}{3}=16 \text{ yds.},$$

$$\text{and } GH=\frac{EF+BC}{2}=\frac{16 \text{ yds.}+20 \text{ yds.}}{2}=18 \text{ yds.};$$

$$\begin{aligned}\therefore \text{area of } AEFD &= \frac{AD+EF}{2} \times AK = \frac{14 \text{ yds.}+16 \text{ yds.}}{2} \times 4 \text{ yds.} \\ &= 15 \times 4 \text{ sq. yds.} = 60 \text{ sq. yds.};\end{aligned}$$

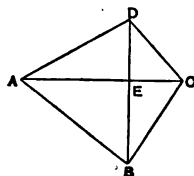
$$\begin{aligned}\text{area of } EGHF &= \frac{EF+GH}{2} \times KL = \frac{16 \text{ yds.}+18 \text{ yds.}}{2} \times 4 \text{ yds.} \\ &= 17 \times 4 \text{ sq. yds.} = 68 \text{ sq. yds.}\end{aligned}$$

$$\begin{aligned}\text{Lastly area of } GBCH &= \frac{GH+BC}{2} \times LM = \frac{18 \text{ yds.}+20 \text{ yds.}}{2} \times 4 \text{ yds.} \\ &= 19 \text{ yds.} \times 4 \text{ yds.} = 76 \text{ sq. yds.}\end{aligned}$$

(Note. Hence, if in a trapezoid two straight lines are drawn across the figure parallel to the parallel sides, and dividing each of the other sides into three equal parts, the smaller of the two former straight lines is equal to  $\frac{1}{3}$  of the sum of twice the smaller parallel side and the other parallel side; and the longer of the two straight lines is equal to  $\frac{2}{3}$  of the sum of twice the longer parallel side and the other parallel side.)

17. Here  $AC=26$  ft.,  $DB=24$  ft., and angles at  $E$  are right angles; area of  $ABCD$ =area of

$$\begin{aligned}\triangle ABC + \text{area of } \triangle ADC &= \frac{1}{2} \times AC \times EB + \frac{1}{2} \times AC \times DE \\ &= \frac{1}{2} AC (DE+EB) = \frac{1}{2} AC \times DB = \frac{1}{2} \times 26 \text{ ft.} \times 24 \text{ ft.} \\ &= 312 \text{ sq. ft.}\end{aligned}$$



18. Let  $ABCD$  be a rhombus, of which  
 $AC=110$  yds.,  $BD=88$  yds.

Now (Euc. I. 8, 4, and Def. 10) angles at  $E$  are right angles. Then (as in preceding), area of  $ABCD$

$$= \frac{1}{2} AC \times DB = \frac{1}{2} \times 110 \times 88 \text{ sq. yds.} = 4840 \text{ sq. yds.} = 1 \text{ ac.}$$

19. (Vide diag. to 18). Here  $AC = 64$  yds.,  $BD = 36$  yds.;

$$\therefore \text{area of } ABCD = \frac{1}{2} \times 64 \times 36 \text{ sq. yds.} = 1152 \text{ sq. yds.}$$

$$\therefore \text{cost of turfing} = 1152 \times 4d. = 1152 \times \frac{2}{3} = \pounds 768 = \pounds 19. 4s.$$

20. (Vide diag. to 18). Here area of  $ABCD = 52204$  sq. ft., and

$$BD = 248 \text{ ft.}; \text{ Required } AC.$$

Now area of  $ABCD = \frac{1}{2} \times AC \times BD = AC \times 124 \text{ ft.} = (\text{by hyp.}) 52204 \text{ sq. ft.};$

$$\therefore AC = \frac{52204}{124} \text{ ft.} = 421 \text{ ft.}$$

21. Here  $AB = 28$  ft.,  $BC = 45$  ft.,  $CD = 51$  ft.,  $DA = 52$  ft., and  $AC = 53$  ft. Now  $53^2 = 28^2 + 45^2$ ;  $\therefore AC^2 = AB^2 + BC^2$ ;

$\therefore \angle ABC$  is a right  $\angle$  (Euc. I. 48);  $\therefore$  area of

$$\triangle ABC = \frac{1}{2} \times AB \times BC = \frac{1}{2} \times 28 \text{ ft.} \times 45 \text{ ft.} = 630 \text{ sq. ft.};$$

and area of  $\triangle ADC = \frac{1}{2} \sqrt{[(53 + 52)^2 \text{ sq. ft.} - 51^2 \text{ sq. ft.}]}$

$$\{51^2 \text{ sq. ft.} - (53 - 52)^2 \text{ sq. ft.}\}]$$

$$= \frac{1}{2} \sqrt{[(105^2 - 51^2) \text{ sq. ft.} \times (51^2 - 1) \text{ sq. ft.}]}$$

$$= \frac{1}{2} \sqrt{(156 \times 54 \text{ sq. ft.} \times 52 \times 50 \text{ sq. ft.})}$$

$$= \frac{1}{2} \sqrt{(52 \times 3 \times 9 \times 3 \times 2 \text{ sq. ft.} \times 52 \times 25 \times 2 \text{ sq. ft.})}$$

$$= \frac{1}{2} \sqrt{(52^2 \times 3^2 \times 3^2 \times 2^2 \times 5^2 \text{ sq. ft.})}$$

$$= \frac{1}{2} \times 52 \times 3 \times 3 \times 2 \times 5 \text{ sq. ft.} = 1170 \text{ sq. ft.};$$

$$\therefore \text{area of } ABCD = \text{area of } \triangle ABC + \text{area of } \triangle ADC = (630 + 1170) \text{ sq. ft.}$$

$$= 1800 \text{ sq. ft.}$$

22. Here  $AB = 48$  ch.,  $BC = 20$  ch.,  $AC = 52$  ch.,  $DE = 30$  ch.

Now  $52^2 = 48^2 + 20^2$ ;  $\therefore AC^2 = AB^2 + BC^2$ ; wherefore  $\angle ABC$  is a rt.  $\angle$ ;  $\therefore$  area of  $ABCD = \text{area of } \triangle ABC + \text{area of } \triangle ADC$

$$= \frac{1}{2} AB \times BC + \frac{1}{2} AC \times DE = \frac{1}{2} \times 48 \times 20 \text{ sq. ch.}$$

$$+ \frac{1}{2} \times 52 \times 30 \text{ sq. ch.} = 1260 \text{ sq. ch.}$$

23. Here  $AB = 27$  ft.,  $BC = 36$  ft.,  $CD = 30$  ft.,  $DA = 25$  ft., and  $\angle ABC$  is a rt.  $\angle$ . (By Euc. I. 47)

$$AC = \sqrt{(AB^2 + BC^2)} = \sqrt{(27^2 + 36^2) \text{ sq. ft.}} = \sqrt{(9^2 (3^2 + 4^2) \text{ sq. ft.})}$$

$$= \sqrt{(9^2 \times 25 \text{ sq. ft.})} = 9 \times 5 \text{ ft.} = 45 \text{ ft.};$$

Now area of  $\triangle ABC = \frac{1}{2} \times BC \times AB = \frac{1}{2} \times 36 \times 27 \text{ sq. ft.} = 486 \text{ sq. ft.}$

and area of  $\triangle ADC = \frac{1}{2} \sqrt{[(30 + 25)^2 \text{ sq. ft.} - 45^2 \text{ sq. ft.}]}$

$$\{45^2 \text{ sq. ft.} - (30 - 25)^2 \text{ sq. ft.}\} = \frac{1}{2} \sqrt{[(55^2 - 45^2) \text{ sq. ft.} \times (45^2 - 5^2) \text{ sq. ft.}]}$$

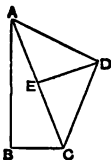
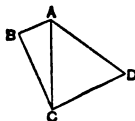
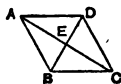
$$= \frac{1}{2} \sqrt{(100 \times 10 \text{ sq. ft.} \times 50 \times 40 \text{ sq. ft.})}$$

$$= \frac{1}{2} \sqrt{(100 \times 10 \text{ sq. ft.} \times 2 \times 25 \times 10 \times 4 \text{ sq. ft.})}$$

$$= \frac{1}{2} \sqrt{(10^2 \times 10^2 \times 5^2 \times 2^2 \times 2 \text{ sq. ft.})} = \frac{1}{2} \times 10 \times 10 \times 5 \times 2 \sqrt{2} \text{ sq. ft.}$$

$$= 250 \times 1.414213 \text{ sq. ft.} = 353.553 \text{ sq. ft. nearly.}$$

$$\therefore \text{area of } ABCD = (486 + 353.553) \text{ sq. ft.} = 839.553 \text{ sq. ft.}$$



24. Here  $AB=BC=5$  ft.,  $\angle ABC=60^\circ$ ,  $CD=4$  ft.,  $AD=3$  ft.  
(By Euc. I. 5, 6 and 82), the triangle  $ABC$  is equilateral; hence  
 $AB=BC=AC=5$  ft.; again  $5^2=4^2+3^2$ ;  $\therefore AC^2=CD^2+AD^2$ ;  
 $\therefore \angle ADC$  is a rt. angle (Euc. I. 48);  $\therefore$  area of

$$\triangle ADC = \frac{1}{2} AD \times DC = \frac{1}{2} \times 3 \times 4 \text{ sq. ft.} = 6 \text{ sq. ft.}$$

Now draw  $AE$  perpendicular to  $BC$  (Euc. I. 12); then (by Euc. I. 5, and 26)  $BE=EC=\frac{1}{2}BC$ ;  $\therefore BE^2=\frac{1}{4}BC^2=\frac{1}{4}AB^2$ ;  $\therefore$  (Euc. I. 47)

$$AE = \sqrt{(AB^2 - BE^2)} = \sqrt{(AB^2 - \frac{1}{4}AB^2)} = \sqrt{(\frac{3}{4}AB^2)} = \frac{AB}{2} \sqrt{3};$$

$\therefore$  area of equil.  $\triangle ABC$

$$= \frac{1}{2} BC \times AE = \frac{1}{2} AB \times \frac{AB}{2} \sqrt{3} = \frac{AB^2}{4} \sqrt{3} = \frac{5^2}{4} \sqrt{3} = 10.825 \text{ sq. ft.};$$

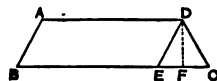
$$\therefore \text{area of } ABCD = (10.825 + 6) \text{ sq. ft.} = 16.825 \text{ sq. ft.}$$

(Note. Hence, the area of an equilateral  $\triangle = \frac{\text{side}^2}{4} \sqrt{3}$ . See Article 65, Example (2) of the Mensuration.)

25. Here  $AD=80$  ft.,  $BC=92$  ft.,

$$AB=DC=10 \text{ ft.}$$

Draw  $DE$  parallel to  $AB$  (Euc. I. 31), and  $DF$  perpendicular to  $EC$  (Euc. I. 12).



$$\therefore EC = BC - BE = BC - AD \text{ (Euc. I. 34)} = (92 - 80) \text{ ft.} = 12 \text{ ft.};$$

and  $DE=AB$  (Euc. I. 34)  $=DC$  (by hyp.)  $=10$  ft.;  $\therefore DE=10$  ft.;  $\therefore$  (by Euc. I. 26)  $EF=FC=\frac{1}{2}EC=6$  ft. Now (by Euc. I. 47)

$$DF = \sqrt{(DE^2 - EF^2)} = \sqrt{(100 - 36) \text{ sq. ft.}} = \sqrt{(64 \text{ sq. ft.})} = 8 \text{ ft.};$$

$\therefore$  area of  $ABCD$

$$= \frac{AD+BC}{2} \times DF = \frac{80 \text{ ft.} + 92 \text{ ft.}}{2} \times 8 \text{ ft.} = 86 \times 8 \text{ sq. ft.} = 688 \text{ sq. ft.}$$

26. Here  $AB=845$  ft.,  $BC=618$  ft.,  
 $CD=610$  ft., and  $\angle DAB$  is a rt.  $\angle$ .

Draw  $CE$  perpendicular to  $AB$  (Euc. I. 12);

$$\therefore EB = AB - AE = AB - CD \text{ (Euc. I. 34)}$$

$$= (845 - 610) \text{ ft.} = 235 \text{ ft.};$$

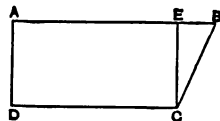
now (by Euc. I. 47)  $CE = \sqrt{(BC^2 - BE^2)}$

$$= \sqrt{(618^2 - 235^2) \text{ ft.}} = \sqrt{(618 + 235) \text{ ft.} \times (618 - 235) \text{ ft.}} = \sqrt{(853 \times 383) \text{ sq. ft.}}$$

$$= \sqrt{(4 \times 81 \times 2) \times (2 \times 17 \times 17) \text{ sq. ft.}} = \sqrt{(2^2 \times 9^2 \times 2^2 \times 17^2 \text{ sq. ft.})}$$

$$= 2 \times 9 \times 2 \times 17 \text{ ft.} = 612 \text{ ft.};$$

$$\therefore \text{area of } ABCD = \frac{AB+CD}{2} \times CE = \frac{845+610}{2} \times 612 \text{ sq. ft.} = 506480 \text{ sq. ft.}$$



27. Here  $AB=165$  ft.,  $CD=123$  ft.,  $CF=100$  and

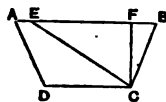
$$AE = \frac{1}{2}(AB - CD) = \frac{1}{2}(165 - 123) \text{ ft.} = 21 \text{ ft.}$$

$$\therefore EB = AB - AE = (165 - 21) \text{ ft.} = 144 \text{ ft.};$$

$\therefore$  area of

$$\triangle EBC = \frac{1}{2} EB \times FC = \frac{1}{2} \times 144 \times 100 \text{ sq. ft.} = 7200 \text{ sq. ft.};$$

$$\text{and area of } ADCE = \frac{AE + DC}{2} \times FC = \frac{21 + 123}{2} \text{ ft.} \times 100 \text{ ft.} = 7200 \text{ sq. ft.}$$



28. Let  $ABCD$  be a rhombus, so that  $BD=234$  ft.,  $AC=88$  ft.;  $\therefore$  area of  $ABCD = \frac{1}{2} BD \times AC$  (vide 17 and 18),  $= \frac{1}{2} \times 234 \times 88 \text{ sq. ft.} = 10296 \text{ sq. ft.}$  (Now by *Eucl. I. 8*, 4, and *def. 10*), the angles at  $F$  are right angles, and  $BF=FD = \frac{1}{2} BD = 117$  ft., and similarly

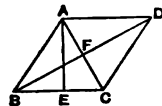
$$AF=FC = \frac{1}{2} AC = 44 \text{ ft.}; \therefore (\text{by } \textit{Eucl. I. 47}),$$

$$AB = \sqrt{(AF^2 + FB^2)} = \sqrt{\{(1936 + 13689) \text{ sq. ft.}\}} = \sqrt{(15625 \text{ sq. ft.})} = 125 \text{ ft.}$$

Lastly, (by *Eucl. I. 8*),  $\triangle ABC = \triangle ADC$ ;  $\therefore$  area of  $ABCD = 2$  area of  $ABC$

$$= \frac{2 \times BC \times AE}{2} = AE \times 125 \text{ ft.} = (\text{as we have found above}) 10296 \text{ sq. ft.};$$

$$\therefore AE = 10296 \text{ sq. ft.} \div 125 \text{ ft.} = 82.368 \text{ ft.}$$



29. The other diagonal  $= 2$  area  $\div 672$  ft. (vide 20)

$$= 854144 \text{ sq. ft.} \div 336 \text{ ft.} = 1054 \text{ ft.}$$

The side and the height may now be found as in *Example 28*.

30. Here  $AB=228$  ft.,  $BC=704$  ft.,  $\angle ABC$  is a rt. angle,

$$AD=DC, \text{ and } \angle ADC=60^\circ.$$

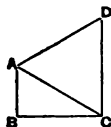
Now (by *Eucl. I. 5*, 32, and 6),  $\triangle ADG$  is equilateral;

$$\therefore AD=DC=AC; \text{ and (by } \textit{Eucl. I. 47}) AC = \sqrt{(AB^2 + BC^2)}$$

$$= \sqrt{\{(51984 + 495616) \text{ sq. ft.}\}} = \sqrt{(547600 \text{ sq. ft.})} = 740 \text{ sq. ft.}$$

$$\therefore \text{area of } ABCD = \frac{1}{2} BC \times AB + \frac{AC^2}{4} \sqrt{3} \text{ (vide 24)}$$

$$= (114 \times 704 + \frac{547600}{4} \sqrt{3}) \text{ sq. ft.} = (80256 + 136900 \sqrt{3}) \text{ sq. ft.}$$



## CHAPTER XV.

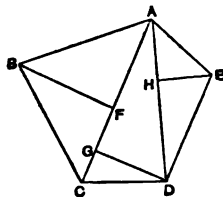
1. Area of  $ABCDE$

$$= \text{area of } \triangle ABC + \text{area of } \triangle ACD + \text{area of } \triangle ADE$$

$$= \frac{1}{2} AC \times BF + \frac{1}{2} AC \times DG + \frac{1}{2} AD \times EH$$

$$= \left( \frac{16 \times 8.4}{2} + \frac{16 \times 4.6}{2} + \frac{12 \times 5}{2} \right) \text{ sq. ft.}$$

$$= (67.2 + 36.8 + 30) \text{ sq. ft.} = 134 \text{ sq. ft.}$$

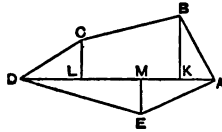


2.  $LK = DA - (DL + KA)$

$$= 15.3 \text{ ft.} - (3.9 + 2.7) \text{ ft.} = (15.3 - 6.6) \text{ ft.} = 8.7 \text{ ft.}$$

Area of  $ABCDE$  = area of  $\triangle DLG$  + area of trapezoid  $CLKB$  + area of  $\triangle ABK$  + area of  $\triangle AED$

$$\begin{aligned} &= \frac{1}{2} DL \times CL + \frac{CL + BK}{2} \times LK \\ &\quad + \frac{1}{2} BK \times AK + \frac{1}{2} AD \times EM \\ &= \left( 10.725 + \frac{13.1}{2} \times 8.7 + 10.26 + 32.895 \right) \text{ sq. ft.} \\ &= 110.865 \text{ sq. ft.} \end{aligned}$$

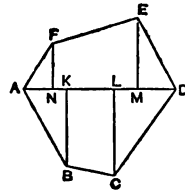


3.  $KL = AD - (AK + DL)$

$$= 18.4 \text{ ft.} - (4.7 + 5.3) \text{ ft.} = 8.4 \text{ ft.; and}$$

$MN = AD - (AN + DM) = 18.4 \text{ ft.} - (4.1 + 4.9) \text{ ft.} = 9.4 \text{ ft.};$   
 $\therefore$  area of  $ABCDEF$  = area of  $\triangle ANF$  + area of trapezoid  $FNME$  + area of  $\triangle EMD$  + area of  $\triangle AKB$  + area of trapezoid  $KBCL$  + area of  $\triangle DLC$

$$\begin{aligned} &= \frac{1}{2} AN \times NF + \frac{FN + EM}{2} \times MN + \frac{1}{2} MD \times ME \\ &\quad + \frac{1}{2} AK \times KB + \frac{KB + LC}{2} \times KL + \frac{1}{2} DL \times LC \\ &= \left( \frac{1}{2} \times 4.1 \times 4 + \frac{4+6}{2} \times 9.4 + \frac{1}{2} \times 4.9 \times 6 + \frac{1}{2} \times 4.7 \times 5 + \frac{5+7}{2} \times 8.4 + \frac{1}{2} \times 5.3 \times 7 \right) \text{ sq. ft.} \\ &= (8.2 + 47 + 14.7 + 11.75 + 50.4 + 18.55) \text{ sq. ft.} = 150.6 \text{ sq. ft.} \end{aligned}$$

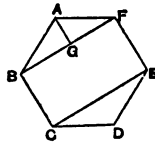


4. From  $A$  draw  $AG$  perpendicular to  $BF$ . Then (by Euc. I. 5 and 26)  $BG = GF = \frac{1}{2} BF = 32.2 \text{ ft.}$

$$\begin{aligned} \therefore (\text{by Euc. I. 47}) \quad AG &= \sqrt{(AB^2 - BG^2)} \\ &= \sqrt{\{(57.8 + 32.2)(57.8 - 32.2) \text{ sq. ft.}\}} = \sqrt{(90 \times 25.6 \text{ sq. ft.})} \\ &= \sqrt{(2304 \text{ sq. ft.})} = 48 \text{ ft.; and (by Euc. I. 8)} \end{aligned}$$

$$\triangle ABF = \triangle CDE; \therefore \triangle ABF + \triangle CDE = 2 \triangle ABF;$$

$$\begin{aligned} \therefore \text{area of } ABCDEF &= \text{area of } \triangle ABF + \text{area of } \triangle CDE + \text{area of rectangle } BE \\ &= \frac{1}{2} \times 2 \times BF \times AG + BF \times BC = BF(AG + BC) = 64.4(48 + 57.8) \text{ sq. ft.} \\ &= 6813.52 \text{ sq. ft.} \end{aligned}$$

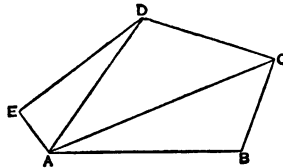


5. Join  $A, D$ . Then (by Euc. I. 47)

$$\begin{aligned} AD &= \sqrt{(DE^2 + AE^2)} = \sqrt{\{(144 + 25) \text{ sq. ft.}\}} \\ &= \sqrt{(169 \text{ sq. ft.})} = 13 \text{ ft.} \end{aligned}$$

Now area of  $\triangle AED = \frac{1}{2} \times AE \times ED$

$$= \frac{1}{2} \times 5 \times 12 \text{ sq. ft.} = 30 \text{ sq. ft.};$$



and area of  $\triangle ADC$

$$\begin{aligned}
 &= \frac{1}{2} \sqrt{[(13+10)^2 \text{ sq. ft.} - 17^2 \text{ sq. ft.}]\{17^2 \text{ sq. ft.} - (13-10)^2 \text{ sq. ft.}\}} \\
 &= \frac{1}{2} \sqrt{(23^2 - 17^2) \text{ sq. ft.} \times (17^2 - 3^2) \text{ sq. ft.}} \\
 &= \frac{1}{2} \sqrt{\{40 \times 6 \text{ sq. ft.} \times 20 \times 14 \text{ sq. ft.}\}} \\
 &= \frac{1}{2} \sqrt{(20 \times 2 \times 2 \times 8 \text{ sq. ft.} \times 20 \times 14 \text{ sq. ft.})} \\
 &= 10 \sqrt{42 \text{ sq. ft.}} = 10 \times 6.48074 \text{ sq. ft.} = 64.8074;
 \end{aligned}$$

and area of  $\triangle ABC = \frac{1}{2} \sqrt{[(17+14)^2 - 7^2] \text{ sq. ft.} \times \{7^2 - (17-14)^2\} \text{ sq. ft.}}$   
 $= \frac{1}{2} \sqrt{\{(31^2 - 7^2) \text{ sq. ft.} \times (7^2 - 3^2) \text{ sq. ft.}\}} = \frac{1}{2} \sqrt{\{(88 \times 24 \text{ sq. ft.}) \times (10 \times 4 \text{ sq. ft.})\}}$   
 $= \frac{1}{2} \sqrt{(88 \times 4 \times 6 \text{ sq. ft.} \times 10 \times 4 \text{ sq. ft.})} = \sqrt{2280 \text{ sq. ft.}} = 47.7493 \text{ sq. ft.};$   
 $\therefore \text{area of } ABCDE = (80 + 64.8074 + 47.7493) \text{ sq. ft.} = 142.5567 \text{ sq. ft.}$   
 $= 142.557 \text{ sq. ft. nearly.}$

6. (Vide diag. to the following.) It is proved in Euc. IV. 15, that  $\triangle AOB$ ,  $BOC$ ,  $COD$ ,  $DOE$ ,  $EOF$ , and  $FOA$  are all equal equilateral  $\triangle$ 's;  $\therefore$  the sum of their areas =  $6 \times$  area of one of them;

$\therefore$  area of  $ABCDEF = 6 \times$  area of  $\triangle COD$

$$= 6 \times \frac{CD^2}{4} \sqrt{3} = 6 \times \frac{14^2}{4} \times 1.73205 \text{ sq. ft.} = 1039.23 \text{ sq. ft.}$$

Or; it is shown in Art. 167 of the Mensuration that the area of a hexagon, whose side is one foot, is  $\frac{3}{2} \sqrt{3}$ ; therefore the area of a hexagon whose side is 20 feet, is  $20^2 \times \frac{3}{2} \sqrt{3}$ .

7. It is proved in Euc. IV. 15, that  $CD = OC = \frac{1}{2}$  diameter of circle = 50 ft.  $\therefore$  (as in preceding) area of  $ABCDEF = 6 \times \frac{14^2}{4} \sqrt{3} \text{ sq. ft.} = 3750 \times 1.73205 \text{ sq. ft.} = 6495.1875 \text{ sq. ft.} = 6495.2 \text{ sq. ft. nearly.}$

8. (As in 6) area of field =  $6 \times \frac{14^2}{4} \sqrt{3} \text{ sq. ch.}$   
 $= 150 \times 1.73205 \text{ sq. ch.} = 259.8075 \text{ sq. ch.} = 259.81 \text{ sq. ch. nearly.}$

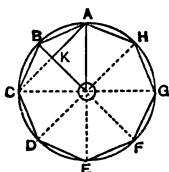
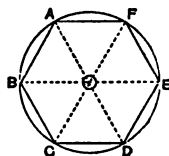
9. Join the angular points  $A, B, C, D, E, F, G$ , and  $H$  to  $O$ , the centre of the circle. Then (by Euc. I. 8), the eight  $\triangle AOB, BOC, COD, DOE, EOF, FOG, GOH$ , and  $HOA$  are all equal to one another; and  $\therefore$  area of octagon =  $8 \times$  area of  $\triangle AOB$ . Now join  $A, C$ . (By Euc. I. 8, 4), the angles at  $K$  are right angles, and  $KA = KC = \frac{1}{2} AC$ . But angle  $AOB$  is also a right angle as the arc  $AC$  is one-fourth of the circumference,

$$\therefore AC \text{ (by Euc. I. 47)} = \sqrt{(AO^2 + OC^2)} = \sqrt{(AO^2 + AO^2)}$$

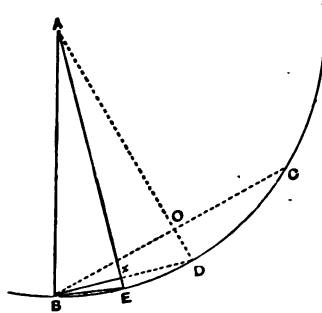
$$= \sqrt{2AO^2} = AO \sqrt{2} = 1 \text{ ft.} \times \sqrt{2} = \sqrt{2} \text{ ft.}; \therefore AK = \frac{\sqrt{2}}{2} \text{ ft.};$$

$$\therefore \text{area of } \triangle AOB = \frac{1}{2} OB \times AK = \frac{1}{2} \times 1 \text{ ft.} \times \frac{\sqrt{2}}{2} \text{ ft.} = \frac{\sqrt{2}}{4} \text{ sq. ft.};$$

$$\therefore \text{area of the octagon} = 8 \times \frac{\sqrt{2}}{4} \text{ sq. ft.} = 2\sqrt{2} \text{ sq. ft.}$$



10. Let  $BEDC$  be a circle, the radius of which,  $AB=1$  ft. Let  $BC$  be the side of a regular hexagon inscribed (by Euc. IV. 15) in the circle. Then  $BC=AB=1$  ft. (vide Euc. IV. 15). Draw  $AO$  perpendicular to  $AC$  (Euc. I. 12), and produce it to meet the circumference in  $D$ . Then (by Euc. III. 2),  $BO=OC=\frac{1}{2}BC=\frac{1}{2}$  ft. Join  $B, D$ ; then (by Euc. III. 28), it may be shewn that arc  $BED$ =arc  $DC=\frac{1}{2}$  arc  $BDC$ . But by construction, arc  $BDC=\frac{1}{2}$  of the whole circumference of the circle;  $\therefore$  arc  $BED=\frac{1}{4}$  of the whole circumference. Hence  $BD$  is the side of a regular duodecagon inscribed in the circle. In the same manner, by drawing  $AX$  perpendicular to  $BD$ , and producing it to meet the circumference in  $E$ , it may be shewn that  $BX=XD=\frac{1}{2}BD$ . Now join  $B, E$ , and  $BE$  is the side of a regular polygon of 24 sides inscribed in the circle. And (as in 6 and 9) we can shew that area of this polygon =  $24 \times$  area of  $\triangle ABE$ . (By Euc. I. 47),



$$AO = \sqrt{(AB^2 - BO^2)} = \sqrt{\left\{(1 - \frac{1}{4}) \text{ sq. ft.}\right\}} = \sqrt{\left(\frac{3}{4} \text{ sq. ft.}\right)} = \frac{\sqrt{3}}{2} \text{ ft.};$$

$$\therefore OD = AD - AO = \left(1 - \frac{\sqrt{3}}{2}\right) \text{ ft.} = \frac{2 - \sqrt{3}}{2} \text{ ft.};$$

$$\begin{aligned} \therefore BD^2 &= (BO^2 + OD^2) = \left(\frac{1}{2} \text{ ft.}\right)^2 + \left(\frac{2 - \sqrt{3}}{2} \text{ ft.}\right)^2 = \frac{1}{4} \text{ sq. ft.} + \frac{4 - 4\sqrt{3} + 3}{4} \text{ sq. ft.} \\ &= \frac{8 - 4\sqrt{3}}{4} \text{ sq. ft.} = (2 - \sqrt{3}) \text{ sq. ft.} = (2 - 1.732050803) \text{ sq. ft.} = .267949197 \text{ sq. ft.}; \end{aligned}$$

$$\therefore BD = \sqrt{(.267949197 \text{ sq. ft.})} = .51764 \text{ ft.}; \therefore BX = \frac{1}{2} BD = .25882 \text{ ft.}$$

$$\text{Now area of } \triangle ABE = \frac{1}{2} AE \times BX = \frac{1}{2} \times 1 \times .25882 \text{ sq. ft.} = .12941 \text{ sq. ft.};$$

$$\therefore \text{area of the regular polygon of 24 sides} = 24 \times .12941 \text{ sq. ft.} = 3.10584 \text{ sq. ft.}$$

Or; it is shown in Art. 99 of the Mensuration, by successive calculation, that  $BD = .51764$  ft.; and the solution may be completed as above.

## CHAPTER XVI.

$$1. \text{ Area of circle} = \frac{1}{2} \times 21^2 \text{ sq. ft.} = 22 \times 3 \times 21 \text{ sq. ft.} = 1386 \text{ sq. ft.}$$

$$4. \text{ Area of circle} = 3.1416 \times 25^2 \text{ sq. ft.} = 3.1416 \times 625 \text{ sq. ft.} = 1963.5 \text{ sq. ft.}$$

$$7. \text{ Area} = \frac{1}{2} \times \text{radius}^2;$$

$$\therefore \text{radius}^2 = \text{area} \times \frac{2}{\pi} = 100 \text{ sq. ft.} \times \frac{2}{\pi} = \frac{200}{\pi} \text{ sq. ft.} = 31.83 \text{ sq. ft.}$$

$$\therefore \text{radius} = \sqrt{(31.83 \text{ sq. ft.})} = 5.64 \text{ ft.}$$

$$10. (\text{As in 7}) \text{radius}^2 = \frac{\text{area}}{3.1416} = \frac{500}{3.1416} \text{ sq. ft.} = 159.154570 \text{ sq. ft.};$$

$$\therefore \text{radius} = \sqrt{(159.154570 \text{ sq. ft.})} = 12.616 \text{ ft.}$$

13. Area of ring = area of outer circle - area of inner circle  
 $= 8 \cdot 1416 \times 16^2 \text{ sq. ft.} - 8 \cdot 1416 \times 14^2 \text{ sq. ft.} = 8 \cdot 1416 (16^2 - 14^2) \text{ sq. ft.}$   
 $= 8 \cdot 1416 (16 + 14) (16 - 14) \text{ sq. ft.} = 8 \cdot 1416 \times 30 \times 2 \text{ sq. ft.} = 188 \cdot 496 \text{ sq. ft.}$

16. Area of outer circle = area of ring + area of inner circle =  $100 \text{ sq. in.} + 8 \cdot 1416 \times 14^2 \text{ sq. in.} = (100 + 615 \cdot 7536) \text{ sq. in.} = 715 \cdot 7536 \text{ sq. in.}$ ; but radius<sup>2</sup> of outer circle (as in 7) = area of outer circle  $\div 8 \cdot 1416$   
 $= 715 \cdot 7536 \text{ sq. in.} \div 8 \cdot 1416 = 227 \cdot 830914 \text{ sq. in.}$ ;  
 $\therefore$  radius of outer circle =  $\sqrt{(227 \cdot 830914 \text{ sq. in.})} = 15 \cdot 094 \text{ in.}$

17. Area of inner circle = area of outer circle - area of ring  
 $= 8 \cdot 1416 \times 18^2 \text{ sq. ft.} - 300 \text{ sq. ft.} = (1017 \cdot 8784 - 300) \text{ sq. ft.} = 717 \cdot 8784 \text{ sq. ft.}$ ;  
 $\therefore$  (as in preceding) radius of inner circle  
 $= \sqrt{(717 \cdot 8784 \text{ sq. ft.} \div 8 \cdot 1416)} = \sqrt{(228 \cdot 507258 \text{ sq. ft.})} = 15 \cdot 116 \text{ ft.}$

18. Area of whole circle =  $28 \text{ sq. yds.} = 28 \times 9 \text{ sq. ft.}$ ;  
 $\therefore$  radius =  $\sqrt{(28 \times 9 \text{ sq. ft.} \div 8 \cdot 1416)} = \sqrt{(80 \cdot 213903 \text{ sq. ft.})} = 8 \cdot 956 \text{ ft.}$

19. Circum. =  $2 \times 8 \cdot 1416 \times \text{radius}$ ;

$$\therefore \text{radius} = \frac{\text{circum.}}{2 \times 8 \cdot 1416} = \frac{700}{2 \times 8 \cdot 1416} \text{ ft.} = \frac{850}{8 \cdot 1416} \text{ ft.};$$

$$\therefore \text{area of circle} = 8 \cdot 1416 \times \left( \frac{850}{8 \cdot 1416} \text{ ft.} \right)^2 = 8 \cdot 1416 \times \frac{850 \times 850}{8 \cdot 1416 \times 8 \cdot 1416} \text{ sq. ft.}$$

$$= \frac{122500}{8 \cdot 1416} = 38993 \text{ sq. ft.}$$

Or; use the rule in Art. 176 of the Mensuration;

$$\text{area} = \frac{700^2}{4 \times 8 \cdot 1416} = \frac{122500}{8 \cdot 1416} = 38993.$$

21. Radius =  $\sqrt{(\text{area} \div 8 \cdot 1416)}$  (vide 10)

$$= \sqrt{(\frac{1}{2} \text{ ac.} \div 8 \cdot 1416)} = \sqrt{(2420 \times 9 \text{ sq. ft.} \div 8 \cdot 1416)},$$

$$\therefore \text{circum.} = 2 \times 8 \cdot 1416 \times \frac{\sqrt{2420 \times 9}}{\sqrt{8 \cdot 1416}} \text{ ft.} = 2 \times \sqrt{8 \cdot 1416} \times \sqrt{(2420 \times 9)} \text{ ft.}$$

$$= \sqrt{(4 \times 8 \cdot 1416 \times 2420 \times 9)} \text{ ft.} = \sqrt{(273696 \cdot 1920)} \text{ ft.} = 523 \cdot 16 \text{ ft. nearly.}$$

22. Area of rectangle =  $400 \times 256 =$  (by hyp.) area of circle;

$$\therefore \text{radius of circle} = (400 \times 256 \div 8 \cdot 1416)$$

$$\therefore \text{circum.} = 2 \times 8 \cdot 1416 \times \sqrt{(400 \times 256 \div 8 \cdot 1416)}$$

$$= (\text{as in 21}) \sqrt{(4 \times 8 \cdot 1416 \times 400 \times 256)} = \sqrt{(1286799 \cdot 36)} = 1134 \cdot 4 \text{ nearly.}$$

23. Area of 1<sup>st</sup> circle =  $8 \cdot 1416 \times 8^2 \text{ sq. ft.}$ ;

$$\therefore \text{area of 2<sup>nd</sup> circle} = \frac{1}{2} \times 8 \cdot 1416 \times 8^2 \text{ sq. ft.};$$

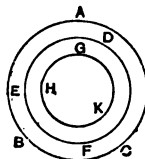
$$\therefore \text{radius of latter} = \sqrt{(\frac{1}{2} \times 8 \cdot 1416 \times 8^2 \text{ sq. ft.} \div 8 \cdot 1416)} = \sqrt{\left( \frac{8^2}{2} \text{ sq. ft.} \right)}$$

$$= \frac{8}{\sqrt{2}} \text{ ft.} = \frac{8\sqrt{2}}{\sqrt{2} \times \sqrt{2}} \text{ ft.} = \frac{8\sqrt{2}}{2} \text{ ft.} = 4 \times 1 \cdot 4142 \text{ ft.} = 5 \cdot 657 \text{ ft. nearly.}$$

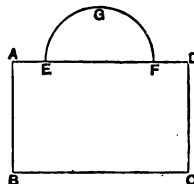
24. Radius required =  $\sqrt{\left(\frac{18^2}{5} \text{ sq. in.}\right)}$  (vide preceding) =  $\sqrt{\left(\frac{12^2}{5} \text{ sq. in.}\right)}$   
 =  $\sqrt{(64 \cdot 8 \text{ sq. in.})} = 8 \cdot 05 \text{ in. nearly.}$

25. Area of  $GHK = \frac{1}{3}$  area of  $ABC$ ;  
 and area of  $DEF = \frac{2}{3}$  area of  $ABC$ ;

$\therefore$  radius of  $GHK = \sqrt{\left(\frac{10^2}{3} \text{ sq. in.}\right)}$  (vide 23) =  $\frac{10}{3} \sqrt{3} \text{ in.}$   
 =  $\frac{10}{3} \times 1 \cdot 732 \text{ in.} = 5 \cdot 77 \text{ in.}$ , and radius of  $DEF$   
 =  $\sqrt{\left(\frac{10^2 \times 2}{3} \text{ sq. in.}\right)} = \sqrt{2} \times 5 \cdot 77 \text{ in.} = 8 \cdot 16 \text{ in.}$



26. Area of whole room  
 = area of rectangle  $AC$  + area of semicircle  $EGF$   
 =  $(25\frac{1}{2} \times 14\frac{1}{2}) \text{ sq. ft.} + \frac{1}{2} \times 3 \cdot 1416 \times (2\frac{1}{2})^2 \text{ sq. ft.}$   
 =  $(\frac{101}{2} \times \frac{29}{2}) \text{ sq. ft.} + \frac{3 \cdot 1416 \times 441}{2 \times 4} \text{ sq. ft.}$   
 =  $(\frac{2929}{2} + 3927 \times 441) \text{ sq. ft.}$   
 =  $(366 \cdot 125 + 173 \cdot 1807) \text{ sq. ft.} = 539 \cdot 3057 \text{ sq. ft.}$



27. Area of plate  
 =  $3 \cdot 1416 \times (\frac{3}{2} \text{ ft.})^2 = 3 \cdot 1416 \times 18^2 \text{ sq. in.};$   
 $\therefore$  pressure =  $3 \cdot 1416 \times 18 \times 18 \times 15 \text{ lbs.} = \frac{3 \cdot 1416 \times 18 \times 18 \times 15}{28 \times 4} \text{ cwt.}$   
 =  $1122 \times 9 \times 9 \times 15 \text{ cwt.} = 136 \cdot 323 \text{ cwt.} = 136 \text{ cwt. nearly.}$

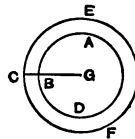
28. Area of court =  $3 \cdot 1416 \times 20^2 \text{ sq. ft.} = 3 \cdot 1416 \times 400 \text{ sq. ft.};$   
 $\therefore$  expense of paving =  $3 \cdot 1416 \times 400 \times 2s. 3d. = 3 \cdot 1416 \times 400 \times \frac{2 \cdot 3}{8} \text{ } \pounds$   
 =  $3 \cdot 1416 \times \frac{232}{2} = \pounds 141 \cdot 372 = \pounds 141 \cdot 7 \cdot 44s.$

29. Here  $GB = \frac{68 \text{ ft. } 10 \text{ in.}}{2} = 34 \frac{1}{2} \text{ ft.};$

and  $BC = 22 \text{ in.} = 1 \frac{1}{2} \text{ ft.}; \therefore CG = (\frac{68}{2} + \frac{1}{2}) \text{ ft.} = 34 \frac{1}{2} \text{ ft.}$

We are required to find area of the ring enclosed between the circumferences of circles  $ABD$  and  $ECF$ . The area of this ring = area of circle  $ECF$  - area of circle  $ABD$

=  $3 \cdot 1416 \times (34 \frac{1}{2} \text{ ft.})^2 - 3 \cdot 1416 \times (1 \frac{1}{2} \text{ ft.})^2$   
 =  $3 \cdot 1416 \times (\frac{137}{2} + \frac{1}{2}) \times (\frac{137}{2} - \frac{1}{2}) \text{ sq. ft.} = 3 \cdot 1416 \times 2 \frac{1}{2} \times \frac{1}{2} \text{ sq. ft.}$   
 =  $\frac{5236 \times 2332}{8} \text{ sq. ft.} = \frac{1221 \cdot 0352}{8} \text{ sq. ft.} = 407 \cdot 01 \text{ sq. ft. nearly.}$



30. (Vide diag. to preceding.) Here  $CG = 50 \text{ ft.}, BC = 10 \text{ ft.};$

$\therefore BG = CG - CB = (50 - 10) \text{ ft.} = 40 \text{ ft.};$

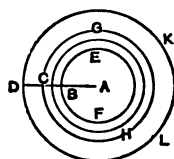
$\therefore$  area of circular ride = area of circle  $ECF$  - area of circle  $ABD$

=  $3 \cdot 1416 \times 50^2 \text{ sq. ft.} - 3 \cdot 1416 \times 40^2 \text{ sq. ft.} = 3 \cdot 1416 (50^2 - 40^2) \text{ sq. ft.}$   
 =  $3 \cdot 1416 \times 90 \times 10 \text{ sq. ft.};$

$\therefore$  cost =  $3 \cdot 1416 \times 90 \times 10 \times 4d. = 3 \cdot 1416 \times 90 \times 10 \times \frac{4}{16} \text{ } \pounds = \pounds 47 \cdot 124 = \pounds 47 \cdot 2 \cdot 48s.$

T. M. K.

31. Here circular ring enclosed between the circumferences of the circles  $GCH$  and  $EBF$ , represents the gravel walk. Hence  $DA=20$  yds.,  $DC=1$  yd., and  $CB=1$  yd.;  $\therefore AB=(20-2)$  yds. = 18 yds., and  $AC=(20-1)$  yds. = 19 yds.;  $\therefore$  area of gravel walk



$$\begin{aligned} &= \text{area of circle } GCH - \text{area of circle } EBF \\ &= 3.1416 \times 19^2 \text{ sq. yds.} - 3.1416 \times 18^2 \text{ sq. yds.} \\ &= 3.1416 (19^2 - 18^2) \text{ sq. yds.} = 3.1416 \times 37 \times 1 \text{ sq. yds.}; \end{aligned}$$

$$\begin{aligned} \therefore \text{area of portion to be turfed} &= 3.1416 \times 20^2 \text{ sq. yds.} - 3.1416 \times 37 \text{ sq. yds.} \\ &= 3.1416 (400 - 37) \text{ sq. yds.} = 3.1416 \times 363 \text{ sq. yds.}; \end{aligned}$$

$$\begin{aligned} \therefore \text{cost of turfing} &= 3.1416 \times 363 \times 4d. = 3.1416 \times 363 \times \frac{4}{4} \\ &= \frac{95.0834}{5} = \pounds 19.00668 = \pounds 19. 0s. 1.6032d. = \pounds 19. 0s. 1\frac{1}{2}d. \text{ nearly.} \end{aligned}$$

32. (Vide diag. to 29.) Here circum. of  $ECF=500$  ft., and that of  $ABD=420$  ft.;  $\therefore GC = \frac{500}{2 \times 3.1416}$  ft. (vide Chap. VIII. 9)  $= \frac{250}{3.1416}$  ft.;

$$\begin{aligned} \text{and } GB &= \frac{420}{2 \times 3.1416} \text{ ft.} = \frac{210}{3.1416} \text{ ft.}; \therefore \text{area of circular ring or road} \\ &= 3.1416 \times \left\{ \left( \frac{250}{3.1416} \right)^2 - \left( \frac{210}{3.1416} \right)^2 \right\} \text{ sq. ft.} = \frac{3.1416}{(3.1416)^2} \times (250^2 - 210^2) \text{ sq. ft.} \\ &= \frac{1}{3.1416} \times 460 \times 40 \text{ sq. ft.} = \frac{2300}{.3927} \text{ sq. ft.} = 5856.8 \text{ sq. ft.} = 5857 \text{ sq. ft. nearly.} \end{aligned}$$

33. Area of circle  $= 3.1416 \times 80^2 \text{ sq. ft.} = 20106.24 \text{ sq. ft.}$  (by hyp.) area of square;  $\therefore$  side of square  $= \sqrt{(20106.24 \text{ sq. ft.})} = 141.8 \text{ sq. ft. nearly.}$

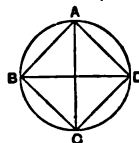
$$\begin{aligned} 34. \text{Area of square} &= 80^2 \text{ sq. ft.} = 6400 \text{ sq. ft.} = (\text{by hyp.}) \text{area of circle}; \\ \therefore \text{radius of circle} &= \sqrt{(6400 \text{ sq. ft.} \div 3.1416)} = \frac{1}{3.1416} \times \sqrt{(6400 \text{ sq. ft.} \times 3.1416)} \\ &= \frac{1}{3.1416} \times \sqrt{(20106.24 \text{ sq. ft.})} = \frac{141.8}{3.1416} \text{ ft.} = 45.1 \text{ ft.} \end{aligned}$$

35. Diam. of circle  $=$  side of square (vide Euc. IV. 8)  $= 16$  ft.;  $\therefore$  area of space required  $=$  area of square  $-$  area of circle  $= 16^2 \text{ sq. ft.} - 3.1416 \times 8^2 \text{ sq. ft.} = 8^2 (2^2 - 3.1416) \text{ sq. ft.} = 54.9376 \text{ sq. ft.}$

36.  $AC$  is the diam. of the circle (Euc. IV. 9). Now (by Euc. I. 47)  $AC^2 = AB^2 + BC^2 = 2AB^2$ ;

$$\therefore AC = AB\sqrt{2} = 18 \text{ ft.} \times \sqrt{2};$$

$$\begin{aligned} \therefore \text{area of space required} &= 3.1416 (9\sqrt{2})^2 \text{ sq. ft.} - 18^2 \text{ sq. ft.} \\ &= (3.1416 \times 162 - 324) \text{ sq. ft.} = (508.9392 - 324) \text{ sq. ft.} \\ &= 184.9392 \text{ sq. ft.} \end{aligned}$$



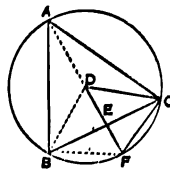
37. (By Euc. I. 47.) Hypotenuse

$$= \sqrt{\{(27^2 + 43^2) \text{ sq. ft.}\}} = \sqrt{\{(729 + 1849) \text{ sq. ft.}\}} = \sqrt{(2578 \text{ sq. ft.})};$$

$$\begin{aligned} \therefore \text{area of circle} &= 3.1416 \times \left\{ \frac{1}{2} \sqrt{(2578 \text{ sq. ft.})} \right\}^2 = 3.1416 \times \frac{2578}{2} \text{ sq. ft.} \\ &= 7854 \times 2578 \text{ sq. ft.} = 2024.7612 \text{ sq. ft.} = 2024.8 \text{ sq. ft. nearly.} \end{aligned}$$

38. Area of semicircle  $= \frac{1}{2} \times 3.1416 \times \text{radius}^2 = \frac{1}{2} \times 3.1416 \times \frac{1}{4} \text{ of Diam.}^2$   
 $= .8927 \times \text{Diam.}^2 = (\text{by hyp.}) 645 \text{ sq. ft.};$   
 $\therefore \text{Diam.} = \sqrt{(645 \text{ sq. ft.} \div .8927)} = \sqrt{(215 \text{ sq. ft.} \div .1809)}$   
 $= \sqrt{(1042.4752 \text{ sq. ft.})} \text{ nearly} = 40.52 \text{ ft.};$   
 $\therefore \text{perimeter of semicircle} = \text{circum. of semicircle} + \text{diam.}$   
 $= \frac{1}{2} \times 3.1416 \times 40.52 \text{ ft.} + 40.52 \text{ ft.} = 40.52 (1.5708 + 1) \text{ ft.} = 40.52 \times 2.5708 \text{ ft.}$   
 $= 104.168816 \text{ ft.} = 104.2 \text{ ft. nearly.}$

39. Let  $ABCD$  be an equil.  $\Delta$  inscribed in the circle  $ABC$ , the centre of which is  $D$ . From  $D$  draw  $DE$  perpendicular to  $BC$ , and produce it to meet the circum. in  $F$ . Join  $D, C$ ; and  $C, F$ . Now by joining  $D, A$ ; and  $D, B$ , we can shew that  $\Delta ADB = \Delta ADC = \Delta CDB$  (by Euc. I. Def. 15 and prop. 8); but (by Euc. I. 26)  $\Delta DBC = 2 \Delta DEC$ ;  $\therefore$  whole  $\Delta ABC = 6 \Delta DEC$ . Now since (by hyp.) arc  $BFC$  is  $\frac{1}{2}$  of the whole circum. of circle  $ABC$ ;  $\therefore FC$  is  $\frac{1}{2}$  of this circum. (by joining  $B, F$ , and applying Euc. I. 8 and III. 28);  $\therefore FC$  is the side of a regular hexagon inscribed in the circle  $ABC$ ; wherefore  $\Delta DFC$  is equilateral (by Euc. IV. 15) wherefore  $DF = FC = DC = 1 \text{ ft.}$  Then (by Euc. I. 5 and 26)  $\Delta DEC = \Delta FEC = \frac{1}{2} \Delta DFC$ ;  $\therefore$  area of  $\Delta ABC$  (see note to Example 24 of Chapter xiv.)



$$= 6 \Delta DFC = 6 \times \frac{DC^2}{4} \cdot \sqrt{3} = 6 \times \frac{1}{4} \text{ ft.} \times 1.732 = 6 \times .433 \text{ ft.} = 1.299 \text{ sq. ft.};$$

$$\text{and area of circle} = 3.1416 \times (1 \text{ ft.})^2 = 3.1416 \text{ sq. ft.};$$

$$\therefore \text{area of space required} = (3.1416 - 1.299) \text{ sq. ft.} = 1.8426 \text{ sq. ft.}$$

(Note. Hence area of an equil.  $\Delta$  inscribed in a circle  $= 8 \times$  area of an equil.  $\Delta$ , each of the equal sides of which is equal to the radius of the circle, i. e., in other words,  $= 8 \times \frac{\text{radius}^2}{4} \sqrt{3}.$ )

40. To be solved like Example 37.

41. Perimeter of rectangle  $= 2 (8 + 7) \text{ ft.} = 80 \text{ ft.} = (\text{by hyp.})$  perimeter, i. e. circum. of circle;  $\therefore$  radius of circle  $= (80 \text{ ft.} \div 2 \times 3.1416) = (15 \text{ ft.} \div 3.1416)$ ;  
 $\therefore \text{area of circle} = 3.1416 \times \frac{225}{(3.1416)^2} \text{ sq. ft.} = \frac{225}{3.1416} \text{ sq. ft.} = 71.62 \text{ sq. ft. nearly.}$

42. Perimeter of  $\Delta = (18 + 14 + 15) \text{ ft.} = 42 \text{ ft.} = \text{perimeter, i. e. circum. of circle; } \therefore$  radius of circle  $= (42 \text{ ft.} \div 2 \times 3.1416) = (21 \text{ ft.} \div 3.1416)$ ;  
 $\therefore \text{area of circle} = 3.1416 \times \frac{21 \times 21}{(3.1416)^2} \text{ sq. ft.} = \frac{21}{3.1416} \text{ sq. ft.} = 140.874 \text{ sq. ft.}$

43. Perimeter of rectangle  $= 2 (18 + 10) \text{ ft.} = 56 \text{ ft.} = \text{perimeter, i. e. circum. of circle; } \therefore$  area of circle (as in 41)

$$= \frac{28 \times 28}{3.1416} \text{ sq. ft.} = \frac{14}{.0681} \text{ sq. ft.} = 249.55 \text{ sq. ft.};$$

and area of rectangle  $= 18 \times 10 \text{ sq. ft.} = 180 \text{ sq. ft.}; \therefore$  area of circle  $>$  area of a rectangle of the same perimeter.

$$\begin{aligned}
 45. \text{ Area of } \Delta &= \frac{1}{2} \sqrt{\{(10+9)^2 - 17^2\} \text{ sq. ft.} \times \{17^2 - (10-9)^2\} \text{ sq. ft.}} \\
 &= \frac{1}{2} \sqrt{\{19^2 - 17^2\} \text{ sq. ft.} \times \{17^2 - 1^2\} \text{ sq. ft.}} \\
 &= \frac{1}{2} \sqrt{\{36 \times 2\} \text{ sq. ft.} \times \{18 \times 16 \text{ sq. ft.}\}} \\
 &= \frac{1}{2} \sqrt{36 \times 2 \text{ sq. ft.} \times 2 \times 9 \times 16 \text{ sq. ft.}} = \frac{1}{2} \sqrt{6^2 \times 2^2 \times 3^2 \times 4^2 \text{ sq. ft.}} \\
 &= \frac{1}{2} \times 6 \times 2 \times 3 \times 4 \text{ sq. ft.} = 36 \text{ sq. ft.}
 \end{aligned}$$

Again, perimeter of  $\Delta = (9+10+17) \text{ ft.} = 36 \text{ ft.} = (\text{by hyp.})$  perimeter, i. e. circum. of circle;  $\therefore$  area of circle (as in 42)

$$\begin{aligned}
 &= \frac{18 \times 18}{8 \cdot 1416} \text{ sq. ft.} = \frac{27}{\cdot 2618} \text{ sq. ft.} = 103 \cdot 13 \text{ sq. ft.};
 \end{aligned}$$

$\therefore$  area of circle  $>$  area of  $\Delta$  of same perimeter.

47. Area of rectangle  $= 15 \times 12 \text{ sq. ft.} = 180 \text{ sq. ft.} = (\text{by hyp.})$  area of circle;  $\therefore$  radius of circle  $= \sqrt{(180 \text{ sq. ft.} \div 3 \cdot 1416)}$ ;  $\therefore$  perimeter, i. e. circum.

$$\begin{aligned}
 \text{of circle} &= 2 \times 3 \cdot 1416 \times \frac{\sqrt{180}}{\sqrt{3 \cdot 1416}} \text{ ft.} = \sqrt{(4 \times 3 \cdot 1416 \times 180 \text{ sq. ft.})} \\
 &= \sqrt{(2261 \cdot 9520 \text{ sq. ft.})} = 47 \cdot 56 \text{ ft. nearly.}
 \end{aligned}$$

But perimeter of rectangle  $= 2(15+12) \text{ ft.} = 54 \text{ ft.}$ ;  $\therefore$  perimeter of circle  $<$  perimeter of rectangle of same area.

$$\begin{aligned}
 49. \text{ Area of } \Delta &= \frac{1}{2} \sqrt{\{(6+5)^2 - 7^2\} \text{ sq. ft.} \times \{7^2 - (6-5)^2\} \text{ sq. ft.}} \\
 &= \frac{1}{2} \sqrt{\{11^2 - 7^2\} \text{ sq. ft.} \times \{7^2 - 1\} \text{ sq. ft.}} = \frac{1}{2} \sqrt{(18 \times 4 \text{ sq. ft.} \times 8 \times 6 \text{ sq. ft.})} \\
 &= \frac{1}{2} \sqrt{(9 \times 2 \times 4 \text{ sq. ft.} \times 2 \times 4 \times 6 \text{ sq. ft.})} = \frac{1}{2} \sqrt{(3^2 \times 2^2 \times 4^2 \times 6 \text{ sq. ft.})} \\
 &= \frac{1}{2} \times 3 \times 2 \times 4 \sqrt{6} \text{ sq. ft.} = 6 \times 2 \cdot 45 \text{ sq. ft.} = 14 \cdot 7 \text{ sq. ft.};
 \end{aligned}$$

$\therefore$  (by hyp.) area of circle also  $= 14 \cdot 7 \text{ sq. ft.}$ ;  $\therefore$  (as in preceding) perimeter of circle  $= \sqrt{(4 \times 14 \cdot 7 \times 3 \cdot 1416 \text{ sq. ft.})} = \sqrt{(184 \cdot 72608 \text{ sq. ft.})} = 13 \cdot 591 \text{ ft.}$  But perimeter of  $\Delta = (5+6+7) \text{ ft.} = 18 \text{ ft.}$ ;  $\therefore$  perimeter of circle  $<$  perimeter of  $\Delta$  of same area.

51. (Vide diag. to 36.)  $AC$  or the diam. of the circle

$$= \frac{4}{3 \cdot 1416} \text{ ft.} = \frac{1}{\cdot 7854} \text{ ft.} = 1 \cdot 2732 \text{ ft.}$$

Now (by Euc. I. 47)  $AB^2 + BC^2$ , i. e.  $2AB^2 = AC^2 = (1 \cdot 2732 \text{ ft.})^2$ ;

$$\therefore AB^2 = \frac{1}{2} (1 \cdot 2732 \text{ ft.})^2 = \cdot 8105191 \text{ ft.} = \cdot 81 \text{ sq. ft. nearly.}$$

But area of square  $ABCD = AB^2$ ;  $\therefore$  area of  $ABCD = \cdot 81 \text{ sq. ft. nearly.}$

#### CHAPTER XVII.

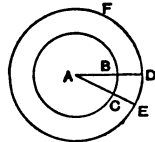
We shall in the following examples take  $a$ ,  $a'$ , and  $A$ , to represent respectively the area of the sector, the area of the circle, and the angle of the sector.

1. (By Euc. VI. 33),  $a : a' :: 25^\circ : 360^\circ$ ;

$$\therefore a = \frac{a' \times 25^\circ}{360^\circ} = \frac{3 \cdot 1416 \times 24^2 \times 25}{360} \text{ sq. ft.} = 125 \cdot 664 \text{ sq. ft.}$$

4. Area of sector  $DAE$

$$= \frac{\text{area of circle } DEF \times 40^\circ}{360^\circ} \text{ (vide 1)}$$



$$\begin{aligned}
 &= \frac{3 \cdot 1416 \times 15^2 \times 40}{360} \text{ sq. ft.} = 3 \cdot 1416 \times 25 \text{ sq. ft.}; \text{ and area of sector } BAC \\
 &= \frac{\text{area of circle } ABC \times 40^\circ}{360^\circ} = \frac{3 \cdot 1416 \times 10^2 \times 40}{360} \text{ sq. ft.} = 3 \cdot 1416 \times 1\frac{1}{3} \text{ sq. ft.}; \\
 \therefore \text{ area of } BCDE &= \text{area of sector } DAE - \text{area of sector } BAC \\
 &= (3 \cdot 1416 \times 25 - 3 \cdot 1416 \times 1\frac{1}{3}) \text{ sq. ft.} = 3 \cdot 1416 (25 - 1\frac{1}{3}) \text{ sq. ft.} \\
 &= \frac{130 \cdot 9}{3} \text{ sq. ft.} = 43 \cdot 63 \text{ sq. ft. nearly.}
 \end{aligned}$$

$$\begin{aligned}
 6. \quad &(\text{By Euc. VI. 33}) \quad a' : a :: 360^\circ : 50^\circ; \\
 \therefore a' &= \frac{150 \text{ sq. ft.} \times 360^\circ}{50^\circ} = 1080 \text{ sq. ft.};
 \end{aligned}$$

$$\begin{aligned}
 \therefore \text{ radius of circle} &= \sqrt{(1080 \text{ sq. ft.} \div 3 \cdot 1416)} = \sqrt{(45 \text{ sq. ft.} \div 1309)} \\
 &= \sqrt{(343 \cdot 7738 \text{ sq. ft.})} = 18 \cdot 54 \text{ ft.}
 \end{aligned}$$

$$\begin{aligned}
 7. \quad \text{Area of circle} &= \frac{360}{40} \times \text{area of sector} = 9 \times 230 = 2070 \text{ sq. ft.}; \\
 \therefore \text{ radius} &= \sqrt{2070 \div 3 \cdot 1416} = 25 \cdot 67 \text{ ft.};
 \end{aligned}$$

$$\begin{aligned}
 \text{and,} \quad \text{arc of sector} \times \text{radius} &= 2 \times \text{area of sector}; \\
 \therefore \text{ arc of sector} &= \frac{2 \times 230}{25 \cdot 67} = 17 \cdot 92 \text{ ft.}; \\
 \therefore \text{ whole perimeter of sector} &= 17 \cdot 92 + 2 \times 25 \cdot 67 \\
 &= 69 \cdot 26 \text{ ft.}
 \end{aligned}$$

$$\begin{aligned}
 8. \quad &(\text{By Euc. VI. 33}), \quad A : 360^\circ : a : a'; \\
 \therefore A &= \frac{a}{a'} \times 360^\circ = \frac{45 \times 360^\circ}{3 \cdot 1416 \times 64} = \frac{675^\circ}{8 \cdot 3776} = 80^\circ \cdot 57.
 \end{aligned}$$

9 and 10. Use the rule, Arc  $\times$  radius  $= 2 \times$  Area (Art. 181).

$$\begin{aligned}
 11. \quad &(\text{By Euc. VI. 33}), \quad A : 360^\circ : a : a'; \\
 \therefore A &= \frac{a}{a'} \times 360^\circ = \frac{125 \times 360^\circ}{400} = \frac{225^\circ}{2} = 112^\circ \cdot 5.
 \end{aligned}$$

$$\begin{aligned}
 12. \quad &\text{Radius of circle} \\
 &= \sqrt{(700 \text{ sq. ft.} \div 3 \cdot 1416)} = \sqrt{(25 \text{ sq. ft.} \div 1122)} = \sqrt{(222 \cdot 8163 \text{ sq. ft.})} = 14 \cdot 92 \text{ ft.} \\
 \text{But } \frac{\text{arc} \times \text{radius}}{2} &= \text{area of sector} = 115 \text{ sq. ft.};
 \end{aligned}$$

$$\therefore \text{ arc} = \frac{2 \times 115}{14 \cdot 92} \text{ ft.} = \frac{115}{7 \cdot 46} \text{ ft.} = 15 \cdot 41 \text{ ft.}$$

13. Let  $ABCF$  be a circle. Here  $BC = 58$  in., draw the diam.  $AF$  perpendicular to  $BC$ . Hence

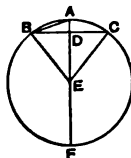
$$AE = EB = 100 \text{ in.};$$

$$\therefore BD = DC \text{ (Euc. III. 8)} = \frac{1}{2} BC = 29 \text{ in.}$$

Now (by Euc. I. 47),

$$\begin{aligned}
 DE &= \sqrt{(BE^2 - BD^2)} = \sqrt{\{(100^2 - 29^2) \text{ sq. in.}\}} \\
 &= \sqrt{(129 \times 71 \text{ sq. in.})} = \sqrt{(9159 \text{ sq. in.})};
 \end{aligned}$$

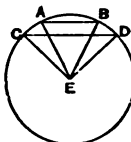
$$\therefore AD = AE - DE = 100 \text{ in.} - \sqrt{(9159 \text{ sq. in.})}. \quad \text{Again (by Euc. I. 47)}$$





$\therefore$  area of segment  $AB = \frac{1}{2}$  area of circle  $ACDB - 25\sqrt{3}$  sq. ft.;  $\therefore$  area of segments  $AB$  and  $CD$  together  $= \frac{1}{2}$  area of circle  $ACDB - 50\sqrt{3}$  sq. ft.;  $\therefore$  area of zone  $ACDB =$  area of circle  $ACDB -$  area of segments  $AB$  and  $CD =$  area of circle  $-\frac{1}{2}$  area of circle  $+ 50\sqrt{3}$  sq. ft.  $= \frac{1}{2}$  area of circle  $+ 50\sqrt{3}$  sq. ft.  $= (\frac{1}{2} \times 3 \cdot 1416 \times 100 + 50 \times 1 \cdot 732)$  sq. ft.  $= (209 \cdot 44 + 86 \cdot 6)$  sq. ft.  $= 296 \cdot 04$  sq. ft.

18. Here  $EC = EA = EB = ED = 12$  ft.,  $\angle AEB = 60^\circ$ , and  $\angle CED = 90^\circ$ ;  $\therefore \triangle AEB$  is equilateral (by Euc. I. 5, 32, and 6). Now (by Euc. VI. 33) area of sector  $AEB : \text{area of circle } ABDC :: 60^\circ : 360^\circ$ ;  $\therefore$  area of sector  $AEB = \frac{1}{6}$  area of circle; and area of  $\triangle AEB$



$$= \frac{AE^2}{4} \sqrt{3} = \frac{144}{4} \sqrt{3} \text{ sq. ft.} = 36\sqrt{3} \text{ sq. ft.} = 36 \times 1 \cdot 732 \text{ sq. ft.};$$

$\therefore$  area of segment  $AB = \frac{1}{6}$  area of circle  $- 36 \times 1 \cdot 732$  sq. ft.

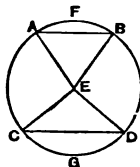
Again, area of sector  $CED : \text{area of circle } ABDC :: 90^\circ : 360^\circ$ ;  $\therefore$  area of sector  $CED = \frac{1}{4}$  area of circle  $ABDC$ ; and area of  $\triangle CED$

$$= \frac{EC \times ED}{2} = \frac{EC^2}{2} = \frac{144}{2} \text{ sq. ft.} = 72 \text{ sq. ft.};$$

$\therefore$  area of segment  $CABD = \frac{1}{4}$  area of circle  $ABDC - 72$  sq. ft.;

$\therefore$  area of zone  $CABD = \text{area of segment } CABD - \text{area of segment } AB$   
 $= \frac{1}{4}$  area of circle  $- 72$  sq. ft.  $- \frac{1}{6}$  area of circle  $+ 36 \times 1 \cdot 732$  sq. ft.  
 $= \text{area of circle } (\frac{1}{4} - \frac{1}{6}) - 36 (2 - 1 \cdot 732)$  sq. ft.  
 $= \frac{1}{12} \times 3 \cdot 1416 \times 144$  sq. ft.  $- 36 \times 268$  sq. ft.  
 $= (37 \cdot 6992 - 9 \cdot 648)$  sq. ft.  
 $= 28 \cdot 0512$  sq. ft.

19. Here  $EA = EB = EC = ED = 12$  ft.,  $\angle AEB = 60^\circ$ , and  $\angle CED = 90^\circ$ ;  $\therefore \triangle AEB$  is equilateral (by Euc. I. 5, 32, and 6). Then (as in preceding), area of segment  $AFB = \frac{1}{6}$  area of circle  $- 36 \times 1 \cdot 732$  sq. ft.; and area of segment  $CGD = \frac{1}{4}$  area of circle  $- 72$  sq. ft.  $\therefore$  Area of zone  $ACDB = \text{area of whole circle} - (\text{area of segment } AFB + \text{area of segment } CGD) = \text{area of circle} - (\frac{1}{6} \text{ area of circle} + \frac{1}{4} \text{ area of circle} - 36 \times 1 \cdot 732 \text{ sq. ft.} - 72 \text{ sq. ft.})$

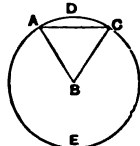


$$= \text{area of circle} - \frac{5}{12} \text{ area of circle} + 36 (1 \cdot 732 + 2) \text{ sq. ft.}$$

$$= \frac{7}{12} \text{ area of circle} + 36 \times 3 \cdot 732 \text{ sq. ft.} = (\frac{7}{12} \times 3 \cdot 1416 \times 144 + 36 \times 3 \cdot 732) \text{ sq. ft.}$$

$$= (263 \cdot 8944 + 134 \cdot 352) \text{ sq. ft.} = 398 \cdot 2464 \text{ sq. ft.} = 398 \cdot 25 \text{ sq. ft. nearly.}$$

20. Here  $AB = BC = CA = 15$  ft.;  $\therefore \triangle ABC$  is equilateral. Wherefore area of segment  $ADC = \frac{1}{2}$  area of circle  $- \frac{AB^2}{4} \sqrt{3}$



(vide 18)  $= (\frac{1}{2} \times 3 \cdot 1416 \times 225 - \frac{1}{4} \times 225 \times 1 \cdot 7320508) \text{ sq. ft.}$   
 $= 225 (5236 - 4330127) \text{ sq. ft.} = 225 \times 0905873 \text{ sq. ft.}$

$$= 20 \cdot 382 \text{ sq. ft. nearly.}$$

And area of segment  $AEC = \text{area of whole circle} - \text{area of segment } ADC = (3 \cdot 1416 \times 225 - 20 \cdot 382) \text{ sq. ft.}$

$$= (706 \cdot 86 - 20 \cdot 382) \text{ sq. ft.} = 686 \cdot 478 \text{ sq. ft.}$$

$$\begin{aligned}
 21. \text{ Area of segment} &= \frac{1}{2} \times 5 \text{ ft.} \times \sqrt{\left\{\frac{1}{2}(17 \cdot 3205)^2 + \frac{1}{2} \times 5^2\right\} \text{ sq. ft.}} \\
 &= \frac{1}{2} \text{ ft.} \times \sqrt{\{1299 \cdot 99972025 + 10\} \text{ sq. ft.}} \\
 &= \frac{1}{2} \text{ ft.} \times \sqrt{\{74 \cdot 9999300625 + 10\} \text{ sq. ft.}} \\
 &= \frac{1}{2} \text{ ft.} \times \sqrt{\{84 \cdot 9999300625 \text{ sq. ft.}\}} \\
 &= \frac{1}{2} \text{ ft.} \times 9 \cdot 21954 \text{ sq. ft.} = \frac{184 \cdot 3908}{8} \text{ sq. ft.} \\
 &= 61 \cdot 4636 \text{ sq. ft.} = 61 \cdot 464 \text{ sq. ft. nearly.}
 \end{aligned}$$

(Note. Questions of this kind may be solved without applying the rule given in the text book, thus: (vide diag. to 13). Here  $BC$  is given.  $\therefore BD$  is known, for it is  $\frac{1}{2}BC$  (by *Eucl. III. 3*); and as  $AD$  is also given, we can find  $AB$  (by *Eucl. I. 47*), for it  $= \sqrt{(AD^2 + DB^2)}$ . Hence length of arc  $BAC$  is known, because it  $= \frac{8AB - BC}{3}$ . Again (by joining  $B, F$  and applying

*Eucl. VI. 8 Cor.*) we have  $AF = \frac{AB^2}{AD}$ ; wherefore the radius  $AE, BE$ , or  $EC$  is known. Now we can find the area of the sector  $BEC$ , for it  $= \frac{1}{2} \times \text{length of arc } BAC \times BE$ . Lastly we can find area of  $\triangle BEC$  since we know its three sides. Hence area of segment  $BAC$  is found, because it  $= \text{area of sector } BEC - \text{area of } \triangle BEC$ .)

## CHAPTER XVIII.

- Area  $= \{(3 + 63) + 2(15 + 35) + 4(8 + 24 + 48)\} \text{ ft.} \times \frac{1}{2} \text{ of } 1 \text{ ft.}$   
 $= (66 + 100 + 320) \text{ ft.} \times \frac{1}{2} \text{ ft.} = 483 \text{ sq. ft.} = 162 \text{ sq. ft.}$
- Area  $= \{(4 + 140) + 2 \times 36 + 4(14 + 76)\} \text{ ft.} \times \frac{1}{2} \text{ of } 1 \text{ ft.}$   
 $= (144 + 72 + 860) \text{ ft.} \times \frac{1}{2} \text{ ft.} = 538 \text{ sq. ft.} = 192 \text{ sq. ft.}$
- Area  $= \{2(32 + 32) + 4(20 + 36 + 20)\} \text{ ft.} \times \frac{1}{2} \times 2 \text{ ft.}$   
 $= (128 + 304) \text{ ft.} \times \frac{1}{2} \text{ ft.} = 432 \times \frac{1}{2} \text{ sq. ft.} = 288 \text{ sq. ft.}$
- Area  $= \{2(4 + 8) + 4(1 \cdot 25 + 6 \cdot 75 + 6 \cdot 25)\} \text{ ft.} \times \frac{1}{2} \text{ of } 1 \text{ ft.}$   
 $= (24 + 57) \text{ ft.} \times \frac{1}{2} \text{ ft.} = 81 \times \frac{1}{2} \text{ sq. ft.} = 27 \text{ sq. ft.}$
- Area  $= \{(6 \cdot 082 + 6 \cdot 557) + 2(6 \cdot 245 + 6 \cdot 408)$   
 $+ 4(6 \cdot 164 + 6 \cdot 325 + 6 \cdot 481)\} \text{ ft.} \times \frac{1}{2} \times \frac{1}{2} \text{ ft.} = (12 \cdot 639 + 25 \cdot 296 + 75 \cdot 88) \text{ ft.} \times \frac{1}{2} \text{ ft.}$   
 $= 113 \cdot 815 \times \frac{1}{2} \text{ sq. ft.} = 56 \cdot 9075 \text{ sq. ft.}$
- Area  $= \{(2 \cdot 714 + 2 \cdot 884) + 2 \times 2 \cdot 802 + 4(2 \cdot 759 + 2 \cdot 844)\} \text{ ft.} \times \frac{1}{2} \times \frac{1}{2} \text{ ft.}$   
 $= (5 \cdot 598 + 5 \cdot 604 + 22 \cdot 412) \text{ ft.} \times \frac{1}{2} \text{ ft.} = 33 \cdot 614 \times \frac{1}{2} \text{ sq. ft.} = 8 \cdot 4035 \text{ sq. ft.}$
- Area  $= \{(14 \cdot 2 + 13 \cdot 7) + 2(15 \cdot 3 + 14 \cdot 5) + 4(14 \cdot 9 + 15 \cdot 1 + 14 \cdot 1)\} \text{ ft.} \times \frac{1}{2} \times 3 \text{ ft.}$   
 $= (27 \cdot 9 + 59 \cdot 6 + 176 \cdot 4) \text{ ft.} \times 1 \text{ ft.} = 263 \cdot 9 \text{ sq. ft.}$
- Area  $= \{(0 + 19 \cdot 52) + 2(2 \cdot 48 + 6 \cdot 24 + 11 \cdot 76)$   
 $+ 4(1 \cdot 11 + 4 \cdot 17 + 8 \cdot 75 + 15 \cdot 33)\} \text{ ft.} \times \frac{1}{2} \times 1 \text{ ft.} = (19 \cdot 52 + 40 \cdot 96 + 117 \cdot 44) \text{ ft.} \times \frac{1}{2} \text{ ft.}$   
 $= 177 \cdot 92 \times \frac{1}{2} \text{ sq. ft.} = 59 \cdot 306 \text{ sq. ft.} = 59 \cdot 307 \text{ sq. ft. nearly.}$

9. Area =  $\{(10 \cdot 204 + 7 \cdot 692) + 2(9 \cdot 434 + 8 \cdot 771 + 8 \cdot 197) + 4(9 \cdot 804 + 9 \cdot 090 + 8 \cdot 475 + 7 \cdot 937)\}$  ft.  $\times \frac{1}{3}$  of 1 ft.  
 =  $(17 \cdot 896 + 52 \cdot 804 + 141 \cdot 224)$  ft.  $\times \frac{1}{3}$  ft. =  $211 \cdot 924 \times \frac{1}{3}$  sq. ft. =  $70 \cdot 641\bar{3}$  sq. ft.
10. Area =  $\{(2 \cdot 4849 + 2 \cdot 9957) + 2(2 \cdot 6391 + 2 \cdot 7726 + 2 \cdot 8904) + 4(2 \cdot 5649 + 2 \cdot 7081 + 2 \cdot 8332 + 2 \cdot 9444)\}$  ft.  $\times \frac{1}{3}$  of 1 ft.  
 =  $(5 \cdot 4806 + 16 \cdot 6042 + 44 \cdot 2024)$  ft.  $\times \frac{1}{3}$  ft. =  $66 \cdot 2872 \times \frac{1}{3}$  sq. ft. =  $22 \cdot 0957\bar{3}$  sq. ft.
11. Area =  $\{(0 + 1) + 2(-6 + \cdot 8 + \cdot 9165 + \cdot 9798) + 4(\cdot 4359 + \cdot 7141 + \cdot 8660 + \cdot 9539 + \cdot 9950)\}$  ft.  $\times \frac{1}{3}$  of 1 ft.  
 =  $(1 + 6 \cdot 5926 + 15 \cdot 8596)$  ft.  $\times \frac{1}{3}$  ft. =  $2 \cdot 34522 \times \frac{1}{3}$  sq. ft. =  $\cdot 78174$  sq. ft.
12. Area  
 =  $\{(\frac{1}{16} + \frac{1}{16}) + 2(\frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8}) + 4(\frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4})\}$  ft.  $\times \frac{1}{3}$  of 1 ft.  
 =  $10\{(\frac{1}{16} + \frac{1}{16}) + 2(\frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8}) + 4(\frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4})\}$  ft.  $\times \frac{1}{3}$  ft.  
 =  $10(\frac{2}{16} + 2 \times \frac{4}{16} + 4 \times \frac{5}{16})$  ft.  $\times \frac{1}{3}$  ft.  
 =  $(15 + \cdot 5456 + 1 \cdot 3838)$  ft.  $\times \frac{1}{3}$  ft. =  $2 \cdot 0794 \times \frac{1}{3}$  sq. ft. =  $\cdot 6931$  sq. ft.

CHAPTER XIX.

1. 10 ft. on field = 1 in. on plan  $\therefore (10 \text{ ft.})^2$  or 100 sq. ft. on field = 1 sq. in. on plan;  $\therefore 1$  sq. ft. on field =  $\frac{1}{100}$  sq. in. on plan;  $\therefore 3600$  sq. yds. or  $3600 \times 9$  sq. ft. on field =  $\frac{1}{100} \times 3600 \times 9$ , i.e. 324 sq. in. on plan.
2. 20 ft. on field = 1 in. on plan;  $\therefore 400$  sq. ft. on field = 1 sq. in. on plan;  $\therefore 1$  sq. ft. on field =  $\frac{1}{400}$  sq. in. on plan;  $\therefore 6$  ac. or  $6 \times 4840 \times 9$  sq. ft. on field =  $\frac{1}{400} \times 6 \times 4840 \times 9$ , i.e. 653.4 sq. in. on plan.
3. 1 sq. in. on plan represents 1 sq. yd. or  $9 \times 144$  sq. in.;  $\therefore 1$  in. on plan represents  $\sqrt{9 \times 144}$  sq. in.), i.e. 36 in.
4. 1 sq. ft. i.e. 144 sq. in. on plan represent 10 ac., i.e.  
 $10 \times 4840 \times 9 \times 144$  sq. in.;  
 1 sq. in. on plan represents  $10 \times 4840 \times 9$  sq. in.;  $\therefore 1$  in. on plan represents  $\sqrt{(48400 \times 9 \text{ sq. in.})}$ , i.e. 660 in.
5. 10,000 sq. in. on field = 1 sq. in. on plan;  $\sqrt{(10000 \text{ sq. in.})}$  i.e. 100 in. on field = 1 in. on plan;  $\therefore 1$  in. on field =  $\frac{1}{100}$  in. on plan;  $\therefore 20$  yds. i.e.  $20 \times 3 \times 12$  in. on field =  $\frac{1}{100} \times 20 \times 3 \times 12$ , i.e. 7.2 in. on plan.
6. 100,000,000 sq. in. on estate = 1 sq. in. on plan;  $\therefore \sqrt{(100000000 \text{ sq. in.})}$  i.e. 10000 in. on estate = 1 in. on plan;  $\therefore 1$  in. on estate =  $\frac{1}{10000}$  in. on plan;  $\therefore 1$  mile on estate =  $\frac{1}{10000} \times 1760 \times 3 \times 12$ , i.e. 6.336 in. on plan.
7. Let  $l$  and  $b$  represent respectively the length and breadth of the rectangle. Then, by hyp.,  $b : l :: 2 : 3$ ;  $\therefore b = \frac{2}{3}l$ . But  $l \times b$ , i.e.  $l \times \frac{2}{3}l$  or  $\frac{2}{3}l^2$  = area of rectangle = 210 sq. ft.;  $\therefore l^2 = \frac{3}{2} \times 210$  sq. ft. = 315 sq. ft.;  $\therefore l = \sqrt{(315 \text{ sq. ft.})} = 17 \cdot 748$  ft.; and  $b = \frac{2}{3}l = \frac{2}{3} \times 17 \cdot 748$  ft. =  $11 \cdot 832$  ft.

8. Let  $a$ ,  $b$  and  $c$  represent the sides of the  $\Delta$ . Then, by hyp.,

$$a : b :: 18 : 15; \therefore b = \frac{5}{6}a; \text{ and } a : c :: 18 : 14; \therefore c = \frac{7}{9}a.$$

$$\therefore \text{area of } \Delta = \frac{1}{2} \sqrt{\{(\frac{5}{6}a + \frac{7}{9}a)^2 - a^2\} \{a^2 - (\frac{5}{6}a - \frac{7}{9}a)^2\}}$$

$$= \frac{1}{2} \sqrt{\{(\frac{17}{18}a)^2 - a^2\} \{a^2 - (\frac{1}{18}a)^2\}} = \frac{1}{2} \sqrt{\{(\frac{17}{18}a \times \frac{17}{18}a - \frac{1}{18}a \times \frac{1}{18}a)\}}$$

$$= \frac{1}{2} \sqrt{\left(6 \times 7 \times 16 \times 7 \times 2 \times 6 \times 2 \times \frac{a^4}{13^4}\right)} = \frac{1}{2} \times \sqrt{\left(6^2 \times 7^2 \times 4^2 \times 2^2 \times \frac{a^4}{13^4}\right)}$$

$$= \frac{1}{2} \times 6 \times 7 \times 4 \times 2 \times \frac{a^2}{13^2} = \frac{6 \times 7 \times 2 \times a^2}{169}.$$

But, by hyp., area of  $\Delta = 24276$  sq. ft.

$$\therefore \frac{6 \times 7 \times 2 \times a^2}{169} = 24276 \text{ sq. ft.};$$

$$\therefore a^2 = 24276 \times \frac{169}{6 \times 7 \times 2} \text{ sq. ft.} = 289 \times 169 \text{ sq. ft.};$$

$$\therefore a = \sqrt{(289 \times 169 \text{ sq. ft.})} = \sqrt{(17^2 \times 13^2 \text{ sq. ft.})} = 17 \times 13 \text{ ft.} = 221 \text{ ft.};$$

$$\text{and } b = \frac{5}{6}a = \frac{5}{6} \times 221 \text{ ft.} = 255 \text{ ft.}; \text{ and } c = \frac{7}{9}a = \frac{7}{9} \times 221 \text{ ft.} = 238 \text{ ft.}$$

Or thus; The area of the triangle whose sides are 18, 14, 15 feet, is 84 square feet. Thus by Art. 235,

$$84 : 24276 :: \text{square of } 18 : \text{square of corresponding side.}$$

10. Let  $x$  represent the side of the equil.  $\Delta$ ;  $\therefore 3x = \text{perimeter of the equil. } \Delta = (\text{by hyp.}), \text{ perimeter of square} = 4 \times \text{side of square}; \therefore \text{side of square} = \frac{3}{4}x$ ;  $\therefore \text{area of square} = \frac{9}{16}x^2$ . And area of equil.  $\Delta = \frac{x^2}{4}\sqrt{3}$ ;  $\therefore \text{area}$

$$\text{of square : area of equil. } \Delta :: \frac{9}{16}x^2 : \frac{\sqrt{3}}{4}x^2. \therefore \text{area of square} = \frac{9}{16}x^2 \times \frac{4}{\sqrt{3} \times x^2} \\ \times \text{area of equil. } \Delta = \frac{9}{4\sqrt{3}} \times \text{area of equil. } \Delta = \frac{3\sqrt{3}}{4} \times \text{area of equil. } \Delta = \frac{3 \times 1.732}{4} \\ \times \text{area of equil. } \Delta = 1.299 \times \text{area of equil. } \Delta.$$

11. Let  $x = \text{side of square}$ ;  $\therefore 4x = \text{perimeter of square} = \text{by hyp. perimeter of hexagon}$ . But  $6 \times \text{side of hexagon} = \text{perimeter of hexagon}$ ;  $\therefore \text{side of hexagon} = \frac{4}{6}x = \frac{2}{3}x$ .  $\therefore \text{area of hexagon} = 6 \times \frac{2}{3}x \times \frac{\sqrt{3}}{4}$  (vide Chap. XV. 6)  $= \frac{2}{3}\sqrt{3} \times x^2$ ; and area of square  $= x^2$ ;  $\therefore \text{area of hexagon : area of square} :: \frac{2}{3}\sqrt{3}x^2 : x^2$ ; i.e. as  $\frac{2}{3}\sqrt{3} : 1$ , i.e. as  $1.73205 : 1$ , i.e. as  $1.1547 : 1$ ;  $\therefore \text{area of hexagon} = 1.1547 \times \text{area of square}$ .

12. Let  $x = \text{radius of circle}$ ;  $\therefore 2 \times 8.1416 \times x = \text{perimeter of circle} = (\text{by hyp.}) \text{ perimeter of square} = 4 \times \text{side of square}$ ;  $\therefore \text{side of square}$

$$= \frac{2 \times 8.1416}{4}x = \frac{8.1416}{2}x.$$

$$\therefore \text{area of square} = \frac{(8.1416)^2}{4}x^2. \text{ And area of circle} = 8.1416 \times x^2; \therefore \text{area of circle : area of square} :: 8.1416x^2 : \frac{(8.1416)^2}{4}x^2, \text{ i.e. } 1 : \frac{8.1416}{4}, \text{ i.e. as } 1 : .7854;$$

$$\therefore \text{area of circle} = \frac{1}{.7854} \times \text{area of square} = 1.2732 \times \text{area of square}.$$

13. Let  $x$  = radius of circle.  $\therefore 2 \times 3.1416 \times x$  = perimeter of circle = (by hyp.) perimeter of hexagon =  $6 \times$  side of hexagon;  $\therefore$  side of hexagon =  $\frac{2 \times 3.1416}{6} \times x = \frac{3.1416}{3} x$ ;  $\therefore$  area of hexagon =  $6 \times \frac{(3.1416)^2}{9 \times 4} \sqrt{3} \times x^2$  (vide Chap. XV. 6) =  $\frac{(3.1416)^2}{6} \sqrt{3} \times x^2$ ; and area of circle =  $3.1416 x^2$ ;  $\therefore$  area of circle : area of hexagon ::  $3.1416 x^2 : \frac{(3.1416)^2}{6} \sqrt{3} \times x^2$ ; i.e. as  $1 : \frac{3.1416}{6} \sqrt{3}$ ;  $\therefore$  area of circle =  $\frac{6}{3.1416 \times \sqrt{3}} \times$  area of hexagon =  $\frac{2\sqrt{3}}{3.1416} \times$  area of hexagon =  $\frac{1.7320508}{1.5708} \times$  area of hexagon =  $1.1026 \times$  area of hexagon.

14. Area of equil.  $\Delta = \frac{\text{side}^2}{4} \sqrt{3}$  = (by hyp.) 100 sq. ft.;  
 $\therefore$  side<sup>2</sup> =  $\frac{400}{\sqrt{3}}$  sq. ft. =  $\frac{400 \sqrt{3}}{3}$  sq. ft. =  $\frac{400 \times 1.7320508}{3}$  sq. ft. = 230.94011 sq. ft.;  
 $\therefore$  side =  $\sqrt{(230.94011 \text{ sq. ft.})}$  = 15.197 ft. nearly.

15. Area of equil.  $\Delta = \frac{150^2}{4} \sqrt{3}$  sq. ft. = (by hyp.) area of hexagon =  $6 \times \frac{\text{side}^2}{4} \sqrt{3}$  (vide Chap. XV. 6);  $\therefore \frac{3}{4} \times \text{side}^2$  of hexagon  $\times \sqrt{3} = \frac{150^2}{4} \sqrt{3}$  sq. ft.;  
side<sup>2</sup> of hexagon =  $\frac{150^2}{6}$  sq. ft. = 3750 sq. ft.;  
 $\therefore$  side of hexagon =  $\sqrt{(3750 \text{ sq. ft.})}$  = 61.237 ft.

16. Here  $\angle ABC = 90^\circ$ , and area of segment  $ADC = 50$  sq. ft.

(By Euc. VI. 33), area of sector  $ABC$  : area of circle ::  $90^\circ : 360^\circ$ ;  $\therefore$  area of sector  $ABC = \frac{1}{4}$  area of circle =  $\frac{1}{4} \times 3.1416 \times AB^2 = .7854 \times AB^2$ ;

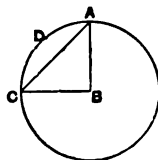
and area of  $\Delta ABC = \frac{1}{2} BC \times AB = \frac{1}{2} AB^2 = .5 \times AB^2$ ;

$\therefore$  area of segment  $ADC = .7854 \times AB^2 - .5 \times AB^2$

=  $AB^2 \times .2854$  = (by hyp.) 50 sq. ft.;

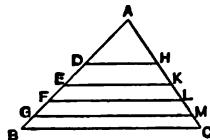
$\therefore AB^2 = 50 \text{ sq. ft.} \div .2854 = 175.192712 \text{ sq. ft.}$  nearly.

$\therefore AB = \sqrt{(175.192712 \text{ sq. ft.})}$  = 13.236 ft.



17. Here  $AB = 15$  ft. Let  $DH$ ,  $EK$ ,  $FL$ , and  $GM$  divide  $\Delta ABC$  into 5 equal parts, and be parallel to  $BC$ . Then  $\Delta ADH = \frac{1}{5} \Delta ABC$ ;  $\Delta AEK = \frac{4}{5} \Delta ABC$ ;  $\Delta AFL = \frac{9}{5} \Delta ABC$ ; and  $\Delta AGM = \frac{16}{5} \Delta ABC$ . We are required to find  $AD$ ,  $AE$ ,  $AF$ , and  $AG$ . Now (by Euc. VI. 19)

$\Delta ADH : \Delta ABC :: AD^2 : AB^2$ .



$$\therefore AD^2 = \frac{\Delta ADH \times AB^2}{\Delta ABC} = \frac{\frac{1}{2} \Delta ABC \times 225 \text{ sq. ft.}}{\Delta ABC} = 45 \text{ sq. ft.}$$

$$\therefore AD = \sqrt{(45 \text{ sq. ft.})} = 6.7082 \text{ ft.};$$

$$\text{again, } \Delta AEK : \Delta ABC :: AE^2 : AB^2; \therefore AE^2 = \frac{\frac{2}{3} \Delta ABC \times 225}{\Delta ABC} = 90 \text{ sq. ft.}$$

$$\therefore AE = \sqrt{(90 \text{ sq. ft.})} = 9.4868 \text{ ft. In the same manner}$$

$$AF = \sqrt{(\frac{1}{3} \times 225 \text{ sq. ft.})} = \sqrt{(135 \text{ sq. ft.})} = 11.61895 \text{ ft.} = 11.6190 \text{ ft. nearly};$$

$$\text{and } AG = \sqrt{(\frac{1}{3} \times 225 \text{ sq. ft.})} = \sqrt{(180 \text{ sq. ft.})} = 13.4164 \text{ ft.}$$

18. Let  $x$  = side of equil.  $\Delta$ ;  $\therefore$  area of equil.  $\Delta = \frac{x^2}{4} \sqrt{3} = \frac{x^2}{4} \times 1.73205$   
 = (by hyp.) area of square = side<sup>2</sup> of square;

$$\therefore \text{side of square} = \sqrt{\left(\frac{x^2}{4} \times 1.73205\right)} = \frac{x}{2} \times 1.816 = x \times .658;$$

$$\therefore \text{perimeter of square} = 4 \times x \times .658 = 2.632x; \text{ and perimeter of equil. } \Delta = 3x;$$

$$\therefore \text{perimeter of equil. } \Delta : \text{perimeter of square} :: 3x : 2.632x; \therefore \text{perimeter of equil. } \Delta = \frac{3}{2.632} \times \text{perimeter of square} = 1.139, \text{ i.e. } 1.14 \text{ nearly} \times \text{perimeter of square.}$$

19. Let  $ABCD$  be a trapezoid, of which  $AB = 16 \text{ ft.}$ ,  
 $CD = 20 \text{ ft.}$ ,  $IE = 5 \text{ ft.}$ . Let  $FG$  divide the trapezoid into two  
 equal trapezoids. Now area of  $ACDB$

$$= \frac{AB + CD}{2} \times IE = \frac{16 + 20}{2} \text{ ft.} \times 5 \text{ ft.} = 90 \text{ sq. ft.};$$

$\therefore$  area of  $AFGB$  and of  $FCDG = 45 \text{ sq. ft.}$  Produce  $CA$  and  
 $DB$  to meet at  $K$ . From  $K$  draw the perpendicular  $KILE$ .  
 Now from similar  $\Delta KAB$ , and  $KCD$  (vide Euc. VI. 4),

$$IK : AB :: KE : CD; \text{ i.e. } IK : 16 \text{ ft.} :: IK + 5 \text{ ft.} : 20 \text{ ft.};$$

$$\therefore 20 \text{ ft.} \times IK = 16 \text{ ft.} \times IK + 80 \text{ sq. ft.}; \therefore 4 \text{ ft.} \times IK = 80 \text{ sq. ft.};$$

$$\therefore IK = 20 \text{ ft.}; \therefore \text{area of } \Delta KAB = \frac{1}{2} AB \times IK = \frac{1}{2} \times 16 \times 20 \text{ sq. ft.} = 160 \text{ sq. ft.};$$

$$\therefore \text{area of } \Delta KFG = \text{area of } \Delta KAB + \text{area of } AFGB;$$

$$= (160 + 45) \text{ sq. ft.} = 205 \text{ sq. ft.}$$

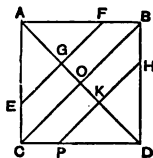
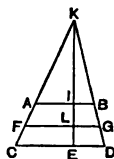
Then (by Euc. VI. 19),  $\Delta KAB : \Delta KFG :: KI^2 : KL^2$ ;

$$\therefore KL^2 = \frac{205 \times 400}{160} \text{ sq. ft.} = 512 \frac{1}{2} \text{ sq. ft.};$$

$$\therefore KL = \sqrt{(512 \frac{1}{2} \text{ sq. ft.})} = 22 \frac{1}{2} \sqrt{82} \text{ ft.} = 5 \times \frac{1}{2} \times 9.0553 \text{ ft.} = 22.638 \text{ ft. nearly.}$$

$$\text{Hence } IL = KL - KI = (22.638 - 20) \text{ ft.} = 2.638 \text{ ft.}$$

20. Let  $ABCD$  be a square, each of the sides of which  
 = 12 ft. Let the lines  $EF$  and  $PH$  divide the square into  
 three equal parts, and be parallel to  $BC$ . Join  $A, D$ .  
 We are to find  $GK$ . (By Euc. I. 4),  $\Delta ABC = \Delta BCD$ ,  
 but, by hyp.,  $\Delta AEF = \Delta PDH$ ;  $\therefore$  remainder  $ECBF$   
 = remainder  $BCPH$  (Euc. I. Ax. 3) =  $\frac{1}{2}$   $ECPHBF = \frac{1}{2}$  of  
 $ACDB = \frac{1}{2} \times 144 \text{ sq. ft.} = 72 \text{ sq. ft.}$  Again area of  $\Delta AEF$   
 =  $\frac{1}{3}$  area of  $ABCD = \frac{1}{3} \times 144 \text{ sq. ft.} = 48 \text{ sq. ft.};$



and area of  $\triangle ABC = (48 + 24)$  sq. ft. = 72 sq. ft. Now (by Euc. VI. 19),  
 $\triangle ABC : \triangle AFE :: AO^2 : AG^2$ ; but  $AO^2$  is (by Euc. I. 47)

$$= \frac{AC^2}{2} = 144 \text{ sq. ft.} = 72 \text{ sq. ft.}; \therefore 72 \text{ sq. ft.} : 48 \text{ sq. ft.} :: 72 \text{ sq. ft.} : AG^2;$$

$$\therefore AG^2 = 48 \text{ sq. ft.};$$

$$\therefore AG = \sqrt{48 \text{ sq. ft.}} = \sqrt{16 \times 3 \text{ sq. ft.}} = 4\sqrt{3} \text{ ft.} = 4 \times 1.732 \text{ ft.} = 6.928 \text{ ft.};$$

$\therefore GO = AO - AG = \sqrt{72 \text{ ft.}} - 6.928 \text{ ft.} = (8.485 - 6.928) \text{ ft.} = 1.557 \text{ ft.}$  In the same manner it may be shewn that  $KO$  likewise = 1.557 ft.;

$$\therefore GK = (1.557 + 1.557) \text{ ft.} = 3.114 \text{ ft.}$$

21. (By Art. 167), area of polygon = 3 sq. in., when radius of circumscribing circle is 1 in., and (by Art. 99), side of this polygon = .51764 in.;  $\therefore$  perimeter of polygon =  $12 \times .51764 \text{ in.} = 6.21168 \text{ in.}$ ; and this, by hyp., also = circum. or perimeter of circle =  $2 \times 3.1416 \times \text{radius}$ ;

$$\therefore \text{radius} = \{6.21168 \div (2 \times 3.1416)\} \text{ in.} = \frac{3.10584}{3.1416} \text{ in.};$$

$$\therefore \text{area of circle} = 3.1416 \times \left(\frac{3.10584}{3.1416} \text{ in.}\right)^2 = \frac{9.6462}{3.1416} \text{ sq. in. nearly};$$

$$\therefore \text{area of circle} : \text{area of polygon} :: \frac{9.6462}{3.1416} \text{ sq. in.} : 3 \text{ sq. in.};$$

$$\therefore \text{area of circle} = \frac{3.2154}{3.1416} \times \text{area of polygon.}$$

## CHAPTER XXII.

1. Vol. of cube =  $(2 \text{ ft. } 8 \text{ in.})^3 = \left(\frac{3}{2} \text{ ft.}\right)^3 = \frac{27}{8} \text{ cub. ft.} = 18 \text{ cub. ft. } 1664 \text{ in.}$

5. Vol. of parallelepiped =  $4 \text{ ft. } 8 \text{ in.} \times 3 \text{ ft. } 6 \text{ in.} \times 2 \text{ ft. } 4 \text{ in.}$   
 $= 1\frac{4}{5} \times \frac{7}{2} \times \frac{1}{2} \text{ cub. ft.} = \frac{49}{2} \text{ cub. ft.} = 38 \text{ cub. ft. } 192 \text{ in.}$

9. Vol. of parallelepiped = area of base  $\times$  height  
 $= 16 \text{ sq. ft.} \times 4 \text{ ft. } 3 \text{ in.} = 16 \times 4\frac{1}{2} \text{ cub. ft.} = 68 \text{ cub. ft.}$

13. Area of base  $\times$  height, i. e.  $8 \text{ sq. ft.} \times \text{height} = \text{Vol. of parallelepiped}$   
 $= 6 \text{ cub. ft.}; \therefore \text{height} = \frac{6 \text{ cub. ft.}}{8 \text{ sq. ft.}} = \frac{3}{4} \text{ ft.} = 9 \text{ in.}$

17. Area of base

$$= \frac{\text{Vol. of parallelepiped}}{\text{height}} \text{ (vide 13)} = \frac{15 \text{ cub. ft.}}{9 \text{ in.}} = 15 \text{ cub. ft.} \div \frac{3}{4} \text{ ft.} = 20 \text{ sq. ft.}$$

21. Vol. of parallelepiped =  $6 \times 6 \times 6 \text{ cub. ft.} = 6 \times 6 \times 6 \times 1728 \text{ cub. in.}$ ; and 1 gallon =  $277\frac{1}{8} \text{ cub. in.} = 11\frac{1}{8} \text{ cub. in.}$ ;  $\therefore$  no. of gallons in the given volume =  $6 \times 6 \times 6 \times 1728 \div 11\frac{1}{8} = 14\frac{1}{8} \times 11\frac{1}{8} = 1346.2 = 1346$  (to the nearest gallon).

25. Vol. of parallelepiped =  $5 \times 5 \times 5$  cub. ft.; and 1 cub. foot of water weighs 1000 oz. Av., i. e.  $\frac{1000}{16 \times 28 \times 4}$  cwt.  $\therefore$  no. of cwts. of water in the above vol. =  $5 \times 5 \times 5 \times \frac{1000}{16 \times 28 \times 4} = \frac{15625}{112} = 69.7 = 70$  nearly.

29. Vol. of cube 6 in. long =  $(6 \text{ in.})^3 = 216$  cub. in.; sum of the vols. of the other 3 cubes =  $(3 \text{ in.})^3 + (4 \text{ in.})^3 + (5 \text{ in.})^3 = (27 + 64 + 125)$  cub. in. = 216 cub. in.;  $\therefore$  vol. of cube 6 in. long = sum of the vols. of 3 cubes which are respectively 3 in., 4 in., and 5 in. long.

30. Vol. of parallelepiped  
 = 94 ch. 50 lbs.  $\times$  1 ch. 5 lbs.  $\times$   $31\frac{1}{2}$  lbs. = 94.5 ch.  $\times$  1.05 ch.  $\times$  315 ch.  
 = 31.255875 cub. ch.

31. Since vol. of 200 beams = 1000 cub. ft.;  $\therefore$  vol. of 1 beam = 5 cub. ft.;  
 $\therefore$  length of beam =  $\frac{5 \text{ cub. ft.}}{3 \text{ in.} \times 10 \text{ in.}}$  (vide 18) = 5 cub. ft.  $\div \frac{1}{36}$  sq. ft. = 24 ft.

32. Vol. of wall = 90 ft.  $\times$  18 in.  $\times$  8 ft. =  $90 \times \frac{3}{4} \times 8$  cub. ft. =  $90 \times 12$  cub. ft.; and vol. of 1 brick = 9 in.  $\times \frac{3}{4}$  in.  $\times$  3 in. =  $\frac{3}{4} \times \frac{3}{4} \times \frac{1}{4}$  cub. ft.;  $\therefore$  no. of bricks whose vol. will be 90  $\times$  12 cub. ft.

$$= 90 \times 12 \div (\frac{3}{4} \times \frac{3}{4} \times \frac{1}{4}) = 15360 \text{ bricks.}$$

33. Vol. of box = 3 ft. 6 in.  $\times$  3 ft.  $\times$  2 ft. =  $\frac{7}{2} \times 6$  cub. ft. = 21 cub. ft.; and vol. of 1 book = 8 in.  $\times 2\frac{1}{2}$  in.  $\times \frac{3}{4}$  in. =  $\frac{3}{2}$  ft.  $\times \frac{7}{8}$  ft.  $\times \frac{3}{8}$  ft. =  $\frac{7}{16}$  cub. ft.;  $\therefore$  no. of books that can be packed in box =  $21 \div \frac{7}{16} = 384$  books.

34. Here vol. of gold = 1 cub. ft. = 1728 cub. in.;  $\therefore$  thickness of gold  
 =  $\frac{1728 \text{ cub. in.}}{432000000 \text{ sq. in.}}$  (vide 18) =  $\frac{1}{250000}$  in. = .000004 in.

35. Vol. of cube =  $(39.37 \text{ in.})^3 = \frac{61023.377953}{1728}$  cub. ft. = 35.314454 cub. ft.

36. Vol. of block of stone =  $48 \times 30 \times 15$  cub. in. Now since this vol. weighs 27 cwt., i. e.  $27 \times 4 \times 28$  lbs.;  $\therefore$  100 cub. in. will weigh

$$100 \times 27 \times 4 \times 28 \div (48 \times 30 \times 15) \text{ lbs., that is 14 lbs.}$$

37. Vol. of block of marble =  $9\frac{1}{2} \times 2\frac{1}{4} \times 2$  cub. ft. =  $\frac{19 \times 9}{4}$  cub. ft.; and weight of 1 cub. foot of marble = 2.716  $\times$  weight of 1 cub. foot of water =  $2.716 \times 1000$  oz. Av. = 2716 oz.;  $\therefore$  vol. of the block will weigh

$$\frac{19 \times 9}{4} \times 2716 \text{ oz.} = 116109 \text{ oz.}$$

38. Vol. of vessel in 1<sup>st</sup> case =  $(14.04 \text{ in.})^3 = 2767.587264$  cub. in.; and 10 gallons = 2772.74 cub. in.;  $\therefore$  vessel in 1<sup>st</sup> case holds less than 10 gallons. Again, vol. of vessel in 2<sup>nd</sup> case =  $(14.05 \text{ in.})^3 = 2773.505125$  cub. in.;  $\therefore$  vessel in 2<sup>nd</sup> case contains more than 10 gallons.

39. Here, empty portion of reservoir left after the water has been drawn off is 24 ft. 8 in. long, 12 ft. 9 in. wide, and 1 ft. high.

∴ its vol. =  $24\frac{2}{3} \times 12\frac{3}{4} \times 1$  cub. ft. =  $7\frac{1}{2} \times \frac{51}{4}$  cub. ft. =  $\frac{371}{8}$  cub. ft. =  $314\frac{1}{8}$  cub. ft.  
= quantity of water drawn off.

40. 260 gallons =  $260 \times 277\frac{1}{2}$  cub. in. =  $65 \times 1109$  cub. in. This then is the vol. of the empty portion of the cistern left after 260 gallons are drawn off;

∴ height of this portion =  $\frac{65 \times 1109 \text{ cub. in.}}{13\frac{1}{2} \text{ ft.} \times 9\frac{1}{2} \text{ ft.}}$  (vide preceding and 13)

$$= \frac{65 \times 1109 \text{ cub. in.}}{27 \times 6 \text{ in.} \times 89 \times 3 \text{ in.}} = \frac{719955}{1458} \text{ in.} = 3 \cdot 8 \text{ in.}$$

41. Vol. of required no. of leaves

$$= 20 \text{ sq. yds.} \times 1 \text{ in.} = 20 \times 9 \times 144 \text{ sq. in.} \times 1 \text{ in.} = 180 \times 144 \text{ cub. in.}$$

Now 1 cub. foot, i. e. 1728 cub. in. of gold weighs 10 cwt. 95 lbs., i. e. 1215 × 16 oz.;

∴ vol. of 1 oz. of gold =  $\frac{1728}{1215 \times 16}$  cub. in. But, by hyp., 1 leaf weighs 1 oz.;

∴ vol. of 1 leaf =  $\frac{1728}{1215 \times 16}$  cub. in. ∴ no. of leaves required to make up a

$$\text{volume of } 180 \times 144 \text{ cub. in.} = 180 \times 144 \div \frac{1728}{1215 \times 16} = 291600 \text{ leaves.}$$

42. 1 cub. fathom = 216 cub. ft.; and 1 cub. ft. of water weighs 1000 oz.;

∴ 1 cub. fathom of water weighs 216 × 1000 oz. which

$$= \frac{216 \times 1000}{16 \times 28 \times 4 \times 20} \text{ tons} = \frac{45}{7} \text{ tons} = 6 \cdot 02 \text{ tons} = 6 \text{ tons nearly.}$$

43. Let  $x$ ,  $y$ , and  $z$  represent respectively the length, breadth, and height of the 2<sup>nd</sup> parallelepiped. Then  $x + \frac{x}{2}$ , i. e.  $\frac{3}{2}x$  = length of 1<sup>st</sup> parallelepiped, and  $\frac{2}{3}y$  and  $\frac{4}{3}z$  = respectively its breadth and height. ∴ vol. of 2<sup>nd</sup> parallelepiped =  $xyz$ ; and vol. of 1<sup>st</sup> parallelepiped

$$= \frac{3}{2}x \times \frac{2}{3}y \times \frac{4}{3}z = \frac{4}{3}xyz = 3\frac{1}{3}xyz.$$

∴ the 1<sup>st</sup> parallelepiped is more than 3 times the second.

44. Let  $x$ ,  $y$ ,  $z$  represent respectively the length, breadth, and height of the 2<sup>nd</sup> parallelepiped. Then  $x + \frac{1}{2}x$ , i. e.  $\frac{3}{2}x$  = length of 1<sup>st</sup> parallelepiped, and  $\frac{2}{3}y$  and  $\frac{4}{3}z$  = respectively its breadth and height. ∴ vol. of 2<sup>nd</sup> parallelepiped =  $xyz$ ; and vol. of 1<sup>st</sup> parallelepiped =  $\frac{3}{2}x \times \frac{2}{3}y \times \frac{4}{3}z = 2xyz$  nearly. ∴ 1<sup>st</sup> parallelepiped is nearly twice the second.

45. Let  $x$ ,  $y$ ,  $z$  represent respectively the length, breadth, and height of the 2<sup>nd</sup> parallelepiped. Then  $x + \frac{1}{2}x$ , i. e.  $\frac{3}{2}x$  = length of 1<sup>st</sup> parallelepiped,  $y + \frac{1}{2}y$ , i. e.  $\frac{3}{2}y$  = its breadth, and  $z + \frac{1}{2}z$ , i. e.  $\frac{3}{2}z$  = its height. ∴ vol. of 2<sup>nd</sup> parallelepiped =  $xyz$ ; and vol. of 1<sup>st</sup> parallelepiped

$$= \frac{3}{2}x \times \frac{3}{2}y \times \frac{3}{2}z = \frac{27}{8}xyz = 3\frac{3}{8}xyz;$$

∴ 1<sup>st</sup> parallelepiped is half as large again as the second.

46. Inside length of bottom = whole outward length - the thickness of the wood at both the ends = 3 ft. - 2 in. = 34 in.; and inside height of box = the outward height - thickness of wood at the bottom of the box = 3 ft. - 1 in. = 35 in. Now vol. of box were it full

$$= 3 \times 3 \times 3 \text{ cub. ft.} = 27 \times 1728 \text{ cub. in.} = 46656 \text{ cub. in.}$$

And vol. of inside or empty portion =  $34 \times 34 \times 35 \text{ cub. in.} = 40460 \text{ cub. in.}$ ;  
 $\therefore$  vol. of the wood alone =  $(46656 - 40460) \text{ cub. in.} = 6196 \text{ cub. in.}$

47. Internal length = 6 ft. -  $2 \times \frac{1}{2} \text{ in.}$  (vide preceding) = 71 in. Internal breadth = 5 ft. -  $2 \times \frac{1}{2} \text{ in.} = 59 \text{ in.}$ , and internal height = 3 ft. -  $\frac{1}{2} \text{ in.}$  (observe there is no lid) =  $2\frac{1}{2} \text{ in.}$ ; now vol. of box when full

$$= 6 \times 5 \times 3 \text{ cub. ft.} = 90 \times 1728 \text{ cub. in.} = 155520 \text{ cub. in.};$$

and vol. of internal or empty portion =  $71 \times 59 \times 2\frac{1}{2} \text{ cub. in.} = 148709\frac{1}{2} \text{ cub. in.}$ ;  
 $\therefore$  vol. of material alone =  $(155520 - 148709\frac{1}{2}) \text{ cub. in.} = 6810\frac{1}{2} \text{ cub. in.}$

48. Whole vol. of box when full =  $18 \times 10 \times 6 \text{ cub. in.} = 1080 \text{ cub. in.}$ ;  
 now internal length of box =  $(18 - 2 \times \frac{1}{2} \text{ in.})$  (vide 46), = 17 in., and internal breadth =  $(10 - 2 \times \frac{1}{2} \text{ in.}) = 9 \text{ in.}$ ; and internal height =  $(6 - 2 \times \frac{1}{2} \text{ in.}) = 5 \text{ in.}$  (Here, in the case of the internal height, we have to subtract the thickness of both the cover and the bottom of the box from the whole external height.)  
 $\therefore$  vol. of inside or empty portion of box =  $17 \times 9 \times 5 \text{ cub. in.} = 765 \text{ cub. in.}$ ;  
 $\therefore$  vol. of wood alone =  $(1080 - 765) \text{ cub. in.} = 315 \text{ cub. in.}$  Now since, by hyp., 315 cub. in. of wood weigh 15 lbs.;  $\therefore$  1 cub. in. of wood weighs  $\frac{15}{315} \text{ lbs.}$ , i.e.  $\frac{1}{21} \text{ lb.}$  Again, when the box is filled with sand it weighs 100 lbs., and the wood of the box weighs 15 lbs.;  $\therefore$  the sand alone weighs  $(100 - 15) \text{ lbs.}$ , i.e. 85 lbs. But the vol. of the sand is the same as the vol. of the empty portion of the box, i.e. it is 765 cub. in.;  $\therefore$  765 cub. in. of sand weigh 85 lbs.;  $\therefore$  1 cub. in. of sand weighs  $\frac{85}{765} \text{ lbs.}$ , i.e.  $\frac{1}{9} \text{ lb.}$

49. Internal length of box =  $(2 \text{ ft. } 10 \text{ in.} - 2 \times 1 \text{ in.})$  (vide 46),

$$= 2 \text{ ft. } 8 \text{ in.} = \frac{8}{3} \text{ ft.};$$

internal breadth =  $(2 \text{ ft. } 5 \text{ in.} - 2 \times 1 \text{ in.}) = 2 \text{ ft. } 3 \text{ in.} = \frac{5}{3} \text{ ft.}$ ; and internal height =  $(1 \text{ ft. } 7 \text{ in.} - 1 \text{ in.}) = 1 \text{ ft. } 6 \text{ in.} = \frac{5}{2} \text{ ft.}$   $\therefore$  vol. of empty portion of box, or, in other words, vol. the box could hold =  $\frac{8}{3} \times \frac{5}{3} \times \frac{5}{2} \text{ cub. ft.} = 9 \text{ cub. ft.}$

50. Area of base =  $\frac{40 \text{ cub. ft. } 1440 \text{ in.}}{3 \text{ ft. } 4 \text{ in.}}$  (vide 13)

$$= 40\frac{1}{2} \text{ cub. ft.} \div 3\frac{1}{3} \text{ ft.} = 24\frac{1}{2} \times \frac{3}{2} \text{ sq. ft.} = 36\frac{1}{2} \text{ sq. ft.} = \text{side}^2 \text{ of square base};$$

$$\therefore \text{side of square base} = \sqrt{(36\frac{1}{2} \text{ sq. ft.})} = 6\frac{1}{2} \text{ ft.} = 3 \text{ ft. } 6 \text{ in.}$$

51. Here  $ABCD$  and  $EFHG$  each

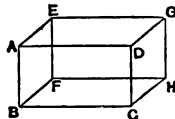
$$= 2 \text{ sq. ft. } 117 \text{ in.} = 21\frac{3}{4} \text{ sq. ft.} = 4\frac{5}{8} \text{ sq. ft.};$$

$AEGD$  and  $BFHC$  each

$$= 2 \text{ sq. ft. } 12 \text{ in.} = 2\frac{1}{4} \text{ sq. ft.} = 1\frac{1}{2} \text{ sq. ft.};$$

and  $ADFE$  and  $DCHG$  each

$$= 1 \text{ sq. ft. } 96 \text{ in.} = 1\frac{1}{2} \text{ sq. ft.} = \frac{5}{8} \text{ sq. ft.}$$



Now  $AB \times BF = \text{area of } ABFE = \frac{1}{2} \text{ sq. ft.}; \therefore BF = \frac{1}{2} \text{ sq. ft.} \times \frac{1}{AB}$ . Again

$$BF \times BC = \text{area of } BFHC = \frac{1}{2} \text{ sq. ft.}; \therefore BF = \frac{1}{2} \text{ sq. ft.} \times \frac{1}{BC};$$

$$\therefore \frac{1}{2} \text{ sq. ft.} \times \frac{1}{AB} = \frac{1}{2} \text{ sq. ft.} \times \frac{1}{BC}; \therefore 60 BC = 75 AB; \therefore BC = \frac{5}{4} AB.$$

Lastly,  $AB \times BC = \text{area of } ABCD = \frac{1}{2} \text{ sq. ft.};$

$$\text{i. e. } AB \times \frac{5}{4} AB \text{ or } \frac{5}{4} AB^2 = \frac{1}{2} \text{ sq. ft.};$$

$$\therefore AB^2 = \frac{2}{5} \text{ sq. ft.}; \therefore AB = \sqrt{\left(\frac{2}{5} \text{ sq. ft.}\right)} = \frac{2}{5} \text{ ft.}$$

$\therefore$  vol. of parallelepiped

$$= \text{area of } BFHC \times AB = \frac{1}{2} \text{ sq. ft.} \times \frac{2}{5} \text{ ft.} = \frac{1}{5} \text{ cub. ft.} = 3 \text{ cub. ft. } 216 \text{ in.}$$

Or thus; If the edges of the rectangular parallelepiped be  $a, b, c$  feet, the three areas are  $ab, bc, ca$ . The product of these is  $a^2b^2c^2$ , which is the square of the volume  $abc$ .

Hence the rule; Multiply the three given areas together, and the square root of the product will be the volume.

52. Vol. of cube = length<sup>3</sup>;

$$\therefore \text{length} = \sqrt[3]{(\text{volume})} = \sqrt[3]{(2 \text{ cub. ft.})} = 1.26 \text{ ft. nearly.}$$

54. Vol. of 4000 gallons of water =  $4000 \times 277\frac{1}{2} \text{ cub. in.} = 1109000 \text{ cub. in.};$

$$\therefore \text{length of cubical vessel} = \sqrt[3]{(1109000 \text{ cub. in.})} = 103.51 \text{ in.}$$

56. The no. of cub. in. that weigh  $\frac{1}{2}$  ton or  $\frac{1}{2} \times 20 \times 4 \times 28 \text{ lbs.}$

$$= 10 \times 4 \times 28 \times 100 + 14 = 8000;$$

$$\therefore \text{length of cube} = \sqrt[3]{(8000 \text{ cub. in.})} = 20 \text{ in.}$$

## CHAPTER XXIII.

1. Vol. of prism =  $6 \text{ sq. ft. } 85 \text{ in.} \times 2 \text{ ft. } 6 \text{ in.} = 6.7\frac{5}{8} \text{ sq. ft.} \times 2\frac{1}{2} \text{ ft.}$

$$= \frac{112\frac{5}{8}}{4} \times \frac{5}{2} \text{ cub. ft.} = \frac{141\frac{5}{8}}{4} \text{ cub. ft.} = 15 \text{ cub. ft. } 1050 \text{ in.}$$

5. Vol. of prism = area of base  $\times$  height. Now area of triangular base (calculated as in Chap. XIII.) =  $42 \text{ sq. in.};$

$$\therefore \text{vol.} = 42 \text{ sq. in.} \times 45 \text{ in.} = \frac{1}{4} \text{ sq. ft.} \times \frac{1}{4} \text{ ft.} = \frac{1}{16} \text{ cub. ft.} = 1 \text{ cub. ft. } 162 \text{ in.}$$

9. Vol. of cylinder =  $3.1416 \times 4 \text{ sq. ft.} \times 3\frac{1}{2} \text{ ft.} = 43.9824 \text{ cubic ft.}$

13. Vol. of prism = area of base  $\times$  height;

$$\therefore \text{height} = \frac{\text{volume}}{\text{area of base}} = \frac{18 \text{ cub. ft. } 708 \text{ in.}}{6 \text{ sq. ft. } 100 \text{ in.}} = 18\frac{7}{10} \text{ cub. ft.} + 6\frac{1}{10} \text{ sq. ft.}$$

$$= \frac{187}{10} \text{ cub. ft.} \div \frac{244}{10} \text{ sq. ft.} = \frac{187}{244} \text{ ft.} = 2 \text{ ft. } 9 \text{ in.} = 33 \text{ in.}$$

$$17. \text{ Area of base of cylinder} = \frac{\text{volume}}{\text{height}} = \frac{10,000 \text{ cub. in.}}{4 \text{ ft. } 2 \text{ in.}} = 200 \text{ sq. in.}$$

But area of base =  $8 \cdot 1416 \times \text{radius}^2$ ;  $\therefore \text{radius}^2 = \frac{200}{8 \cdot 1416} \text{ sq. in.} = 63 \cdot 66 \text{ sq. in.}$ ;  
 $\therefore \text{radius} = \sqrt{(63 \cdot 66 \text{ sq. in.})} = 7 \cdot 979 \text{ in. nearly.}$

21. Vol. of vessel =  $8 \cdot 1416 \times 100 \text{ sq. in.} \times 20 \text{ in.} = 6283 \cdot 2 \text{ cub. in.}$ ; and the volume of one gallon =  $277 \frac{1}{2} \text{ cub. in.} = 277 \cdot 5 \text{ cub. in.}$ ;  $\therefore$  no. of gallons in vessel =  $6283 \cdot 2 \div 277 \cdot 5 = 22 \cdot 6 = 23 \text{ nearly.}$

$$25. \text{ Area of base} = \frac{18+12}{2} \text{ ft.} \times 5 \text{ ft.} = 75 \text{ sq. ft.}; \therefore \text{ vol. of prism} \\ = 75 \text{ sq. ft.} \times 24 \text{ ft.} = 1800 \text{ cub. ft.}$$

26. The figure of the wall is that of a very long prism of which the ends are trapezoids. Now the area of either end

$$= \frac{15+25}{2} \text{ ft.} \times 20 \text{ ft.} = 400 \text{ sq. ft.} \\ = \frac{400}{9} \text{ sq. yards,}$$

and the length of the prism

$$= 1500 \text{ miles} = 1500 \times 1760 \text{ yards.}$$

Thus the number of cubic yards of material contained in the wall

$$= 1500 \times 1760 \times \frac{400}{9} = 117,333,333 \frac{1}{3}.$$

27. The ditch is of the form of a long prism of which each end is a trapezoid whose area =  $\frac{16+20}{2} \times 8 \text{ sq. feet} = 144 \text{ sq. feet.}$

Thus the number of cubic feet to be dug out =  $144 \times 1000 = 144,000$ .

$$23. \text{ Vol. of ditch} = \frac{10+8}{2} \text{ ft.} \times 6 \text{ ft.} \times 40 \text{ ft.} = 2160 \text{ cub. ft.}$$

$$= 2160 \times 1728 \text{ cub. in.} = 3732480 \text{ cub. in.};$$

and the volume of one gallon =  $277 \cdot 274 \text{ cub. in.}$ ;  $\therefore$  no. of gallons required =  $3732480 \div 277 \cdot 274 = 13461 \text{ gallons nearly.}$

$$29. \text{ Vol. of ditch} = \frac{16+24}{2} \text{ ft.} \times 8 \text{ ft.} \times \text{length} \\ = 160 \text{ sq. ft.} \times \text{length} = (\text{by hyp.}) 250000 \text{ cub. ft.}; \\ \therefore \text{ length} = \frac{250000 \text{ cub. ft.}}{160 \text{ sq. ft.}} = 1562 \cdot 5 \text{ ft.}$$

30. Vol. of ditch =  $\frac{5+4}{2} \text{ ft.} \times 4 \text{ ft.} \times \text{length} = 18 \text{ sq. ft.} \times \text{length}$ ; and vol. of 10000 gallons =  $10000 \times 277 \cdot 274 \text{ cub. in.} = 2772740 \text{ cub. in.} = 2772740 \div 1728 \text{ cub. ft.}$   
 $= (\text{by hyp.}) \text{ vol. of ditch}; \therefore \text{ length of ditch} = \frac{2772740 \text{ cub. ft.}}{18 \text{ sq. ft.}} = 154041 \cdot 1 \text{ ft.}$   
 $= 89 \text{ ft. nearly.}$

$$\begin{aligned} 81. \text{ Vol. of well} &= 3 \cdot 1416 \times \left(\frac{3}{4} \text{ ft.}\right)^2 \times 30 \text{ ft.} = 3 \cdot 1416 \times \frac{9}{16} \times 30 \text{ cub. ft.} \\ &= 7854 \times 270 \text{ cub. ft.} = 212 \cdot 058 \text{ cub. ft.} = 212 \cdot 06 \text{ cub. ft. nearly.} \end{aligned}$$

33. Hence the well is half a cylinder;

$$\begin{aligned} \therefore \text{ its vol.} &= \frac{1}{2} \times 3 \cdot 1416 \times \left(\frac{1}{2} \text{ yds.}\right)^2 \times 100 \text{ yds.} = 1 \cdot 5708 \times \frac{1}{4} \times 100 \text{ cub. yds.} \\ &= \frac{1 \cdot 5708}{4} \text{ cub. yds.} = 1745 \cdot 3 \text{ cub. yds.} \end{aligned}$$

$$\begin{aligned} 84. \text{ Vol. of 1 coin} &= 3 \cdot 1416 \times \left(\frac{3}{8} \text{ in.}\right)^2 \times \frac{1}{4} \text{ in.} = 3 \cdot 1416 \times \frac{9}{64} \text{ sq. in.} \times \frac{1}{4} \text{ in.} \\ &= \frac{3 \cdot 1416 \times 9}{64} \text{ cub. in.}; \text{ and vol. of cube} = (3 \text{ in.})^3 = 27 \text{ cub. in.}; \therefore \text{ no. of coins} \\ \text{required} &= 27 \div \frac{3 \cdot 1416 \times 9}{64} = 488 \cdot 9 = 489 \text{ nearly.} \end{aligned}$$

35. Vol. of well

$$\begin{aligned} &= 3 \cdot 1416 \times \left(\frac{3}{4} \text{ ft.}\right)^2 \times 30 \text{ ft.} = \frac{3 \cdot 1416 \times 120}{27} \text{ cub. yds.} = \frac{1 \cdot 0472 \times 40}{3} \text{ cub. yds.}; \\ \therefore \text{ cost} &= \frac{1 \cdot 0472 \times 40}{3} \times 7s. 6d. = \frac{1 \cdot 0472 \times 40}{3} \times £\frac{3}{4} = £5 \cdot 236 \\ &= £5. 4s. 8 \cdot 64d. = £5. 4s. 9d. \text{ nearly.} \end{aligned}$$

38. Vol. of 1 lb. of powder = 30 cub. in.;  $\therefore$  vol. of 10 lbs. = 300 cub. in.;  
 $\therefore$  vol. of gun also = 300 cub. in. But vol. of gun

$$\begin{aligned} &= 3 \cdot 1416 \times \left(\frac{3}{4} \text{ in.}\right)^2 \times \text{length} = 3 \cdot 1416 \times 9 \text{ sq. in.} \times \text{length}; \\ \therefore \text{ length of gun} &= \frac{300 \text{ cub. in.}}{3 \cdot 1416 \times 9 \text{ sq. in.}} = \frac{25}{2 \cdot 3562} \text{ in.} = 10 \cdot 61 \text{ in.} \end{aligned}$$

$$\begin{aligned} 39. \text{ Vol. of wire} &= 3 \cdot 1416 \times \left(\frac{1}{4} \text{ in.}\right)^2 \times \text{length} = \frac{3 \cdot 1416}{16} \text{ sq. in.} \times \text{length} \\ &= (\text{by hyp.}) 1 \text{ cub. ft.} = 1728 \text{ cub. in.} \end{aligned}$$

$$\therefore \text{ length required} = \frac{1728 \times 200}{3 \cdot 1416} \text{ in.} = \frac{115200}{3 \cdot 1416} \text{ in.} = 890061 \text{ in. nearly.}$$

$$\begin{aligned} 41. \text{ Vol. of shell} &= \text{entire vol. of cylinder} - \text{vol. of empty portion} \\ &= 3 \cdot 1416 \times \left(\frac{3}{4} \text{ ft.}\right)^2 \times 7 \text{ ft.} - 3 \cdot 1416 \times \left(\frac{1}{4} \text{ ft.}\right)^2 \times 7 \text{ ft.} = 3 \cdot 1416 \times 7 \text{ ft.} \left(\frac{3}{4} - \frac{1}{4}\right) \text{ sq. ft.} \\ &= 3 \cdot 1416 \times 7 \times \frac{1}{2} \text{ cub. ft.} = \frac{10 \cdot 0793}{6} \text{ cub. ft.} = 1 \cdot 6798 \text{ cub. ft.} \\ &= 1 \cdot 68 \text{ cub. ft. nearly.} \end{aligned}$$

42. Radius of inner surface = (10 - 2) in. = 8 in. The question is then solved as the preceding.

43. Radius of outer surface = (12 + 3) in. = 15 in. The question is then solved as in 41.

44. Inside radius of pipe =  $\frac{3}{4}$  in. ;  $\therefore$  radius of outer surface =  $(\frac{3}{4} + \frac{1}{4})$  in. = 2 in. ;  $\therefore$  vol. of pipe =  $240 \times 3.1416 \times (4 - \frac{1}{2})$  cub. in. (vide 41)  
 =  $240 \times 3.1416 \times \frac{7}{2}$  cub. in. =  $.7854 \times 1680$  cub. in.

But weight of 1 cub. in. of iron =  $4.526$  oz. =  $\frac{2.263}{8}$  lbs. ;

$\therefore$  weight of pipe =  $.7854 \times 1680 \times \frac{2.263}{8}$  lbs.

=  $.7854 \times 210 \times 2.263$  lbs. =  $373.245642$  lbs. =  $373$  lbs. nearly.

46. Since weight of 1728 cub. in. of lead = 11412 oz. =  $11\frac{1}{2}$  lbs. ;

$\therefore$  weight of 1 cub. in. of lead =  $\frac{11412}{16 \times 1728}$  lbs. =  $\frac{1}{16}$  lbs. ; and vol. of pipe

=  $3.1416 \times 8$  yds.  $\times (\frac{3}{4} - 1)$  sq. in. (vide 44 and 41)

=  $3.1416 \times 8 \times 3 \times 12$  in.  $\times \frac{1}{4}$  sq. in. =  $3.1416 \times 360$  cub. in. ;

$\therefore$  weight of pipe =  $3.1416 \times 360 \times \frac{1}{16}$  lbs.

$\therefore$  cost =  $3.1416 \times 360 \times \frac{1}{16} \times 2\frac{1}{2}d.$  =  $3.1416 \times 360 \times \frac{25}{128} \times \frac{1}{4}$

=  $\frac{2 \times 622 \times 4295}{128}$  =  $\pounds 4. 17s. 3d.$  nearly.

47. Vol. of square rod =  $(1 \text{ in.})^2 \times \text{length} = 1$  sq. in.  $\times$  length ; and vol. of round rod =  $3.1416 \times (\frac{1}{2} \text{ in.})^2 \times \text{length} = .7854$  sq. in.  $\times$  length ;  $\therefore$  weight of latter =  $.7854 \times \frac{1}{4}$  lbs. =  $8.2467$  lbs.

48. Hence the base is an equil.  $\Delta$ , each side of which, as also the height of the prism, is 10 in. ;  $\therefore$  vol. of prism

=  $\frac{10^3}{4} \sqrt{3} \times 10$  cub. in. =  $250 \times 1.732$  cub. in. =  $433$  cub. in.

49. Vol. =  $6 \times \frac{1^3}{4} \sqrt{3} \times 1$  cub. ft. (vide Chap. XV. 6)

=  $\frac{3}{2} \times 1.732$  cub. ft. =  $2.598$  cub. ft.

50. Vol. of cylinder in 1<sup>st</sup> case = length  $\times 3.1416 \times \{(\frac{1}{2})^2 - (\frac{3}{4})^2\}$  sq. in. (vide 41) = length  $\times 3.1416 \times \frac{1}{16} \times \frac{7}{4}$  sq. in. = length  $\times 3.1416 \times (\frac{1}{16})^2$  sq. in. ; this is also the vol. of the cylinder in 2<sup>nd</sup> case which =  $3.1416 \times \text{radius}^2 \times \text{length}$  ;  
 $\therefore$  radius<sup>2</sup>  $\times$  length  $\times 3.1416$  = length  $\times 3.1416 \times (\frac{1}{16})^2$  sq. in.

$\therefore$  radius<sup>2</sup> =  $(\frac{1}{16})^2$  sq. in. ;  $\therefore$  radius =  $\frac{1}{16}$  in.

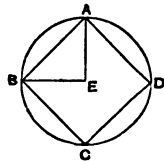
51. The base of the parallelepiped will then be a square inscribed in the circular base of the trunk of the tree ; so that if  $ABCD$  be the base of the parallelepiped, its side  $AB$  (by Euc. IV. 6 and I. 47),

=  $\sqrt{(AE^2 + EB^2)} = \sqrt{(2AE^2)} = AE \sqrt{2}$

= radius of the base of the tree  $\times \sqrt{2}$  =  $\frac{3}{4}$  ft.  $\times \sqrt{2}$  ;  $\therefore$  vol. of parallelepiped =  $AB^2 \times \text{height}$

=  $(\frac{3}{4} \sqrt{2} \text{ ft.})^2 \times 20 \text{ ft.}$  =  $\frac{9}{2} \times 2$  sq. ft.  $\times 20 \text{ ft.}$  =  $90$  cub. ft.

52. Area of triangular base (calculated as in Chap. XIII.) =  $624$  sq. in. ;  
 $\therefore$  vol. of prism =  $624$  sq. in.  $\times 60$  in. =  $37440$  cub. in. = (by hyp.) vol. of cube  
 = side<sup>3</sup> of cube ;  $\therefore$  side of cube =  $\sqrt[3]{(37440 \text{ cub. in.})}$  =  $33.45$  cub. in.



53. Vol. of cylinder

$$= 3 \cdot 1416 \times (4\frac{1}{2} \text{ ft.})^2 \times 4\frac{1}{2} \text{ ft.} = 3 \cdot 1416 \times \frac{17^2}{16} \times \frac{1^2}{2} \text{ cub. ft.} = 17 \times 1848 \times 17^2 \times \frac{19}{4^3} \text{ cub. in.}$$

∴ side of cube required

$$= \sqrt[3]{\left(17^2 \times \frac{1}{4^3} \times 8 \cdot 5112 \text{ cub. ft.}\right)} = \frac{1}{4} \times 1 \cdot 52 \text{ ft. nearly} = 6 \cdot 46 \text{ ft.}$$

54. Vol. of 100000 sovereigns = 100000 × 3·1416 × ( $\frac{1}{16}$  in.)<sup>2</sup> ×  $\frac{1}{16}$  in.

$$= 10^5 \times 100 \times 3 \cdot 1416 \times \frac{49}{16^3} \text{ cub in.} = (\frac{1}{16})^3 \times 15393 \cdot 84 \text{ cub. in.};$$

∴ side of cube required

$$= \sqrt[3]{\{(\frac{1}{16})^3 \times 15393 \cdot 84 \text{ cub. in.}\}} = \frac{1}{16} \times 24 \cdot 88 \text{ in.} = 15 \cdot 55 \text{ in.}$$

55. Here length = 10 × radius;

∴ vol. = 3·1416 × radius<sup>2</sup> × 10 × radius = 31·416 × radius<sup>3</sup> = (by hyp.) 25 cub. ft.;

$$\therefore \text{radius}^3 = 25 \text{ cub. ft.} + 31 \cdot 416 = 795772854 \text{ cub. ft.};$$

$$\therefore \text{radius} = \sqrt[3]{795772854 \text{ cub. ft.}} = 927 \text{ ft. nearly.}$$

56. Volume of 1 gallon = 277 $\frac{1}{2}$  cub. in. =  $\frac{11 \cdot 2^2}{4}$  cub. in.

= (by hyp.) vol. of cylinder = 3·1416 × radius<sup>2</sup> ×  $\frac{1}{2}$  radius;

$$\therefore \text{radius}^3 = 1109 \text{ cub. in.} + (4 \times 1 \cdot 5708) = 176 \cdot 502419 \text{ cub. in.};$$

$$\therefore \text{radius} = \sqrt[3]{(176 \cdot 502419 \text{ cub. in.})} = 5 \cdot 61 \text{ in. nearly.}$$

## CHAPTER XXIV.

1. Circum. GDH = 3·1416 × 2 × AD

$$= 3 \cdot 1416 \times 2 (AC - DC) = 3 \cdot 1416 \times 2AC - 3 \cdot 1416 \times 2DC$$

= circum. FCH or the outer boundary - circum. of cross section (DC being the radius of the cross section); and circum. GDH is the length of the ring.

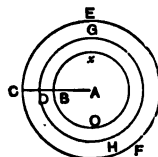
2. (As in preceding) circum. GDH = 2 × 3·1416 × AD

$$= 2 \times 3 \cdot 1416 (AB + BD) = 2 \times 3 \cdot 1416 \times AB + 2 \times 3 \cdot 1416 \times BD$$

= circum. of BxO or inner boundary + circum. of cross section.

3. Vol. of ring = 3·1416 × ( $\frac{1}{4}$  in.)<sup>2</sup> × 20 $\frac{1}{2}$  in. = 3·1416 ×  $\frac{1}{16}$  × 41 cub. in.

$$= \frac{788 \cdot 9343}{16} \text{ cub. in.} = 49 \cdot 3083 \text{ cub. in.} = 49 \cdot 3 \text{ cub. in. nearly.}$$



4. Vol. of ring = 3·1416 × ( $\frac{1 \cdot 1}{2}$  in.)<sup>2</sup> × 16 in. = 3·1416 ×  $\frac{1 \cdot 21}{4}$  × 16 cub. in.

$$= 15 \cdot 205844 \text{ cub. in.} = 15 \cdot 2 \text{ cub. in. nearly.}$$

5. (Vide diag. to 1.) Here  $AB=2.1$  in.;  $AC=2.4$  in.;  $\therefore BC$ , or diameter of cross section  $=AC-AB=(2.4-2.1)$  in.  $=.3$  in.;  $\therefore BD=\frac{1}{2}BC=.15$  in.;  $\therefore$  area of cross section  $=3.1416 \times (.15 \text{ in.})^2 = .071$  sq. in. nearly. Again circum.  $DGH$  or length of ring  $=2 \times 3.1416 \times AD = 2 \times 3.1416 (2.1 + .15)$  in.

$$= 2 \times 3.1416 \times 2.25 \text{ in.} = 14.13 \text{ in. nearly;}$$

$$\therefore \text{vol. of ring} = .071 \times 14.13 \text{ cub. in.} = 1.00323 \text{ cub. in.} = 1 \text{ cub. in. nearly.}$$

6. Area of cross section

$$= 3.1416 \times \left(\frac{3.2}{2} \text{ in.}\right)^2 = 3.1416 \times 2.56 \text{ sq. in.} = 8.042 \text{ sq. in. nearly;}$$

and length of ring

$$= 2 \times 3.1416 (6.15 + 1.6) \text{ in. (vide 2)} = 2 \times 3.1416 \times 7.75 \text{ in.} = 48.694 \text{ in. nearly;}$$

$$\therefore \text{vol. of ring} = 8.042 \text{ sq. in.} \times 48.694 \text{ in.} = 391.597148 \text{ cub. in.}$$

$$= 391.6 \text{ cub. in. nearly.}$$

7. Area of cross section

$$= 3.1416 \times \left(\frac{1.3}{2} \text{ in.}\right)^2 = \frac{.9927 \times 169}{8} \text{ sq. in.} = 8.296 \text{ sq. in. nearly;}$$

and length of ring

$$= 2 \times 3.1416 \left(\frac{1.3}{2} - \frac{1.3}{2}\right) \text{ in. (vide 1)} = 2 \times 3.1416 \times \frac{1.3}{2} \text{ in.} = 49.48 \text{ in. nearly;}$$

$$\therefore \text{vol. of ring} = 8.296 \text{ sq. in.} \times 49.48 \text{ in.}$$

$$= 410.48608 \text{ cub. in.} = 410.5 \text{ cub. in. nearly.}$$

8. Length of ring  $= (15 - 1.6) \text{ in. (vide 1)} = 13.4 \text{ in.}$ ; and radius of cross section  $= \frac{1.6 \text{ in.}}{2 \times 3.1416} = \frac{.8}{3.1416} \text{ in.}$ ;  $\therefore$  area of cross section

$$= 3.1416 \times \frac{.8^2}{(3.1416)^2} \text{ sq. in.} = \frac{.64}{3.1416} \text{ sq. in.};$$

$$\therefore \text{vol. of ring} = \frac{.64}{3.1416} \text{ sq. in.} \times 13.4 \text{ in.} = \frac{1.072}{.3927} \text{ cub. in.} = 2.7 \text{ cub. in. nearly.}$$

9. Area of cross section  $= 3.1416 \times 4 \text{ sq. in.}$ ;  $\therefore$  length of ring

$$= \frac{\text{volume}}{\text{area of cross section}} = \frac{800 \text{ cub. in.}}{3.1416 \times 4 \text{ sq. in.}} = \frac{25}{.3927} \text{ in.} = 63.66 \text{ in.}$$

$$= 63.7 \text{ in. nearly.}$$

10. Area of cross section  $= \frac{\text{vol. of ring}}{\text{length}} = \frac{100 \text{ cub. in.}}{20 \text{ in.}} = 5 \text{ sq. in.};$

$$\therefore \text{radius of cross section} = \sqrt{\left(\frac{5 \text{ sq. in.}}{3.1416}\right)} = \sqrt{(1.591546 \text{ sq. in.})} = 1.261 \text{ in.};$$

$$\therefore \text{circum. of cross section} = 2 \times 3.1416 \times 1.261 \text{ in.} = 7.9231 \text{ in.};$$

$$\therefore \text{inner boundary} + 7.9231 \text{ in.} = 20 \text{ in. (vide 2);}$$

$$\text{i.e. inner diameter} \times 3.1416 + 7.9231 \text{ in.} = 20 \text{ in.};$$

$$\therefore \text{inner diam.} = \frac{(20 - 7.9231) \text{ in.}}{3.1416} = \frac{12.0769}{3.1416} \text{ in.} = 3.84 \text{ in. nearly.}$$

CHAPTER XXV.

$$1. \text{ Vol.} = \frac{1}{3} \times 7 \text{ sq. ft. } 102 \text{ in.} \times 2 \text{ ft. } 5 \text{ in.} = \frac{1}{3} \times 7\frac{1}{2} \text{ sq. ft.} \times 2\frac{1}{2} \text{ ft.} \\ = \frac{1}{3} \times 1\frac{1}{2} \frac{1}{2} \times \frac{1}{2} \frac{1}{2} \text{ cub. ft.} = \frac{1}{3} \frac{1}{2} \frac{1}{2} \frac{1}{2} \text{ cub. ft.} = 6 \text{ cub. ft. } 362 \text{ in.}$$

$$5. \text{ Area of triangular base (calculated as in Chap. XIII.)} \\ = 4\sqrt{6} \text{ sq. ft.} = 4 \times 2.4495 \text{ sq. ft.} = 9.798 \text{ sq. ft.;} \\ \therefore \text{ vol. of pyramid} = \frac{1}{3} \times 9.798 \text{ sq. ft.} \times 6 \text{ ft.} = 19.596 \text{ cub. ft.}$$

$$9. \text{ Vol. of cone} \\ = \frac{1}{3} \times 3.1416 \times (2 \text{ ft.})^2 \times 4 \text{ ft.} = 1.0472 \times 16 \text{ cub. ft.} = 16.7552 \text{ cub. ft.}$$

$$13. \text{ Vol. of pyramid} = \frac{1}{3} \times \text{area of base} \times \text{height} \\ = \frac{1}{3} \times 2 \text{ sq. ft. } 143 \text{ in.} \times \text{height} = \frac{1}{3} \times \frac{1}{2} \frac{1}{2} \text{ sq. ft.} \times \text{height} \\ = (\text{by hyp.}) 17 \text{ cub. ft. } 363 \text{ in.} = 2\frac{1}{2} \frac{1}{2} \text{ cub. ft.;} \\ \therefore \text{ height} = 3 \times 2\frac{1}{2} \frac{1}{2} \text{ cub. ft.} \div \frac{1}{2} \frac{1}{2} \text{ sq. ft.} = \frac{1}{2} \text{ ft.} = 17 \text{ ft. } 3 \text{ in.}$$

$$17. \text{ Area of base of cone} = \frac{3 \times \text{vol.}}{\text{height}} \text{ (vide 13)} \\ = \frac{3 \times 4000 \text{ cub. in.}}{5 \text{ ft.}} = \frac{12000 \text{ cub. in.}}{60 \text{ in.}} = 200 \text{ sq. in.}$$

$\therefore$  radius of base of cone

$$= \sqrt{\left( \frac{200}{3.1416} \text{ sq. in.} \right)} = \sqrt{(63.66 \text{ sq. in.})} = 7.979 \text{ in.}$$

21. Here  $AD = AE = DE = 120 \text{ ft.}$  Then if a perpendicular  $AB$  be let fall from the vertex  $A$  on the base, it will meet the base at the intersection, that is the middle point, of the diagonals.

Thus  $BE = \frac{1}{2} EF$

$$= \frac{1}{2} \sqrt{(ED^2 + DF^2)} = \frac{1}{2} \sqrt{(2ED^2)} = \frac{ED}{2} \sqrt{2}; \therefore EB^2 = \frac{ED^2}{2}.$$

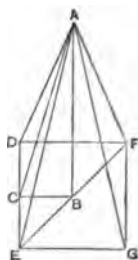
Now  $\angle ABE$  is a right  $\angle$  (Euc. XI. def. 3);

$$\therefore (\text{Euc. I. 47}), AB = \sqrt{(AE^2 - EB^2)} = \sqrt{\left( AE^2 - \frac{ED^2}{2} \right)}$$

$$= \sqrt{\left\{ \left( 120^2 - \frac{120^2}{2} \right) \text{ sq. ft.} \right\}} = \sqrt{\{ 120^2 (1 - \frac{1}{2}) \text{ sq. ft.} \}} \\ = 120 \sqrt{\frac{1}{2}} \text{ ft.} = 1\frac{1}{2} \sqrt{2} \text{ ft.} = 60 \times 1.4142135 \text{ ft.};$$

$$\therefore \text{ vol. of pyramid} = \frac{1}{3} \times 120^2 \text{ sq. ft.} \times 60 \times 1.4142135 \text{ ft.}$$

$$= 288000 \times 1.4142135 \text{ cub. ft.} = 407293.488 \text{ cub. ft.} = 407293.5 \text{ cub. ft. nearly.}$$



22. (Vide diag. to preceding.) Here  $ED = 200$  ft.,  $AE = 150$  ft.,  $\therefore$  (as in preceding)  $AB = \sqrt{\left(AE^2 - \frac{DE^2}{2}\right)} = \sqrt{\left\{\left(150^2 - \frac{200^2}{2}\right)\right\}}$  sq. ft.

$$= \sqrt{\left\{50^2 \left(8^2 - \frac{4^2}{2}\right)\right\}} = 50 \sqrt{\{(9-8)\}} \text{ sq. ft.} = 50 \text{ ft.};$$

$\therefore$  vol of pyramid  $= \frac{1}{3} \times 200^2 \times 50$  cub. ft.  $= \frac{1}{3} \times 2000000$  cub. ft.  $= 666666\frac{2}{3}$  cub. ft.

23. (Vide diag. to 21)  $ED = \sqrt{(20 \cdot 25 \text{ sq. ft.})}$ ;

$\therefore$  (as in 21)  $AB = \sqrt{\left(AE^2 - \frac{ED^2}{2}\right)} = \sqrt{\left[\left\{(30 \cdot 75)^2 - \frac{20 \cdot 25}{2}\right\}\right]}$  sq. ft.

$$= \sqrt{\{945 \cdot 5625 - 10 \cdot 125\}} \text{ sq. ft.} = \sqrt{935 \cdot 4375} \text{ sq. ft.} = 30 \cdot 5849 \text{ ft.}$$

$\therefore$  vol. of pyramid  $= \frac{1}{3} \times 20 \cdot 25 \times 30 \cdot 5849$  cub. ft.  $= 206 \cdot 448075$  cub. ft.  
 $= 206 \cdot 448$  cub. ft. nearly.

24. Hence (as in 21) height

$$= \sqrt{\{(130 \text{ ft.})^2 - (\frac{1}{2} \text{ diagonal})^2\}} = \sqrt{(16900 \text{ sq. ft.} - \frac{1}{4} \text{ diag.}^2)}.$$

Now, (by Euc. I. 47) diag.<sup>2</sup> of base  $= (80^2 + 60^2)$  sq. ft.  $= 10000$  sq. ft.;

$\therefore$  height  $= \sqrt{\{(16900 - 2500) \text{ sq. ft.}\}} = \sqrt{(14400 \text{ sq. ft.})} = 120$  ft.;

$\therefore$  vol. of pyramid  $= \frac{1}{3} \times (80 \times 60) \times 120$  cub. ft.  $= 192000$  cub. ft.

25. (Vide diag. to 21.) Here  $DC = EC = \frac{ED}{2} = 12$  ft. Join  $B, C$ . Then  $C$  being the middle point of  $ED$ , and  $B$  the middle point of  $EF$ , the triangle  $ECB$  is similar to the triangle  $EDF$ , by Euclid VI. 2.

$\therefore BC = \frac{1}{2} DF = 12$  ft.; and  $\angle ABC$  is a rt.  $\angle$  (Euc. XI. def. 8);

$\therefore AB$  (Euc. I. 47)  $= \sqrt{(AC^2 - BC^2)} = \sqrt{\{(21 \cdot 8)^2 - 12^2\}}$  sq. ft.]

$$= \sqrt{\{(33 \cdot 8 \times 9 \cdot 8) \text{ sq. ft.}\}} = \sqrt{\{(2 \times 13 \times 1 \cdot 3 \times 2 \times 7 \times \cdot 7) \text{ sq. ft.}\}}$$

$$= \sqrt{\{(4 \times 9 \cdot 1 \times 9 \cdot 1) \text{ sq. ft.}\}} = 2 \times 9 \cdot 1 \text{ ft.} = 18 \cdot 2 \text{ ft.};$$

$\therefore$  vol of pyramid  $= \frac{1}{3} \times 24^2 \times 18 \cdot 2$  cub. ft.  $= 3494 \cdot 4$  cub. ft.

27. (As in 25), the line joining the middle point of a diagonal with the middle point of the longer side of the rectangle  $= \frac{2}{3}$  ft.  $= 10 \cdot 5$  ft.;

$\therefore$  height  $= \sqrt{\{(28 \cdot 3 \text{ ft.})^2 - (10 \cdot 5 \text{ ft.})^2\}} = \sqrt{(38 \cdot 8 \times 12 \cdot 8 \text{ sq. ft.})}$

$$= \sqrt{(2 \times 13 \times 1 \cdot 3 \times 64 \times \cdot 2 \text{ sq. ft.})} = \sqrt{(64 \times 2 \cdot 6 \times 2 \cdot 6 \text{ sq. ft.})} = 8 \times 2 \cdot 6 \text{ ft.} = 20 \cdot 8 \text{ ft.};$$

$\therefore$  vol. of pyramid  $= \frac{1}{3} \times (21 \times 25) \times 20 \cdot 8$  ft.  $= 3640$  cub. ft.

28. Here the line joining the middle point of a diagonal with the middle point of the shorter side of the rectangle  $= \frac{2}{3}$  ft.  $= 13$  ft.;  $\therefore$  height

$$= \sqrt{\{(24^2 - 13^2) \text{ sq. ft.}\}} = \sqrt{(37 \times 11 \text{ sq. ft.})} = \sqrt{(407 \text{ sq. ft.})} = 20 \cdot 1742 \text{ ft.};$$

$\therefore$  vol. of pyramid  $= \frac{1}{3} \times (18 \times 26) \times 20 \cdot 1742$  cub. ft.  $= 3147 \cdot 1752$  cub. ft.  
 $= 3147 \cdot 18$  cub. ft. nearly.

29. (By Euc. I. 47) height of cone  $= \sqrt{\{(25^2 - 7^2) \text{ sq. ft.}\}}$

$$= \sqrt{(32 \times 18 \text{ sq. ft.})} = \sqrt{(16 \times 2 \times 2 \times 9 \text{ sq. ft.})} = 4 \times 2 \times 3 \text{ ft.} = 24 \text{ ft.};$$

$\therefore$  vol. of cone  $= \frac{1}{3} \times 3 \cdot 1416 \times 7^2 \times 24$  cub. ft.  $= 1231 \cdot 5072$  cub. ft.

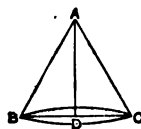
30. Here  $BC$  is diam. of base; and (by hyp.),  $\triangle ABC$  is equilateral;  $\therefore AB=BC=AC=12$  ft., and radius  $BD=6$  ft. Now (by Euc. I. 47),

$$AD = \sqrt{(AB^2 - BD^2)} = \sqrt{\{(12^2 - 6^2) \text{ sq. ft.}\}} = \sqrt{(18 \times 6 \text{ sq. ft.})}$$

$$= \sqrt{(6 \times 6 \times 3 \text{ sq. ft.})} = 6\sqrt{3} \text{ ft.} = 6 \times 1.732 \text{ ft.}$$

$$\therefore \text{vol. of cone} = \frac{1}{3} \times 3.1416 \times 6^2 \times 6 \times 1.732 \text{ cub. ft.}$$

$$= 391.7700864 \text{ cub. ft.} = 391.8 \text{ cub. ft. nearly.}$$



81. (Vide diag. to preceding.) Here  $AB=41$  ft., and  $AD=40$  ft.,  
 $\therefore BD$  (by Euc. I. 47)

$$= \sqrt{(AB^2 - AD^2)} = \sqrt{\{(41^2 - 40^2) \text{ sq. ft.}\}} = \sqrt{(81 \times 1 \text{ sq. ft.})} = 9 \text{ ft.};$$

$$\therefore \text{vol. of cone} = \frac{1}{3} \times 3.1416 \times 9^2 \times 40 \text{ cub. ft.} = 3892.928 \text{ cub. ft.}$$

33. (Vide diag. to 30.) Here  $AB=17$  ft., and  $BD=8$  ft.;  $\therefore$  (by Euc. I. 47)  $AD = \sqrt{(AB^2 - BD^2)} = \sqrt{\{(17^2 - 8^2) \text{ sq. ft.}\}} = \sqrt{(25 \times 9 \text{ sq. ft.})} = 5 \times 3 \text{ ft.} = 15 \text{ ft.};$   
 $\therefore \text{vol. of vessel} = \frac{1}{3} \times 3.1416 \times 8^2 \times 15 \text{ cub. ft.} = 1005.812 \text{ cub. ft.};$  and the vol.

$$\text{of 1 gallon} = 277.274 \text{ cub. in.} = \frac{277.274}{1728} \text{ cub. ft.} = \frac{138.637}{864} \text{ cub. ft.}; \therefore \text{no. of}$$

$$\text{gallons in } 1005.812 \text{ cub. ft.} = \frac{1005.812 \times 864}{138.637} = \frac{868589.568}{138.637} = 6265.2 \text{ gallons}$$

nearly.

$$34. \text{ Vol. of wine glass} = \frac{1}{3} \times 3.1416 \times \left(\frac{3}{4} \text{ in.}\right)^2 \times 3 \text{ in.} = 3.1416 \text{ cub. in.}$$

$$35. \text{ Radius of base} = \frac{16 \text{ ft.}}{2 \times 3.1416} = \frac{8}{3.1416} \text{ ft.};$$

$$\therefore \text{vol. of cone} = \frac{1}{3} \times 3.1416 \times \frac{64}{(3.1416)^2} \times 15 \text{ cub. ft.}$$

$$= \frac{40}{.3927} \text{ cub. ft.} = 101.858 \text{ cub. ft.} = 101.86 \text{ cub. ft. nearly.}$$

36. Here  $ED=3$  ft.,  $FG=2$  ft.,  $AD=5$  ft., and circum. of circle  $BHC=30$  ft.  $\therefore$  radius of circle  $BHC$

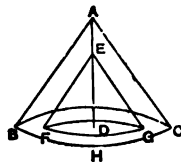
$$= \frac{30 \text{ ft.}}{2 \times 3.1416} = \frac{15}{3.1416} \text{ ft.};$$

$$\therefore \text{vol. of cone } ABC = \frac{1}{3} \times 3.1416 \times \left(\frac{15}{3.1416} \text{ ft.}\right)^2 \times 5 \text{ ft.}$$

$$= \frac{15 \times 25}{3.1416} \text{ cub. ft.} = 119.3659 \text{ cub. ft.};$$

and vol. of cone  $EFG = \frac{1}{3} \times 3.1416 \times 1^2 \text{ sq. ft.} \times 3 \text{ ft.} = 3.1416 \text{ cub. ft.}; \therefore \text{vol. of sand alone} = \text{vol. of cone } ABC - \text{vol. of cone } EFG$

$$= (119.3659 - 3.1416) \text{ cub. ft.} = 116.2243 \text{ cub. ft.}$$



37. Radius of base = 9 ft. + (2 × 3·1416); ∴ area of base  
 = 8·1416 × 81 sq. ft. + (2 × 3·1416)<sup>2</sup> = 27 sq. ft. ÷ (4 × 1·0472);

$$\therefore \text{height} = \text{vol.} \div \frac{1}{3} \text{ area of base} = \frac{45 \times 3 \times 4 \times 1\cdot0472}{2 \times 27} \text{ ft.} = 10\cdot472 \text{ ft.}$$

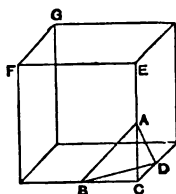
38. The room is in the form of an hexagonal prism having a pyramid with an hexagonal base resting on the top of it. ∴ vol. of room = vol. of prism + vol. of pyramid =  $6 \times \frac{(20 \text{ ft.})^2}{4} \sqrt{3} \times 30 \text{ ft.} + \frac{1}{3} \times 6 \times \frac{(20 \text{ ft.})^2}{4} \sqrt{3} \times 15 \text{ ft.}$

(vide Chap. XV. 6) = (600  $\sqrt{3} \times 30 + 600 \sqrt{3} \times 5$ ) cub. ft.  
 = 600  $\sqrt{3}$  (30 + 5) cub. ft. = 21000 × 1·7320508 cub. ft. = 36373·07 cub. ft. nearly.

39. Here  $\triangle BCD$  is the required pyramid, of which  $\triangle BCD$  is right angled (because  $\angle BCD$  being the angle of a square is a right angle), and  $AC$  is the height; and, by hyp.,  $BC = CD = CA = \frac{1}{2} \text{ ft.} = 10 \text{ ft.}$ ; ∴ vol. of pyramid  $\triangle BCD$

$$= \frac{1}{3} \times \frac{BC \times CD}{2} \times AC = \frac{1}{3} \times \frac{10 \times 10}{2} \times 10 \text{ cub. ft.}$$

$$= 166\frac{2}{3} \text{ cub. ft.} = 166\frac{2}{3} \text{ cub. ft.}$$



40. (As in preceding) vol. of pyramid

$$= \frac{1}{3} \times \frac{6 \times 6}{2} \times 6 \text{ cub. ft.} = 36 \text{ cub. in.};$$

and vol. of whole cube = (14 in.)<sup>3</sup> = 2744 cub. in.; ∴ vol. of solid remaining  
 = (2744 - 36) cub. in. = 2708 cub. in.

41. Vol. of pyramid =  $\frac{1}{3} \times (\frac{1}{4} \text{ yds.})^2 \times \frac{1}{4} \text{ yds.} = \frac{1 \times 1 \times 1}{48} \text{ cub. yds.}$   
 = 3466145 cub. yds. nearly.

42. Vol. of whole spire

$$= \frac{1}{3} \times 6 \times \frac{10^2}{4} \sqrt{3} \times 50 \text{ cub. ft. (vide 38)} = 2500 \sqrt{3} \text{ cub. ft.};$$

and vol. of hollow portion =  $\frac{1}{3} \times 6 \times \frac{9^2}{4} \sqrt{3} \times 45 \text{ cub. ft.} = 1822\cdot5 \sqrt{3} \text{ cub. ft.};$

∴ vol. of stone work alone

$$= (2500 \sqrt{3} - 1822\cdot5 \sqrt{3}) \text{ cub. ft.} = \sqrt{3} (2500 - 1822\cdot5) \text{ cub. ft.}$$

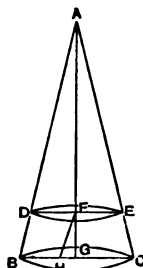
$$= 1\cdot7320508 \times 677\cdot5 \text{ cub. ft.} = 1178\cdot46 \text{ cub. ft. nearly.}$$

## CHAPTER XXVI.

*Note.* The Examples in this Chapter may all be readily solved by the application of the rule in Art. 268 of the text. Here they have been solved by first finding the height of the original cone or pyramid from which the frustum is taken. Then the vol. of the frustum = vol. of whole pyramid or cone of which the frustum is a part - vol. of the smaller pyramid or cone which remains after the frustum has been cut off. This method of solution will be found instructive to the student, though somewhat longer than that furnished by the rule.

$$\begin{aligned} 1. \text{ Vol. required} &= \frac{1}{3} \times 1.5 \text{ ft.} \{ (4.5 + 12.5) \\ &+ \sqrt{(4.5 \times 12.5)} \} \text{ sq. ft.} \times (17 + \sqrt{56.25}) \text{ sq. ft.} \\ &= .5 (17 + 7.5) \text{ cub. ft.} = .5 \times 24.5 \text{ cub. ft.} = 12.25 \text{ cub. ft.} \end{aligned}$$

5. Let  $ABC$  be the cone of which  $DBCE$  is the given frustum, so that  $DF=3$  ft.,  $BG=4$  ft.,  $FG=5\frac{1}{2}$  ft. =  $5\frac{1}{2}$  ft. Now from similar  $\Delta ADF$  and  $ABG$  (by Euc. VI. 4),



$$\begin{aligned} AF : FD :: AG : GB; \\ \therefore AF \times GB = FD \times AG = FD (AF + FG); \\ \therefore AF \times GB = FD \times AF + FD \times FG; \\ \therefore AF (GB - FD) = FD \times FG; \\ \therefore AF = \frac{FD \times FG}{GB - FD} = \frac{3 \text{ ft.} \times 5\frac{1}{2} \text{ ft.}}{4 \text{ ft.} - 3 \text{ ft.}} = 16\frac{1}{2} \text{ ft.}; \end{aligned}$$

$\therefore AG = AF + FG = (16\frac{1}{2} + 5\frac{1}{2})$  ft. = 22 ft.;  $\therefore$  vol. of frustum  $DBCE$  = vol. of cone  $ABC$  - vol. of cone  $ADE$

$$\begin{aligned} &= \frac{1}{3} \times 3 \cdot 1416 \times 4^2 \times 22 \text{ cub. ft.} - \frac{1}{3} \times 3 \cdot 1416 \times 3^2 \times 16\frac{1}{2} \text{ cub. ft.} \\ &= \frac{1}{3} \times 3 \cdot 1416 \times 21 (16 - 16\frac{1}{2}) \text{ cub. ft.} = 3 \cdot 1416 \times 7 \times 16\frac{1}{2} \text{ cub. ft.} = 208 \cdot 4186 \text{ cub. ft.} \end{aligned}$$

(Note. We have proved above that if the cone, of which the frustum is given, be completed, the height of the cone above the smaller end of the frustum is equal to the product of the smaller radius and height of the frustum divided by the difference of the radii of the two ends. In the same manner it may be shewn that the height of the whole cone is equal to the product of the greater radius and height of the frustum divided by the difference of the radii of the two ends.)

9. (Vide diag. to 5.) Here  $DB=5$  ft.,  $DF=7$  ft.,  $BG=10$  ft.; Draw  $FH$  parallel to  $DB$ , meeting  $BG$  in  $H$ . Then (by Euc. I. 34),  $FH=DB=5$  ft., and  $BH=DF=7$  ft.;  $\therefore GH=BG-BH=(10-7)$  ft. = 3 ft.

$$\therefore \text{ (by Euc. I. 47) } FG = \sqrt{(FH^2 - GH^2)} = \sqrt{\{(5^2 - 3^2) \text{ sq. ft.}\}} = \sqrt{(16 \text{ sq. ft.})} = 4 \text{ ft.};$$

$$\therefore \text{ (as in note to 5), } AF = \frac{DF \times FG}{BG - DF} = \frac{7 \text{ ft.} \times 4 \text{ ft.}}{10 \text{ ft.} - 7 \text{ ft.}} = 9\frac{1}{3} \text{ ft.};$$

$$\therefore AG = (9\frac{1}{3} + 4) \text{ ft.} = 13\frac{1}{3} \text{ ft.}; \therefore \text{ vol. of frustum } DBCE$$

$$= \frac{1}{3} \times 3 \cdot 1416 (10^2 \times 13\frac{1}{3} - 7^2 \times 9\frac{1}{3}) \text{ cub. ft. (vide 5)}$$

$$= \frac{1}{3} \times 3 \cdot 1416 (4000 - 1372) \text{ cub. ft.} = \frac{1 \cdot 0472}{3} \times 2628 \text{ cub. ft.} = 917 \cdot 3472 \text{ cub. ft.}$$

10. (Vide diag. to 5.) Here  $DF=\frac{3}{2}$  ft.,  $BG=2$  ft., and  $DB$  or  $FH=8$  ft.;  $\therefore HG$  (as in preceding) = 2 ft. -  $\frac{3}{2}$  ft. =  $\frac{1}{2}$  ft.;  $\therefore FG$  (by Euc. I. 47)

$$= \sqrt{\{8^2 - (\frac{1}{2})^2\}} \text{ sq. ft.} = \sqrt{\{64 - \frac{1}{4}\}} \text{ sq. ft.} = \sqrt{63 \cdot 75} \text{ sq. ft.} = 7 \cdot 9017 \text{ ft.}$$

$$\therefore AF = (\frac{3}{2} \text{ ft.} \times 7 \cdot 9017 \text{ ft.}) \div (2 \text{ ft.} - \frac{3}{2} \text{ ft.}) = 4 \cdot 7410 \text{ ft.};$$

$$\therefore AG = (4 \cdot 7410 + 7 \cdot 9017) \text{ ft.} = 12 \cdot 6427 \text{ ft.} \therefore \text{ vol. of frustum } DBCE$$

$$= \frac{1}{3} \times 3 \cdot 1416 (2^2 \times 12 \cdot 6427 - \frac{9}{4} \times 4 \cdot 7410) \text{ cub. ft.}$$

$$= \frac{1}{3} \times 3 \cdot 1416 (50 \cdot 5708 - 2 \cdot 6668) \text{ cub. ft.} = 1 \cdot 0472 \times 47 \cdot 904 \text{ cub. ft.};$$

$$\therefore \text{ cost} = 1 \cdot 0472 \times 47 \cdot 904 \times £\frac{3}{4} = £60 \cdot 198 \dots = £60. 4s. \text{ nearly.}$$

11. Let  $ABDC$  be the pyramid, of which  $EBDCGF$  is the given frustum, so that  $EF=FG=GE=6$  ft.,  $BD=DC=CB=7$  ft.,  $HK=4$  ft. Now from similar  $\Delta AEF$  and  $ABD$ , (by Euc. VI. 4)  $AB : BD :: AE : EF$ ;  
 $\therefore AB = \frac{AE \times BD}{EF} = \frac{1}{4} \times AE$ ; again from similar  $\Delta AEH$ ,  
 and  $ABK$ ,  $AH : AE :: AK : AB$ ;

$$\text{i.e. } AH : AE :: AH + HK : AB;$$

$$\therefore AH = \frac{AE(AH + HK)}{AB} = \frac{AE(AH + 4 \text{ ft.})}{\frac{1}{4} AE} = \frac{1}{4} AH + 3 \frac{1}{4} \text{ ft.};$$

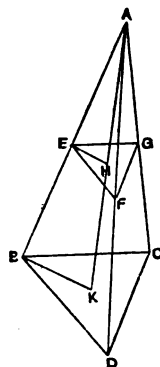
$$\therefore \frac{1}{4} AH = 3 \frac{1}{4} \text{ ft.}; \therefore AH = 24 \text{ ft.}$$

$\therefore AK = (24 + 4) \text{ ft.} = 28 \text{ ft.}; \therefore \text{vol. of given frustum}$   
 $= \text{vol. of pyramid } ABDC - \text{vol. of pyramid } AEF$

$$= \left( \frac{1}{3} \times \frac{7^2}{4} \sqrt{3} \times 28 - \frac{1}{3} \times \frac{6^2}{4} \sqrt{3} \times 24 \right) \text{ cub. ft.}$$

$$= \frac{\sqrt{3}}{12} (4 \times 343 - 216) \text{ cub. ft.} = \frac{1 \cdot 7320508}{3} \times 127 \text{ cub. ft.}$$

$$= 73 \cdot 323 \text{ cub. ft. nearly.}$$



12. Let  $ABCDE$  be the pyramid, of which  $FGBCDEHK$  is the given frustum, so that

$$BC = CD = DE = EB = 30 \text{ ft.};$$

$$FG = GH = HK = KF = 20 \text{ ft.}; FL = LG = 10 \text{ ft.};$$

$BM = MC = 15 \text{ ft.}$ ; and  $LM = 13 \text{ ft.}$  Draw  $OQ$  parallel to  $LM$ , meeting  $MN$  in  $Q$ .  $\therefore OQ = LM$  (by Euc. I. 84)  $= 13 \text{ ft.}$  Now (as in Chap. XXV. 21)

$$MN = \frac{1}{2} BE = 15 \text{ ft.}; \text{ and } LO = \frac{1}{2} FK = 10 \text{ ft.};$$

$$\text{and } QN = MN - MQ = MN - LO \text{ (by Euc. I. 34)} \\ = (15 - 10) \text{ ft.} = 5 \text{ ft.}$$

Now (by Euc. I. 47),  $ON = \sqrt{(OQ^2 - QN^2)}$

$$= \sqrt{\{(169 - 25) \text{ sq. ft.}\}} = \sqrt{(144 \text{ sq. ft.})} = 12 \text{ ft.}$$

Now, join  $A, L$ ; then from similar  $\Delta ALO$  and  $\Delta MN$  (by Euc. VI. 4),  $AO : LO :: AN : MN$ ;

$$\text{i.e., } AO : 10 \text{ ft.} :: AO + 12 \text{ ft.} : 15 \text{ ft.};$$

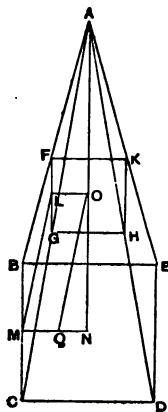
$$\therefore 15 \text{ ft.} \times AO = 10 \text{ ft.} \times AO + 120 \text{ sq. ft.};$$

$$\therefore AO = 24 \text{ ft.}; \therefore AN = 36 \text{ ft.}$$

$\therefore \text{vol. of given frustum}$

$$= \left( \frac{1}{3} \times 30^2 \times 36 - \frac{1}{3} \times 20^2 \times 24 \right) \text{ cub. ft.} = \frac{1}{3} \times 10^2 \times 12 (9 \times 3 - 4 \times 2) \text{ cub. ft.}$$

$$= 100 \times 4 \times 19 \text{ cub. ft.} = 7600 \text{ cub. ft.}$$



13 and 14 are solved like 5.



18. Here  $GE=10$  ft.,  $HB=8$  ft.,

$CF=12$  ft.,  $CD=DF=6$  ft.;

Join  $B, C, E, F$ . Then (as is proved  
Euc. IV. 15),  $BC=BH=8$  ft., and

$EF=EG=10$  ft.;

now, from similar  $\triangle ACB$  and  $AFE$ ,

$AC:CB::AF:FE$ ;

$$\begin{aligned}\therefore AC &= \frac{BC \times AF}{FE} = \frac{8 \text{ ft.} (AC + 12 \text{ ft.})}{10 \text{ ft.}} \\ &= \frac{4 \text{ ft.} \times AC + 48 \text{ sq. ft.}}{5 \text{ ft.}};\end{aligned}$$

$\therefore 5AC = 4AC + 48$  ft.;  $\therefore AC = 48$  ft.;

$\therefore AD = 54$  ft., and  $AF = 60$  ft. Again

$KO = \frac{1}{2} (BH + EG)$

(vide Chap. XIV. 15)  $= 9$  ft.,  $\therefore$  vol. of upper

frustum  $=$  vol. of pyramid whose height is  $AD$   $-$  vol. of pyramid whose height

is  $AC = \left( \frac{1}{3} \times 6 \times \frac{9^2}{4} \sqrt{3} \times 54 - \frac{1}{3} \times 6 \times \frac{8^2}{4} \sqrt{3} \times 48 \right)$  cub. ft. (vide Chap. XV. 6)

$= \frac{1}{2} \times \frac{2}{3} \sqrt{3} \times 6 (9^2 \times 9 - 8^2 \times 8)$  cub. ft.  $= 3\sqrt{3} (729 - 512)$  cub. ft.

$= 3 \times 1.7320508 \times 217$  cub. ft.  $= 1127.6$  cub. ft. nearly.

Again, vol. of remaining frustum  $= \frac{1}{3} \times \frac{2}{3} \sqrt{3} (10^2 \times 60 - 9^2 \times 54)$  cub. ft.

$= \frac{\sqrt{3}}{2} \times 6 (1000 - 729)$  cub. ft.  $= 3 \times 1.7320508 \times 271$  cub. ft.  $= 1408.2$  cub. ft.

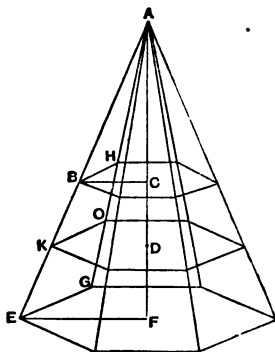
nearly.

*Or thus:* As an example of the rule in the text, Art. 268, we will here find the volume of the frustum  $BHCKOD$ .  $BH=8$  ft.,  $KO=9$  ft.,  $CD=6$  ft. Thus the area of the end  $BCH = \frac{2}{3} \times 8^2 \times \sqrt{3}$  (vide Chap. XV. 6), and the area of the end  $KOD = \frac{2}{3} \times 9^2 \times \sqrt{3}$ .

Thus vol. of frustum  $= \frac{1}{3} \times 6 \left( \frac{2}{3} \times 8^2 \times \sqrt{3} + \frac{2}{3} \times 9^2 \times \sqrt{3} + \frac{2}{3} \times 8 \times 9 \times \sqrt{3} \right)$

$= \frac{1}{3} \times 6 \times \frac{2}{3} \times \sqrt{3} (8^2 + 9^2 + 8 \times 9)$

$= 3\sqrt{3} \times 217 = 651\sqrt{3}$  cub. feet.



## CHAPTER XXVII.

*Note.* A wedge has three parallel edges. If a cross section be made of it by a plane perpendicular to these edges, the section will be a triangle, of which the base is the width of the base of the wedge, and the height is the height of the wedge. The volume of the wedge will be found by multiplying the area of this triangle by one-third the sum of the three parallel edges. Vide Art. 277.

- 1 Length of the edge + twice the length of the base

$$= (27 + 2 \times 27) \text{ inches} = 81 \text{ inches};$$

$$\therefore \text{vol. of wedge} = \frac{81 \times 8 \times 15}{6} = \frac{9720}{6} = 1620 \text{ cub. inches.}$$

2. Length of the edge + twice the length of the base =  $9 + 12 = 21$  feet;

$$\therefore \text{vol.} = \frac{21 \times 8 \times 2}{6} = 21 \text{ cub. feet.}$$

3. Length of the edge + twice the length of the base =  $24 + 30 = 54$  inches;

$$\therefore \text{vol.} = \frac{54 \times 15 \times 24}{6} = 3240 \text{ cub. inches.}$$

4. Use Art. 277. Area of the equilateral base

$$= 4^2 \times \frac{\sqrt{3}}{4} \text{ (vide note, Chap. XIV. 24)} = 4\sqrt{3} \text{ sq. inches};$$

$$\therefore \text{vol.} = \frac{1}{3} \times 4\sqrt{3} \times 4\sqrt{3} = 20\sqrt{3} = 34.64 \text{ cub. inches.}$$

5. Use Art. 279. Area of the rectangular base =  $7 \times 8$  sq. inches;

$$\therefore \text{vol.} = \frac{1}{3} \times 7 \times 8 = 58\frac{2}{3} \text{ cub. inches.}$$

6. Sum of the parallel edges of the wedge =  $2 \times 27 + 21 = 75$  inches;

$$\therefore \text{vol.} = 160 \times \frac{1}{3} = 4000 \text{ cub. inches.}$$

7. Area of the cross section =  $\frac{10^2}{4}\sqrt{3}$  sq. inches (vide note, Chap. XIV. 24),

and sum of the parallel edges =  $25 + 44 = 69$  inches;

$$\therefore \text{vol.} = \frac{1}{3} \times 69 \times \frac{10^2}{4}\sqrt{3} = \frac{69}{12} \times 173.2 = 995.9 \text{ cub. inches.}$$

8. Here  $AD = 15$  inches;  $BF = 24$  inches;  $BC = 7$  inches; and the perpendicular from any point of the edge on the base =  $22$  inches; and the wedge is divided as in the question by the plane  $DGH$ . Thus

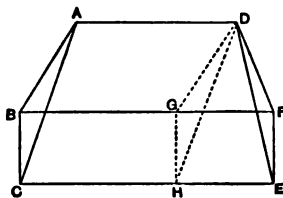
$$GF = BF - AD = 9 \text{ inches.}$$

Then the area of a section of the prism made by a plane perpendicular to the edge is

$$\frac{7 \times 22}{2} \text{ sq. inches} = 77 \text{ sq. inches};$$

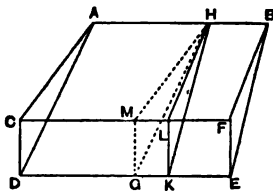
$$\therefore \text{vol. of prism} = 15 \times 77 = 1155 \text{ cub. inches,}$$

$$\text{and vol. of pyramid} = \frac{1}{3} \times 7 \times 9 \times 22 = 462 \text{ cub. inches.}$$



9. Here  $AB=27$  inches,  $CF=33$  inches,  $CD=8$  inches, and the perpendicular from any point of the edge on the base  $=14$  inches. The wedge is divided as in the question by the plane  $MHG$ . We have then

$AH=18$  inches,  $BH=9$  inches,  $MF=15$  inches. Then the area of a section of the prism  $AHCG$  made by a plane perpendicular to the edge is  $\frac{8 \times 14}{2} = 56$  sq. inches;



$$\therefore \text{vol. of prism} = 56 \times 18 = 1008 \text{ cub. inches.}$$

Also, the area of a cross section of the wedge  $HME$  is the same as that of the prism and  $=56$  sq. inches, and the sum of its three parallel edges is  $2 \times 15 + 9 = 39$  inches;

$$\therefore \text{vol. of } HME = \frac{56 \times 39}{8} = 728 \text{ cub. inches.}$$

10. A cross section perpendicular to the edge will be a triangle whose height is that of the wedge, namely 12 inches; its base the breadth of the wedge, namely 5 inches, and  $\therefore$  its area will be  $\frac{5 \times 12}{2} = 30$  sq. inches; and the sum of the three parallel edges of the given wedge  $= 2 \times 27 + 36 = 90$  inches;

$$\therefore \text{vol. of the whole wedge} = 30 \times \frac{1}{2} \times 90 = 900 \text{ cub. inches;}$$

$$\text{and vol. of part cut off} = 30 \times \frac{1}{2} \times 42 = 420 \text{ cub. inches;}$$

$$\therefore \text{vol. of remainder} = 900 - 420 = 480 \text{ cub. inches.}$$

## CHAPTER XXVIII.

1. Area of one rectangular end  $= 400 \times 180 = 72000$  sq. feet; and area of the other rectangular end  $= 350 \times 150 = 52500$  sq. ft. The middle section is a rectangle of which the dimensions are  $\frac{1}{2}(400 + 350)$  and  $\frac{1}{2}(180 + 150)$ , that is, 375 and 165 feet (vide Art. 163), and the area  $= 61875$  sq. feet. Thus the volume of the cavity  $= \frac{1}{3}(72000 + 4 \times 61875 + 52500) = 744000$  cub. feet.

2. Area of the rectangular top  $= 400 \times 180 = 72000$  sq. feet; and area of the rectangular bottom  $= 150 \times 350 = 52500$  sq. feet. The middle section is a rectangle of which the dimensions are  $\frac{1}{2}(400 + 150)$ , and  $\frac{1}{2}(180 + 350)$ , that is 275 and 265 feet, and the area  $= 72875$  sq. feet. Thus the vol. of the cavity  $= \frac{1}{3}(72000 + 4 \times 72875 + 52500) = 832000$  cub. feet.

3. Area of the top rectangle  $= 81 \times 54 = 4374$  sq. inches; and area of the bottom rectangle  $= 42 \times 30 = 1260$  sq. inches. The middle section is a rectangle whose dimensions are  $\frac{1}{2}(81 + 42)$ ,  $\frac{1}{2}(54 + 30)$  inches, and whose area is therefore 2583 sq. inches;

$$\therefore \text{the vol. of the waggon} = \frac{1}{3}(4374 + 4 \times 2583 + 1260) = 125067 \text{ cub. inches.}$$

4. Area of the top rectangle =  $250 \times 16 = 4000$  sq. feet; and area of the bottom rectangle =  $240 \times 14 = 3360$  sq. feet. The middle section is a rectangle whose dimensions are 245 and 15 feet, and whose area = 3675 sq. ft. Thus the vol. of the canal =  $\frac{1}{3} \times \frac{1}{3} (4000 + 4 \times 3675 + 3360) = 16545$  cub. feet; and the number of gallons required to fill it =  $\frac{16545 \times 1728}{277 \cdot 274} = 103110$ .

5. Area of one trapezoidal end =  $\frac{144 + 36}{2} \times 36 = 3240$  sq. feet; and area of the other trapezoidal end =  $\frac{108 + 36}{2} \times 24 = 1728$  sq. ft. The middle section is a trapezoid, of which the corresponding dimensions are  $\frac{1}{3} (144 + 108)$ ,  $\frac{1}{3} (36 + 36)$ ,  $\frac{1}{3} (36 + 24)$ , that is 126, 36, 30 feet, whose area is therefore  $\frac{126 + 36}{2} \times 30 = 2430$  sq. feet.

Thus the volume of the cutting =  $\frac{1}{3} \times 137\frac{1}{2} \times (3240 + 4 \times 2430 + 1728) = 336600$  cub. feet.

6. The length of the section made by the cutting plane =  $\frac{12 + 8}{2} = 10$  ft. (vide Art. 163), and its breadth =  $\frac{6 + 10}{2} = 8$  ft. Thus the vol. of one of the prismoids =  $\frac{1}{3} \times 2 (120 + 4 \times 99 + 80) = 198\frac{2}{3}$  cub. feet., and the vol. of the other =  $\frac{1}{3} \times 2 (80 + 4 \times 63 + 48) = 126\frac{2}{3}$  cub. ft.

7. The length of the upper section =  $\frac{2 \times 10 + 16}{3} = 12$  ft. (vide Chap. XIV. 16), and the breadth =  $\frac{2 \times 8 + 11}{3} = 9$  ft. Thus the volume of the upper prismoid =  $\frac{1}{3} \times 3 (80 + 4 \times 93\frac{1}{2} + 108) = 281$  cub. ft., and the others may be found similarly.

8. In the figure  $EHAC$  is one wedge, and  $BCEG$  is the other. Let  $EH = 20$  ft.,  $BC = 14$  ft.,  $EF = 16$  ft.,  $AB = 12$  ft.; and the height of each wedge will be the same as that of the prismoid, 5 feet. Now in the wedge  $EHAC$  the area of

a cross section =  $\frac{EF \times \text{height}}{2} = 8 \times 5 = 40$  sq. ft.;

and the sum of the three parallel edges =  $20 + 20 + 14 = 54$  ft.;

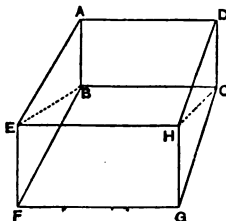
$\therefore$  vol. of wedge =  $40 \times \frac{1}{3} \times 5 = 720$  cub. feet.

Also, in the wedge  $BCEG$  the area of a cross section

=  $\frac{AB \times \text{height}}{2} = 30$  sq. feet;

and the sum of the three parallel edges =  $14 + 14 + 20 = 48$  feet;

$\therefore$  vol. of wedge =  $\frac{30 \times 48}{3} = 480$  cub. feet.



9. The wedge is divided into a prismoid and a wedge, and the dimensions of the figure formed by the cutting plane are,

$$\text{length} = \frac{24+8}{2} = 16 \text{ inches, and breadth } \frac{1}{2} \text{ inches.}$$

Thus the volume of the prismoid  $= \frac{1}{3} \times 8 (7 \times 8 + 4 \times 12 \times \frac{1}{4} + 16 \times \frac{1}{2})$   
 $= 485\frac{1}{3}$  cub. inches.

Also the area of a cross section of the wedge  $= \frac{1}{2} \times 8 \times \frac{1}{2} = 14$  sq. inches;  
 and the sum of its parallel edges  $= 2 \times 16 + 24 = 56$ ;

$$\therefore \text{vol.} = \frac{1}{6} \times 14 = 261\frac{1}{3} \text{ cub. inches.}$$

10. There are two prismoids and a wedge; the dimensions of the section nearest the edge of the wedge are,  $\text{length} = \frac{2 \times 27 + 18}{3} = 24$  inches, and breadth  $= \frac{1}{3}$  of 15 = 5 inches; (vide Chap. XIV. 16). Those of the other section are  $\frac{2 \times 18 + 27}{3} = 21$  inches and  $\frac{1}{3}$  of 15 = 10 inches. Thus area of cross section of the wedge  $= \frac{1}{2} \times 5 \times \frac{1}{3} = 10$  sq. inches, and sum of the parallel edges  $= 2 \times 24 + 27 = 75$ ;  $\therefore \text{vol.} = \frac{1}{6} \times 10 = 250$  cub. inches.

Also vol. of prismoid next to it  $= \frac{1}{3} \times 4 (24 \times 5 + 4 \times \frac{1}{3} \times \frac{1}{3} + 21 \times 10)$   
 $= 670$  cub. inches.

And the volume of the other prismoid may be similarly found.

11. Length of the section  $= \frac{2 \times 12 + 18}{3} = 14$  ft. (vide Chap. XIV. 16), and  
 breadth  $= \frac{2 \times 16 + 10}{3} = 14$  ft.; thus the section is a square.

12. Volume of the prismoid at the larger end  
 $= \frac{1}{3} \times 3 (192 + 4 \times 13 \times 15 + 196) = 584$  cub. feet.

Similarly the other may be found.

## CHAPTER XXIX.

(Note. The vol. of a sphere  $= \frac{1}{6}$  of vol. of circumscribing cylinder, i.e. of a cylinder the diameter of whose base, and whose height, each = diameter of sphere. This is an easily remembered method of expressing the rule of the text. If  $r$  be the radius of the sphere

$$\frac{1}{6} \times 3 \cdot 1416 \times (2r)^2 = \frac{1}{6} \times 3 \cdot 1416 r^2 \times 2r.)$$

1. Vol. of sphere  $= \frac{1}{6} \times 3 \cdot 1416 \times (\frac{1}{2} \text{ in.})^2 \times 11 \text{ in.} = 2618 \times 2662$  cub. in.  
 $= 696 \cdot 9116$  cub. in.

5. Diam. of sphere =  $\frac{6}{8 \cdot 1416}$  ft.;

$\therefore$  vol. of sphere =  $\frac{4}{3} \times 8 \cdot 1416 \times \left(\frac{8}{8 \cdot 1416} \text{ ft.}\right)^3 \times \frac{6}{8 \cdot 1416} \text{ ft.}$   
 $\frac{86}{(8 \cdot 1416)^3} \text{ cub. ft.} = 8 \cdot 65 \text{ cub. ft. nearly.}$

9. Vol. of sphere (were it entire) =  $\frac{4}{3} \times 8 \cdot 1416 \times \left(\frac{5}{8}\right)^3 \times 5 \text{ cub. in.}$ ; and vol. of hollow part =  $\frac{4}{3} \times 8 \cdot 1416 \times 2^3 \times 4 \text{ cub. in.}$ ;  $\therefore$  vol. of shell  
 =  $\frac{4}{3} \times 8 \cdot 1416 (12^3 - 16) \text{ cub. in.} = 2 \times 1 \cdot 0472 \times 2^3 \text{ cub. in.} = 81 \cdot 9396 \text{ cub. in.}$   
 =  $81 \cdot 94 \text{ cub. in. nearly.}$

13. Vol. of bowl

=  $\frac{1}{2} \times \frac{4}{3} \times 8 \cdot 1416 \times 14^3 \times 28 \text{ cub. in.} = 1 \cdot 0472 \times 14^3 \times 28 \text{ cub. in.}$ ;  
 and vol. of 1 gallon =  $277 \cdot 274 \text{ cub. in.}$ ;  $\therefore$  no. of gallons the bowl contains  
 =  $\frac{1 \cdot 0472 \times 14^3 \times 28}{277 \cdot 274} = \frac{2873 \cdot 5168}{188 \cdot 637} = 20 \cdot 78 \text{ nearly.}$

14. Vol. of tank =  $\frac{4}{3} \times \frac{3}{2} \times 8 \cdot 1416 \times 5^3 \times 10 \text{ cub. ft.}$

=  $2618 \times 10^3 \text{ cub. ft.} = 2618 \times 10^3 \times 1728 \text{ cub. in.}$ ;  
 and vol. of 1 gallon =  $277 \cdot 274 \text{ cub. in.}$ ;  
 $\therefore$  no. of gallons in tank =  $\frac{2618 \times 10^3 \times 1728}{277 \cdot 274}$ ;  
 $\therefore$  time required =  $\frac{2618 \times 10^3 \times 1728}{277 \cdot 274 \times 6} \text{ minutes} = \frac{876992}{188 \cdot 637} \text{ minutes} = 272 \text{ minutes}$   
 nearly.

15. Here length of cylinder =  $(29 - 8) \text{ ft.} = 26 \text{ ft.}$ ;  $\therefore$  vol. of solid = vol. of cylinder + 2 vol. of one hemispherical end =  $8 \cdot 1416 \times \left(\frac{5}{8}\right)^3 \times 26 \text{ cub. ft.}$   
 +  $2 \times \frac{1}{2} \times \frac{4}{3} \times 8 \cdot 1416 \times \left(\frac{5}{8}\right)^3 \times 3 \text{ cub. ft.} = 8 \cdot 1416 \times \frac{5}{8} (26 + 2) \text{ cub. ft.}$   
 =  $7854 \times 252 \text{ cub. ft.} = 197 \cdot 9208 \text{ cub. ft.}$

16. Here length of cylinder =  $(22 - \frac{4}{3}) \text{ ft.} = 21 \frac{2}{3} \text{ ft.}$ ;

$\therefore$  vol. of solid =  $8 \cdot 1416 \times \left(\frac{5}{8}\right)^3 \times 21 \frac{2}{3} \text{ cub. ft.} + 2 \times \frac{1}{2} \times \frac{4}{3} \times 8 \cdot 1416 \times \left(\frac{5}{8}\right)^3 \times \frac{5}{2} \text{ cub. ft.}$   
 =  $8 \cdot 1416 \times \frac{5}{8} \times \frac{1}{2} (39 + \frac{10}{3}) \text{ cub. ft.} = \frac{1309 \times 25 \times 127}{4} \text{ cub. ft.}$ ;

and weight of 1 cub. foot of water =  $1000 \text{ oz. Av.} = 1 \frac{1}{4} \text{ lbs.}$ ;  $\therefore$  weight of water  
 =  $\frac{1309 \times 25 \times 127}{4} \times 1 \frac{1}{4} \text{ lbs.} = 6494 \text{ lbs. nearly.}$

17. Quantity of wood cut away = vol. of cube - vol. of sphere

=  $\left(\frac{5}{8} \text{ in.}\right)^3 - \frac{4}{3} \times 8 \cdot 1416 \times \left(\frac{5}{8}\right)^3 \times \frac{5}{8} \text{ cub. in.} = \left(\frac{5}{8}\right)^3 \times (1 - \frac{1}{3} \times 8 \cdot 1416) \text{ cub. in.}$   
 =  $1 \frac{1}{4} \times 4764 \text{ cub. in.} = 48 \cdot 41195 \text{ cub. in.} = 48 \cdot 412 \text{ cub. in. nearly.}$

18. Vol. of shot =  $\frac{2}{3} \times 3 \cdot 1416 \times (3 \text{ in.})^2 \times 6 \text{ in.} = 36 \times 3 \cdot 1416 \text{ cub. in.}$ ; and weight of 1 cub. in. of iron =  $4 \cdot 2 \text{ oz.} = \frac{2 \cdot 1}{8} \text{ lbs.}$

$$\therefore \text{weight of shot} = 36 \times 3 \cdot 1416 \times \frac{2 \cdot 1}{8} \text{ lbs.} = 29 \cdot 688 \text{ lbs.}$$

19. Vol. of 1<sup>st</sup> sphere =  $\frac{4}{3} \times 3 \cdot 1416 \times (2 \text{ in.})^3 \times 4 \text{ in.}$ ; and vol. of 2<sup>nd</sup> sphere =  $\frac{4}{3} \times 3 \cdot 1416 \times (\frac{1}{2} \text{ in.})^3 \times 5 \text{ in.}$  But 1<sup>st</sup> sphere weighs 221·3 oz.;  $\therefore$  2<sup>nd</sup> sphere weighs  $\frac{\frac{4}{3} \times 3 \cdot 1416 \times \frac{1}{8} \times 5}{\frac{4}{3} \times 3 \cdot 1416 \times 4 \times 4} \times 221 \cdot 3 \text{ oz.} = \frac{27662 \cdot 5}{64} \text{ oz.} = 432 \cdot 227 \text{ oz. nearly.}$

20. Vol. of hollow portion

$$= \frac{4}{3} \times 3 \cdot 1416 \times (\frac{1}{4})^3 \times 7 \text{ cub. in.} = 5236 \times 343 \text{ cub. in.};$$

and weight of 30 cub. in. of gunpowder = 1 lb.;  $\therefore$  weight of powder required

$$= \frac{5236 \times 343}{80} \text{ lbs.} = \frac{179 \cdot 5948}{80} \text{ lbs.} = 5 \cdot 98649 \text{ lbs.}$$

22. Internal diam. =  $(10 - 2) \text{ in.} = 8 \text{ in.}$ ;  $\therefore$  vol. of shell

$$= (\frac{4}{3} \times 3 \cdot 1416 \times 5^3 \times 10 - \frac{4}{3} \times 3 \cdot 1416 \times 4^3 \times 8) \text{ cub. in.}$$

$$= \frac{4}{3} \times 3 \cdot 1416 (250 - 128) \text{ cub. in.} = 2 \times 1 \cdot 0472 \times 122 \text{ cub. in.} = \frac{7 \cdot 9849}{54} \text{ cub. ft.};$$

$$\therefore \text{weight} = \frac{7 \cdot 9849}{54} \times 216 \text{ lbs.} = 31 \cdot 9896 \text{ lbs.} = 31 \cdot 94 \text{ lbs. nearly.}$$

24. Vol. of shell

$$= \{\frac{4}{3} \times 3 \cdot 1416 \times (4 \cdot 2)^3 \times 8 \cdot 4 - \frac{4}{3} \times 3 \cdot 1416 \times (3 \cdot 6)^3 \times 7 \cdot 2\} \text{ cub. in.}$$

$$= 2 \times 1 \cdot 0472 \times 16 \times 27 \cdot (7^3 - 6^3) \text{ cub. in.}$$

$$= 1 \cdot 0472 \times 32 \times 27 \times 127 \text{ cub. in.} = 5236 \times 127 \text{ cub. ft.};$$

$\therefore$  weight of shell =  $5236 \times 127 \times 7860 \text{ oz.} = 522 \cdot 667992 \text{ oz.} = 522 \cdot 67 \text{ oz. nearly.}$

25. Inner diam. =  $(13 - \frac{1}{2}) \text{ in.} = \frac{25}{2} \text{ in.}$ ;  $\therefore$  vol. of shell

$$= \frac{4}{3} \times 3 \cdot 1416 \times \{(\frac{13}{2})^3 \times 13 - (\frac{25}{4})^3 \times \frac{25}{2}\} \text{ cub. in.} = 2 \times 1 \cdot 0472 \times \frac{13^3}{4} (1 - \frac{1}{4}) \text{ cub. in.}$$

$$= 5236 \times 2197 \times \frac{3}{4} \text{ cub. in.} = \frac{1309 \times 2197 \times 37}{16} \text{ cub. in.} = \frac{10640 \cdot 7301}{16} \text{ cub. in.};$$

$$\therefore \text{weight of shell} = \frac{10640 \cdot 7301}{16} \times 4 \cdot 4 \text{ oz.} = \frac{11704 \cdot 80311}{4} \text{ oz.} = 2926 \cdot 20077 \text{ oz.}$$

27. Vol. of ball =  $\frac{4}{3} \times 3 \cdot 1416 \times 2^3 \times 4 \text{ cub. in.} = 1 \cdot 0472 \times 32 \text{ cub. in.}$  Inner diam. of shell =  $(20 - 4) \text{ in.} = 16 \text{ in.}$ ; vol. of shell

$$= \frac{4}{3} \times 3 \cdot 1416 \times (10^3 \times 20 - 8^3 \times 16) \text{ cub. in.} = 1 \cdot 0472 \times 32 \times 61 \text{ cub. in.};$$

$$\therefore \text{weight of shell} = \frac{1 \cdot 0472 \times 32 \times 61}{1 \cdot 0472 \times 32} \times 9 \text{ lbs.} = 549 \text{ lbs.}$$

28. Vol. of 1<sup>st</sup> shell  
 $= 2 \times 1.0472 \{ (\frac{1}{2})^3 \times 5 - (\frac{1}{2})^3 \times 3 \}$  cub. in.  $= 1.0472 \times 49$  cub. in. ;  
 and vol. of 2<sup>nd</sup> shell  
 $= 2 \times 1.0472 \times \{ (\frac{1}{2})^3 \times 1\frac{1}{2} - (\frac{1}{2})^3 \times \frac{1}{2} \}$  cub. in.  $= 1.0472 \times 1\frac{1}{2}$  cub. in. ;  
 $\therefore$  weight of latter  $= \frac{1.0472 \times 1323}{1.0472 \times 49 \times 8} \times 1\frac{1}{2}$  lbs.  $= 4\frac{1}{2}$  lbs.  $= 28.6875$  lbs.

29. Vol. of cone  $= \frac{1}{3} \times 3.1416 \times 2^2 \times 7$  cub. in.  $= 1.0472 \times 28$  cub. in. ; and  
 vol. of spherical shell  $= \frac{4}{3} \times 3.1416 \times (2^3 \times 4 - 1 \times 2)$  cub. in.  $= 1.0472 \times 28$  cub. in.  
 Now since the cone and shell are of the same material and volume, they must likewise be of the same weight.

30. Vol. of ball  $= \frac{4}{3} \times 3.1416 \times 2^3 \times 4$  cub. in.  $= 1.0472 \times 32$  cub. in. ; and  
 vol. of pyramid  $= \frac{1}{3} \times \frac{2^2}{4} \times 8$  cub. in.  $= 1.7320508 \times \frac{1}{2}$  cub. in. ;  $\therefore$  weight of  
 pyramid  $= \frac{1.7320508 \times 8}{1.0472 \times 32 \times 3} \times 9$  lbs.  $= 1.24$  lbs.

31. Vol. of cone  
 $= \frac{1}{3} \times 3.1416 \times 4^2$  sq. in.  $\times$  height  $= 1.0472 \times 16$  sq. in.  $\times$  height ;  
 and vol. of sphere  $= \frac{4}{3} \times 3.1416 \times 2^3 \times 4$  cub. in.  $= 1.0472 \times 32$  cub. in. ;  
 $\therefore 1.0472 \times 16$  sq. in.  $\times$  height of cone  $= 1.0472 \times 32$  cub. in. ;  
 $\therefore$  height of cone  $= 2$  in.

32. Vol. of cone  $= \frac{1}{3} \times 3.1416 \times \text{radius}^2 \times 12$  in.  $= 1.0472 \times 12$  in.  $\times$  radius<sup>2</sup> ;  
 and vol. of sphere  $= \frac{4}{3} \times 3.1416 \times 3^3 \times 6$  cub. in.  $= 1.0472 \times 2 \times 3^3 \times 6$  cub. in. ;  
 $\therefore 1.0472 \times 12$  in.  $\times$  radius<sup>2</sup>  $= 1.0472 \times 2 \times 3^3 \times 6$  cub. in. ;  
 $\therefore$  radius<sup>2</sup>  $= 3^3$  sq. in. ;  $\therefore$  radius  $= 3$  in.

33. Radius of base of cone  $= \frac{16}{3.1416}$  ft. ;  $\therefore$  vol. of cone  
 $= \frac{1}{3} \times 3.1416 \times \left( \frac{16}{3.1416} \right)^2$  sq. ft.  $\times$  height  $= \frac{1}{3} \times \frac{16^2}{3.1416}$  sq. ft.  $\times$  height ;  
 and vol. of sphere  $= \frac{4}{3} \times 3.1416 \times 5^3 \times 10$  cub. ft. ;  
 $\therefore$  height of cone  $= \frac{2 \times (3.1416)^2 \times 5^3 \times 10}{16^2}$  ft.  $= 19.27666$  ft.  $= 19.28$  ft. nearly.

34. Vol. of solid  $=$  vol. of cone  $+$  vol. of hemisphere  
 $= \{ \frac{1}{3} \times 3.1416 \times (\frac{5}{2})^2 \times 5 + \frac{1}{2} \times \frac{4}{3} \times 3.1416 \times (\frac{5}{2})^3 \}$  cub. ft.  
 $= 3.1416 \times \frac{5^3}{8} \times 5 (\frac{1}{3} + \frac{1}{2})$  cub. ft.  $= 65.45$  cub. ft.

35. Vol. of Earth  $= \frac{4}{3} \times 3.1416 \times (3950)^3 \times 7900$  cub. miles ; and vol. of  
 Moon  $= \frac{4}{3} \times 3.1416 \times (1080)^3 \times 2160$  cub. miles ;  
 $\therefore$  vol. of Earth  $= (3950)^3 \times 7900 = \left( \frac{395}{108} \right)^3 = \frac{(400-5)^3}{(100+8)^3} = \frac{61629875}{1259712} = 49$   
 nearly.  $\therefore$  the Earth is nearly 49 times larger than the Moon.

86. Vol. of sphere =  $\frac{4}{3} \times 3.1416 \times 10^3 \times 20$  cub. in. = 4188.8 cub. in.; side of cube required =  $\sqrt[3]{(4188.8 \text{ cub. in.})} = 16.12$  in.

87. Vol. of cube =  $(20 \text{ in.})^3 = 8000$  cub. in.; and vol. of sphere

$$= \frac{4}{3} \times 3.1416 \times \left(\frac{\text{diam.}}{2}\right)^3 \times \text{diam.} = \text{diam.}^3 \times .5236;$$

$$\therefore \text{diam.} = \sqrt[3]{(8000 \text{ cub. in.} \div .5236)} = 24.814 \text{ in.}$$

93. Vol. of cylinder =  $3.1416 \times 8^2 \times 12$  cub. in.; and vol. of sphere

$$= \frac{4}{3} \times 3.1416 \times \left(\frac{\text{diam.}}{2}\right)^3 \times \text{diam.} = 3.1416 \times \frac{1}{6} \times \text{diam.}^3;$$

$$\therefore \text{diam. of sphere} = \sqrt[3]{(8^3 \times 12 \times 6 \text{ cub. in.})} = \sqrt[3]{(4608 \text{ cub. in.})} = 16.64 \text{ in.}$$

99. Vol. of hollow sphere =  $15 \times 30$  cub. in. = 450 cub. in.; but internal diam.  $\times \frac{1}{4} \times 3.1416$  = vol. of hollow sphere;  $\therefore$  internal diam.

$$= \sqrt[3]{(450 \times 6 \text{ cub. in.} \div 3.1416)} = \sqrt[3]{(225 \text{ cub. in.} \div .2618)} = 9.5076 \text{ in.}$$

40. Vol. of 1<sup>st</sup> ball =  $\frac{4}{3} \times 1^3 \times 3.1416$  cub. in. = .5236 cub. in.; and vol. of 2<sup>nd</sup> ball =  $\frac{4}{3} \times 3.1416 \times \text{diam.}^3 = .5236 \times \text{diam.}^3$ ;  $\therefore$  weight of latter =  $\text{diam.}^3 \times \frac{1}{16}$  lbs. = (by hyp.) 588 lbs.  $\therefore$  diam. of 2<sup>nd</sup> ball

$$= \sqrt[3]{(588 \times \frac{1}{16} \text{ cub. in.})} = \sqrt[3]{(196 \times 14 \text{ cub. in.})} = \sqrt[3]{(14^3 \text{ cub. in.})} = 14 \text{ in.}$$

41. Vol. of sphere =  $(220.16 \div 6.57)$  cub. in.;

$$\therefore \text{diam.} = \sqrt[3]{\{220.16 \div (6.57 \times .5236)\} \text{ cub. in.}}$$

$$= \sqrt[3]{\{55.04 \div .860013\} \text{ cub. in.}} = \sqrt[3]{(64 \text{ cub. in. nearly})} = 4 \text{ in.}$$

42. Vol. of Stilton cheese =  $3.1416 \times 4^2 \times 14$  cub. in.;  $\therefore$  vol. of Dutch cheese =  $3.1416 \times 4^2 \times 14 \times \frac{2}{3}$  cub. in.  $\therefore$  diam. of latter

$$= \sqrt[3]{(3.1416 \times 56 \times 3 \text{ cub. in.} \div .5236)} = \sqrt[3]{(1008 \text{ cub. in.})} = 10.03 \text{ in.}$$

### CHAPTER XXX.

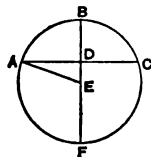
*Note.* (1) Vol. of spherical zone =  $.5236 \times h \{3(r^2 + R^2) + h^2\}$ , and vol. of spherical segment =  $.5236 \times h (3 \times r^2 + h^2)$ , where  $h$  is the height of the zone,  $r$  the radius of the base of the segment, or of one of the ends of the zone, and  $R$  the radius of the other end of the zone.

(2) In the following solutions the diagram represents a section of the sphere made by a plane through the centre at right angles to the base of the segment, or to the ends of the zone. The construction of the diagrams is the same as that employed in the solutions of the examples in Chap. VII.

$$1. \text{ Vol. of zone} = .5236 \times 3 \{3(8^2 + 7^2) + 3^2\} \text{ cub. in.} = .5236 \times 3 \times 348 \text{ cub. in.} \\ = 546.6884 \text{ cub. in.}$$

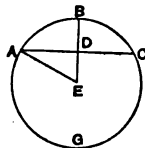
$$3. \text{ Vol. of segment} = .5236 \times 6 (3 \times 4^2 + 6^2) \text{ cub. ft.} = .5236 \times 6 \times 84 \text{ cub. ft.} \\ = 263.8944 \text{ cub. ft.}$$

5. Here  $EA = EB = 6$  ft., and  $DB = 4$  ft.,  $\therefore DE = 2$  ft.;  
 $\therefore$  (Eucl. I. 47),  $AD^2 = AE^2 - ED^2 = (6^2 - 2^2)$  sq. ft. = 32 sq. ft.;  
 $\therefore$  vol. of segment  $ABC = .5236 \times 4 \{3 \times 32 + 4^3\}$  cub. ft.  
 $= .5236 \times 4 \times 112$  cub. ft. = 234.5728 cub. ft.



7. (Vide diag. to 5.) Here  $AD = 12$  ft.,  $AE = 13$  ft.;  
 $\therefore DE = \sqrt{(AE^2 - AD^2)} = \sqrt{\{(13^2 - 12^2)\}$  sq. ft.  $\} = \sqrt{(25 \times 1 \text{ sq. ft.})}$   
 $= 5$  ft.;  $\therefore BD = BE - DE = (13 - 5)$  ft. = 8 ft.;  $\therefore$  vol. of  
segment  $ABC = .5236 \times 8 \{3 \times 12^2 + 8^3\}$  cub. ft.  
 $= .5236 \times 8 \times 496$  cub. ft. = 2077.6448 cub. ft.

9. Here  $AE = EB = 10$  ft.,  $ED = 5$  ft.;  $\therefore$  also  $DB = 5$  ft.;  
 $AD^2 = AE^2 - ED^2 = (10^2 - 5^2)$  sq. ft. = 75 sq. ft.;  
 $\therefore$  vol. of segment  $ABC = .5236 \times 5 \{3 \times 75 + 5^3\}$  cub. ft.  
 $= .5236 \times 5 \times 250$  cub. ft. = 654.5 cub. ft.

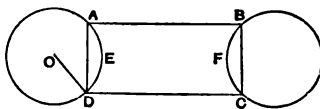


And vol. of segment  $AGC =$  vol. of whole sphere - vol. of  
segment  $ABC = (\frac{4}{3} \times 3.1416 \times 20^3 - 654.5)$  cub. ft.  
 $= (4188.8 - 654.5)$  cub. ft. = 3534.3 cub. ft.

10. (Vide diag. to 5.) Here  $AE = EB = 9$  ft., and  $DF = 2DB$ ; but  
 $DF + DB$ , i. e.  $3DB = BF = 18$  ft.;  $\therefore DB = 6$  ft.;  $\therefore DE = BE - BD = (9 - 6)$  ft.  
 $= 3$  ft.;  $\therefore AD^2 = AE^2 - ED^2 = (9^2 - 3^2)$  sq. ft. = 72 sq. ft.;  $\therefore$  vol. of segment  
 $ABC = .5236 \times 6 \{3 \times 72 + 6^3\}$  cub. ft. =  $.5236 \times 6 \times 252$  cub. ft. = 791.6832 cub. ft.,  
and vol. of segment  $AFC = .5236 \times 12 \{3 \times 72 + 12^3\}$  cub. ft.  
 $= .5236 \times 12 \times 360$  cub. ft. = 2261.952 cub. ft.

11. (Vide diag. to 5.) Here  $AD = 1$  in.,  $AE = EB = \frac{1}{2}$  in.;  
 $\therefore DE = \sqrt{(AE^2 - AD^2)} = \sqrt{\{(\frac{1}{4})^2 - 1^2\}}$  sq. in.  $\} = \sqrt{(1^2 \times \frac{1}{4} \text{ sq. in.})} = \frac{1}{2} \sqrt{65}$  in.  
 $= \frac{1}{2} \times 8.0622$  in. = 2.0156 in. nearly;  $\therefore BD = BE - DE = (2.25 - 2.0156)$  in.  
 $= .2344$  in.;  $\therefore$  vol. of segment  $ABC = .5236 \times .2344 \{3 \times 1^2 + (.2344)^3\}$  cub. in.  
 $= .1227 \times 3.0549$  cub. in. nearly = .87483623 cub. in. = .875 cub. in. very nearly.

12. Here  $AB = 6$  in.,  $AD = 2$  in.,  
 $OD = \frac{1}{2}$  in. Now the segment  $AED$   
has the very same dimensions as the seg-  
ment  $ABC$  in the preceding no.;  $\therefore$  its  
vol. also = .375 cub. in.;  $\therefore$  vol. of dumb-  
bell = vol. of cylinder  $ABCD$  + vol. of  
the two spheres at the ends - vol. of  
the two segments  $AED$  and  $BFC =$  vol. of cylinder  $ABCD$  + 2 vol. of one  
sphere - 2 vol. of segment  $AED$



$= \{3.1416 \times (\frac{AD}{2})^2 \times AB + 2 \times \frac{4}{3} \times 3.1416 \times (\frac{1}{2})^3 - 2 \times .375\}$  cub. in.  
 $= \{3.1416 (6 + 2\frac{1}{4}) - .75\}$  cub. in. =  $(114.2757 - .75)$  cub. in. = 113.5257 cub. in.  
Now vol. of iron ball, the diameter of which is 4 in. =  $\frac{4}{3} \times 3.1416 \times 4^3$  cub. in.  
 $= 33.5104$  cub. in.; and such iron ball weighs 9 lbs.;  $\therefore$  weight of dumb-bell  
 $= \frac{113.5257}{33.5104} \times 9$  lbs. =  $\frac{1021.7313}{33.5104}$  lbs. = 30.49 lbs. = 30.5 lbs. nearly.

13. Here  $BG = GA = GE = \frac{3}{2}$  ft., and

$$BH = HL = LE = 3 \text{ ft.};$$

$$\therefore HG = (\frac{3}{2} - 3) \text{ ft.} = -\frac{3}{2} \text{ ft.}$$

$$\therefore AH^2 = AG^2 - GH^2 = (\frac{5}{2})^2 - (\frac{3}{2})^2 \text{ sq. ft.} = 18 \text{ sq. ft.}$$

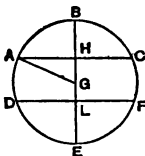
$$\therefore \text{vol. of segment } ABC = .5236 \times 3 (3 \times 18 + 3^2) \text{ cub. ft.}$$

$$= .5236 \times 3 \times 63 \text{ cub. ft.} = 98.9604 = \text{vol. of segment } DEF$$

(by symmetry). Again, vol. of zone  $ADFC$  = vol. of whole sphere - vol. of two segments  $ABC$  and  $DEF$

$$= (\frac{4}{3} \times 3.1416 \times 9^3 - 197.9208) \text{ cub. ft.} = (331.7044 - 197.9208) \text{ cub. ft.}$$

$$= 133.7836 \text{ cub. ft.}$$



14. Here  $AK = KB = 8$  in., and

$$BH = HK = KL = LG = 4 \text{ in.};$$

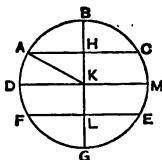
$$\therefore AH^2 = AK^2 - HK^2 = (8^2 - 4^2) \text{ sq. in.} = 48 \text{ sq. in.}; \therefore \text{vol. of segment } ABC = .5236 \times 4 (3 \times 48 + 4^2) \text{ cub. in.}$$

$$= .5236 \times 4 \times 160 \text{ cub. in.} = 335.104 \text{ cub. in.} = \text{vol. of segment } FGE \text{ (by symmetry); again, vol. of zone } ADMC$$

= vol. of hemisphere - vol. of segment  $ABC$

$$= (\frac{1}{2} \times \frac{4}{3} \times 3.1416 \times 16^3 - 335.104) \text{ cub. in.}$$

$$= (1072.3328 - 335.104) \text{ cub. in.} = 737.2288 \text{ cub. in.} = \text{vol. of zone } DFEM.$$



15. Here  $BO = AO = OG = 20$  in.,  $EO = 10$  in.,

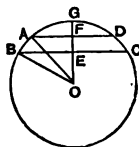
$$FO = 15 \text{ in.}, \therefore EF = 5 \text{ in.}$$

$$\therefore BE^2 = BO^2 - OE^2 = (20^2 - 10^2) \text{ sq. in.} = 300 \text{ sq. in.};$$

$$\text{and } AF^2 = AO^2 - OF^2 = (20^2 - 15^2) \text{ sq. in.} = 175 \text{ sq. in.};$$

$$\therefore \text{vol. of zone } ABCD = .5236 \times 5 \{3 (300 + 175) + 25\} \text{ cub. in.}$$

$$= .5236 \times 5 \times 1450 \text{ cub. in.} = 3796.1 \text{ cub. in.}$$



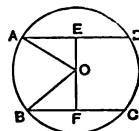
16. Here  $AO = OB = 20$  in.,  $EO = 10$  in.,  $OF = 15$  in.;

$$\therefore \text{(as in preceding), } AE^2 = 300 \text{ sq. in., and } BF^2 = 175 \text{ sq. in.};$$

$$\therefore \text{vol. of zone } ABCD$$

$$= .5236 \times 25 \{3 (300 + 175) + 25\} \text{ cub. in.}$$

$$= .5236 \times 25 \times 2050 \text{ cub. in.} = 26834.5 \text{ cub. in.}$$



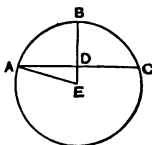
17. Vol. of bowl =  $.5236 \times 9 (3 \times 18^2 + 9^2) \text{ cub. in.}$

$$= .5236 \times 9 \times 1053 \text{ cub. in.}; \text{ and vol. of 1 gallon or 8 pints}$$

$$= 277.274 \text{ cub. in.}; \therefore \text{vol. of 1 pint} = \frac{277.274}{8} \text{ cub. in.};$$

$$\therefore \text{no. of pints in bowl} = \frac{.5236 \times 9 \times 1053 \times 8}{277.274}$$

$$= \frac{19848.6288}{277.274} = 143 \text{ nearly.}$$



18. (Vide diag. to 5.) Here  $AE = EB = \text{radius of sphere}$ ;  $\therefore BD = \frac{1}{2} AE$ ;  
 $\therefore DE = \frac{1}{2} AE$ . Now  $AD^2 = AE^2 - ED^2 = AE^2 - (\frac{1}{2} AE)^2 = \frac{3}{4} AE^2$ ;  
 $\therefore \text{vol. of segment } ABC$   
 $= 5236 \times \frac{1}{2} AE (3 \times \frac{1}{4} AE^2 + \frac{1}{8} AE^2) = 5236 \times \frac{1}{2} \times \frac{1}{8} AE^3$ ;  
 and vol. of sphere whose radius =  $\frac{1}{2} AE$ , (or whose diameter =  $\frac{1}{2} AE$ )  
 $= \frac{1}{8} \times 8 \cdot 1416 \times (\frac{1}{2} AE)^3 = 5236 \times \frac{1}{8} AE^3$ ;  
 $\therefore \text{vol. of segment} = \frac{1}{2} \text{ vol. of such sphere.}$

CHAPTER XXXI.

1. Vol. of water that sinks on the removal of the stone  
 $= 3 \cdot 1416 \times (10 \text{ in.})^2 \times 6 \text{ in.} = 1884 \cdot 96 \text{ cub. in.}$ , which is likewise the vol. of the stone.

2. Vol. required in cubic feet = 4 tons 8 cwt.  $\div$  2716 oz. = 58.06 nearly.

3. Weight of water alone = 8 cwt. - 40 lbs. = 296 lbs.; but 1 gallon weighs 10 lbs.;  $\therefore$  no. of gallons in cask =  $\frac{296}{10} = 29 \cdot 6 = 30$  nearly.

4. Vol. of solid =  $\frac{1}{3} \{ (3 \cdot 72 + 10 \cdot 72) + 2 \times 6 \cdot 96 + 4 (5 \cdot 28 + 8 \cdot 77) \}$  cub. ft.  
 $= (14 \cdot 44 + 13 \cdot 92 + 56 \cdot 20)$  cub. ft. = 84.56 cub. ft.

5. The radii of the sections are respectively,

$$\frac{57}{2 \times 3 \cdot 1416}, \frac{63}{2 \times 3 \cdot 1416}, \frac{69}{2 \times 3 \cdot 1416}, \frac{76}{2 \times 3 \cdot 1416}, \text{ and } \frac{83}{2 \times 3 \cdot 1416} \text{ in.}$$

The areas of the sections will then be respectively,

$$\frac{57^2}{4 \times (3 \cdot 1416)}, \frac{63^2}{4 \times 3 \cdot 1416}, \frac{69^2}{4 \times 3 \cdot 1416}, \frac{76^2}{4 \times 3 \cdot 1416}, \text{ and } \frac{83^2}{4 \times 3 \cdot 1416} \text{ sq. in.}$$

$$\begin{aligned} \therefore \text{vol. of solid} &= \frac{1}{3} \left\{ \frac{1}{4 \times 3 \cdot 1416} [(57^2 + 83^2) + 2 \times 69^2 + 4 (63^2 + 76^2)] \right\} \text{ cub. in.} \\ &= \frac{1}{2 \times 3 \cdot 1416} [(60 - 3)^2 + (80 + 3)^2 + 2 (70 - 1)^2 + 4 \{ (60 + 3)^2 + (70 + 6)^2 \}] \text{ cub. in.} \\ &= \frac{1}{2 \times 3 \cdot 1416} (10138 + 9522 + 38980) \text{ cub. in.} = \frac{1}{2 \times 3 \cdot 1416} \times 58640 \text{ cub. in.} \\ &= \frac{8665}{3927} \text{ cub. in.} = 9333 \text{ cub. in. nearly.} \end{aligned}$$

CHAPTER XXXII.

1. Vol. of 2<sup>nd</sup> ball : vol. of 1<sup>st</sup> ball ::  $(\frac{1}{2} \text{ in.})^3 : (\frac{1}{3} \text{ in.})^3$ ;  $\therefore$  vol. of 2<sup>nd</sup> ball =  $(\frac{1}{2})^3 \times (\frac{1}{3})^3 \times \text{vol. of 1<sup>st</sup> ball} = \frac{21 \times 21 \times 21 \times 8}{64 \times 7 \times 7 \times 7} \times \text{vol. of 1<sup>st</sup> ball} = \frac{21}{8} \times \text{vol. of 1<sup>st</sup> ball}$ ; but 1<sup>st</sup> ball weighs 6 lbs.;  $\therefore$  weight of 2<sup>nd</sup> ball =  $\frac{21}{8} \times 6 \text{ lbs.} = 20 \cdot 25 \text{ lbs.}$

2. If  $x$  represents one of the dimensions of the model,  $9x$  represents the corresponding dimension of the engine.  $\therefore$  vol. of engine : vol. of model  $:: (9x)^3 : x^3$ ; i.e. : 729 : 1;  $\therefore$  vol. of engine = 729  $\times$  vol. of model;  $\therefore$  weight of engine = 729  $\times$  80 lbs. = 58320 lbs.

3. Let  $a, b, c, d$  represent the vols. of the 4 cylinders respectively.

Now  $a : b :: (7 \text{ in.})^3 : (10 \text{ in.})^3$ , i.e. as 343 : 1000;  $\therefore b = \frac{1000}{343} a$ ;

$\therefore a + b = a + \frac{1000}{343} a = \frac{1343}{343} a$ . Again,  $c : a :: (11.03 \text{ in.})^3 : (7 \text{ in.})^3$ , i.e. as 1341.919727 : 343;  $\therefore c = \frac{1341.919727}{343} a$ ; hence  $c$  is less than  $a + b$ . Lastly

$d : a :: (11.04 \text{ in.})^3 : (7 \text{ in.})^3$ , i.e. as 1345.572864 : 343;

$\therefore d = \frac{1345.572864}{343} a$ ;  $\therefore d$  is greater than  $a + b$ .

4. Vol. of smaller pyramid : vol. of whole pyramid  $:: (16 - 4)^3 : 16^3$ ;

$\therefore$  vol. of smaller pyramid =  $\frac{12^3}{16^3} \times 400 \text{ cub. in.} = \frac{12 \times 12 \times 12 \times 400}{16 \times 16 \times 16} \text{ cub. in.}$   
 $= \frac{27}{8} \text{ cub. in.} = 168.75 \text{ cub. in.}$  And vol. of frustum =  $(400 - 168.75) \text{ cub. in.}$   
 $= 231.25 \text{ cub. in.}$

5. Let  $a$  and  $b$  represent respectively the vols., and  $d$  and  $D$  respectively the diameters, of the two cannon balls. Then  $a : b :: 6 \text{ lbs.} : 20 \text{ lbs.}$ ;  
 $\therefore b = \frac{10}{3} a$ . Again,  $a : b :: d^3 : D^3$ ;

$$\therefore D^3 = \frac{bd^3}{a} = \frac{\frac{10}{3} a \times (\frac{7}{3} \text{ in.})^3}{a} = 17\frac{1}{3} \text{ cub. in.} = 142.91\bar{6} \text{ cub. in.};$$

$$\therefore D = \sqrt[3]{(142.91\bar{6} \text{ cub. in.})} = 5.228 \text{ in.}$$

6. Let  $a$  and  $9a$  represent respectively the vols. of the cylinders, and  $b$  the height of the 2<sup>nd</sup> cylinder. Then  $a : 9a :: (4 \text{ ft.})^3 : b^3$ ;

$$\therefore b^3 = 9 \times 64 \text{ cub. ft.} = 576 \text{ cub. ft.}; \therefore b = \sqrt[3]{(576 \text{ cub. ft.})} = 8.320 \text{ ft.}$$

7. (Vide diag. to Chap. XXVI. 16.) Here  $DE = 16 \text{ ft.}$ ,  $BC = 20 \text{ ft.}$ ,  $HL = 5 \text{ ft.}$   $\therefore AH = \frac{DH \times HL}{BL - DH}$  (vide Chap. XXVI. 1 note) =  $\frac{8 \times 5}{10 - 8} \text{ ft.} = 20 \text{ ft.}$   
 $\therefore AL = 25 \text{ ft.}$

Now cone  $ABC$  - cone  $ADE$  = frustum  $DBCE = 2 \times$  frustum  $DFGE$ ;

$\therefore$  frustum  $DFGE = \frac{1}{2}$  (cone  $ABC$  - cone  $ADE$ ).  $\therefore$  cone  $AFG = \frac{1}{2}$  (cone  $ABC$  - cone  $ADE$ ) + cone  $ADE = \frac{1}{2}$  (cone  $ABC$  + cone  $ADE$ )  
 $= \frac{1}{2} \times \frac{1}{3} \times 3.1416 (10^3 \times 25 + 8^3 \times 20) \text{ cub. ft.} = \frac{1}{2} \times 3.1416 \times 1890 \text{ cub. ft.}$

Now vol. of cone  $ADE$  : vol. of cone  $AFG :: AH^3 : AK^3$ ;

$$\therefore AK^3 = \{(\frac{1}{2} \times 3.1416 \times 1890 \times 20^3) \div (\frac{1}{2} \times 3.1416 \times 8^3 \times 20)\} \text{ cub. ft.}$$

$$= 22\frac{1}{2} \text{ cub. ft.} = 11812.5 \text{ cub. ft.};$$

$$\therefore AK = \sqrt[3]{(11812.5 \text{ cub. ft.})} = 22.7744 \text{ ft.}; \therefore HK = 2.7744 \text{ ft.}$$

8. (Vide diag. to Chap. XXVI. 17.) Here  $DL = 8 \text{ ft.}$ ,  $BO = 10 \text{ ft.}$ , and  $LO = 5 \text{ ft.}$  Then (as in preceding),  $AL = 20 \text{ ft.}$ , and  $AO = 25 \text{ ft.}$

Now cone  $ABC$  - cone  $ADE$  = frustum  $DBCE = 3 \times$  frustum  $DFGE$ ;  
 $\therefore$  frustum  $DFGE = \frac{1}{3}$  (cone  $ABC$  - cone  $ADE$ );

$$\begin{aligned}
 \therefore \text{cone } AFG &= \text{cone } ADE + \frac{1}{3} (\text{cone } ABC - \text{cone } ADE) \\
 &= \frac{1}{3} (\text{cone } ABC + 2 \text{ cone } ADE) \\
 &= \frac{1}{3} \times \frac{1}{3} \times 3 \cdot 1416 (10^2 \times 25 + 2 \times 8^2 \times 20) \text{ cub. ft.} \\
 &= \frac{1}{3} \times \frac{1}{3} \times 3 \cdot 1416 \times 5060 \text{ cub. ft.}
 \end{aligned}$$

Now cone  $ADE$  : cone  $AFG$  ::  $AL^2$  :  $AM^2$ ;

$$\therefore AM^2 = \frac{5060 \times 20^2}{3 \times 8^2 \times 20} \text{ cub. ft.} = 219265 \text{ ft.} = 10541 \cdot 6 \text{ cub. ft.};$$

$$\therefore AM = \sqrt[3]{(10541 \cdot 6 \text{ cub. ft.})} = 21 \cdot 9265 \text{ ft.}; \therefore LM = 1 \cdot 9265 \text{ ft.}$$

Again, frustum  $HFGK$  = frustum  $DFGE$  =  $\frac{1}{3}$  (cone  $ABC$  - cone  $ADE$ );  $\therefore$  cone  $AHK$  = cone  $AFG$  +  $\frac{1}{3}$  (cone  $ABC$  - cone  $ADE$ )

$$\begin{aligned}
 &= \text{cone } ADE + \frac{2}{3} (\text{cone } ABC - \text{cone } ADE) = \frac{1}{3} (2 \text{ cone } ABC + \text{cone } ADE) \\
 &= \frac{1}{3} \times \frac{1}{3} \times 3 \cdot 1416 (2 \times 10^2 \times 25 + 8^2 \times 20) \text{ cub. ft.} = \frac{1}{3} \times \frac{1}{3} \times 3 \cdot 1416 \times 6280 \text{ cub. ft.}
 \end{aligned}$$

Now cone  $ADE$  : cone  $AHK$  ::  $AL^2$  :  $AN^2$ ;

$$\therefore AN^2 = \frac{6280 \times 20^2}{3 \times 8^2 \times 20} \text{ cub. ft.} = 13083 \cdot 3 \text{ cub. ft.};$$

$$\therefore AN = \sqrt[3]{(13083 \cdot 3 \text{ cub. ft.})} = 23 \cdot 5635 \text{ ft.}; \therefore LN = 3 \cdot 5635 \text{ ft.}$$

9. Let  $x$  = side of hexagonal base, and  $y$  = height of the pyramid.  $\therefore$  vol. of pyramid =  $\frac{1}{3} \times 6 \times \frac{x^2}{4} \sqrt{3} \times y$  (vide Chap. XV. 6) =  $\cdot 8660x^2y$ . Now the base of the cone will be a circle inscribed in the hexagonal base of the pyramid. The radius of this circle is the height of an equilateral triangle whose side is  $x$ , and  $\therefore = \frac{x}{2} \sqrt{3}$ . The height of the cone is of course =  $y$ .

$$\therefore \text{vol. of cone} = \frac{1}{3} \times 3 \cdot 1416 \times \left(\frac{x}{2} \sqrt{3}\right)^2 \times y = \cdot 7854x^2y;$$

$\therefore$  vol. of portion trimmed off =  $\cdot 8660x^2y - \cdot 7854x^2y = \cdot 0806x^2y$ , which is a little less than  $\cdot 0866x^2y$ , which =  $\frac{\cdot 8660}{10} x^2y$ , i.e.  $\frac{1}{10}$  of the vol. of the pyramid.

10. If circles be inscribed in the square ends of the frustum, these circles will be the ends of the frustum of the cone after the trimming. Now the area of a circle is equal to  $\cdot 7854$  times the area of the square in which it is inscribed. Thus if  $a, b$ , be the areas of the ends of the given frustum, its volume by the rule in Art. 268 is

$$= \frac{1}{3} (a + b + \sqrt{ab}) \times \text{height of frustum};$$

and that of the frustum of the cone

$$= \frac{1}{3} \times \cdot 7854 (a + b + \sqrt{ab}) \times \text{height of the frustum.}$$

Thus the volume of the frustum of the cone

$$= \cdot 7854 \times \text{volume of the frustum of the pyramid.}$$

$\therefore$  part removed =  $(1 - \cdot 7854)$  times the volume of the frustum of the pyramid; and  $1 - \cdot 7854 = \cdot 2146$ , which is rather more than  $\frac{1}{4}$ .

11. (Vide diag. to Chap. XXV. 21.)  $AB = \sqrt{\left( AE^2 - \frac{ED^2}{2} \right)}$  (vide Chap. XXV. 21)  $= \frac{ED}{2} \sqrt{2}$ ;  $\therefore$  vol. of pyramid  $= \frac{1}{3} \times ED^2 \times \frac{ED}{2} \sqrt{2} = ED^3 \times \frac{\sqrt{2}}{6}$ ; but  $ED$  (by hyp.)  $= 1$  ft.;  $\therefore$  vol. of pyramid  $= 1 \times \frac{\sqrt{2}}{6}$  cub. ft.  $= \frac{\sqrt{2}}{6}$  cub. ft.

The latter part of the question follows from the rule in Art. 313.

12. Suppose  $ABC$  of the diagram in Chap. XVI. 39 to be the triangular base of the pyramid; then, as the sloping edges of the pyramid are equal, it may be shown from Euclid I. 47 that the perpendicular from the vertex of the pyramid on the base  $ABC$  will meet it in  $D$ , the centre of the circle circumscribing  $ABC$ . Let  $O$  be the vertex of the pyramid, and therefore  $OD$  the height. Then by Euclid I. 47

$OD^2 + \text{radius } AD^2 = \text{square of an edge of the pyramid.}$

Now from Art. 99 we know that if the radius of the circle be 1 foot, the side of the equilateral triangle is  $\sqrt{3}$  feet; therefore if the side of the triangle is 1 foot, the radius of the circle is  $\frac{1}{\sqrt{3}}$  feet;

$$\text{thus } OD^2 + \left( \frac{1}{\sqrt{3}} \right)^2 = 1 \text{ and } OD^2 = \frac{2}{3};$$

therefore  $OD$ , the height of the pyramid  $= \frac{\sqrt{2}}{\sqrt{3}}$  feet.

Also, the area of the equilateral base of the pyramid (vide Chap. XIV. 24).  $= \frac{1}{2} \sqrt{3}$ , therefore the volume of the pyramid

$$= \frac{1}{3} \times \frac{1}{2} \sqrt{3} \times \frac{\sqrt{2}}{\sqrt{3}} = \frac{\sqrt{2}}{12} \text{ cub. ft.}$$

The latter part of the question follows from the rule in Art. 313.

### CHAPTER XXXIII.

1. Whole surface of cube  $= 6 \times \text{area of one face}$

$$= 6 \times \text{length}^2 = 6 \times \left( \frac{5}{2} \text{ ft.} \right)^2 = \frac{75}{2} \text{ sq. ft.} = 37 \text{ sq. ft. } 72 \text{ in.}$$

5. Surface of parallelepiped

$$= \{ (2 \times 2\frac{1}{2} \times 5) + (2 \times 2\frac{1}{2} \times 5) + (2 \times 3 \times 5) \} \text{ sq. ft.} = 70 \text{ sq. ft.}$$

9. Here bases are right angled (by Euc. I. 48) because  $5^2 = 3^2 + 4^2$ .  $\therefore$  surface of prism

$$= \left\{ 2 \times \frac{3 \times 4}{2} + 3 \times 8 + 4 \times 8 + 5 \times 8 \right\} \text{ sq. ft.} = (12 + 96) \text{ sq. ft.} = 108 \text{ sq. ft.}$$

(observe that the 3 faces are rectangles).

10. Proceed exactly as in 9.

11. Area of triangular base (found as in Chap. XIII.) =  $\frac{5}{2}$  sq. ft.;  
 $\therefore$  surface of prism =  $(2 \times \frac{5}{2} + 7\frac{1}{2} \times \frac{5}{2} + 7\frac{1}{2} \times 8\frac{1}{2} + 7\frac{1}{2} \times 2\frac{1}{2})$  sq. ft.

$$= \{ \frac{5}{2} + 1\frac{1}{2} ( \frac{5}{2} + 1\frac{1}{2} + 2\frac{1}{2} ) \}$$
 sq. ft. =  $(\frac{5}{2} + 50)$  sq. ft. = 51 sq. ft. 96 in.

13. Whole surface of pyramid =  $(2\frac{1}{2}$  ft.)<sup>2</sup> +  $4 \times \frac{1}{2}$  of  $2\frac{1}{2}$  ft.  $\times$   $8\frac{1}{2}$  ft. sq. ft.

$$= (2\frac{1}{2} \times \frac{1}{2} + 2\frac{1}{2} \times 2\frac{1}{2})$$
 sq. ft. =  $2\frac{1}{2} \times 2\frac{1}{2}$  sq. ft. = 24 sq. ft. 47 in.

15. Area of the 4 triangular faces (found as in Chap. XIII.)

$$= 4 \times 1\frac{1}{2}$$
 sq. ft. = 55 sq. ft.

$\therefore$  whole surface of pyramid =  $(8\frac{1}{2})^2 + 55$  sq. ft. = 66 sq. ft. 16 in.

17. (Vide diag. to Chap. XXV. 21.) Here  $DE = 17\frac{1}{2}$  ft., and  $AB = 17\frac{1}{2}$  ft.  
 $\therefore AC = \sqrt{(AB^2 + BC^2)} = \sqrt{(AB^2 + \frac{1}{2} DF^2)}$  (vide Chap. XXV. 25)

$$= \sqrt{[ \{ (\frac{1}{2} DF)^2 + (\frac{1}{2} DF)^2 \} }]$$
 sq. ft. =  $\sqrt{(\frac{1}{2} \times 17\frac{1}{2})^2}$  sq. ft. =  $2\frac{1}{2} \times 17\frac{1}{2}$  ft.

$\therefore$  whole surface of pyramid

$$= (17\frac{1}{2} \text{ ft.})^2 + 4 \times \frac{1}{2} \text{ of } 17\frac{1}{2} \times 2\frac{1}{2} \text{ sq. ft.} = (12\frac{1}{2} \times 12\frac{1}{2} + 2\frac{1}{2} \times 17\frac{1}{2})$$
 sq. ft. = 985 sq. ft. 120 in.

19. Whole surface of frustum

$$= \left( 3^2 + 2^2 + 4 \times \frac{2+3}{2} \times \frac{1}{2} \right)$$
 sq. ft. =  $(9 + 4 + 5)$  sq. ft. = 18 sq. ft.

21. Let the diag. to Chap. XIV. 25 represent one of the faces of the frustum. Here  $AD = 3\frac{1}{2}$  ft.,  $BC = 8\frac{1}{2}$  ft.,  $AB = DC = 5$  in.;

$$\therefore DF = \sqrt{(DE^2 - EF^2)} = \sqrt{ \left\{ DE^2 - \left( \frac{BC - AD}{2} \right)^2 \right\} }$$

$$= \sqrt{ \{ (5^2 - 3^2) \} }$$
 sq. in. = 4 in. =  $\frac{1}{3}$  ft.

$\therefore$  whole surface of frustum =  $\{ (3\frac{1}{2})^2 + (8\frac{1}{2})^2 + 4 \times \frac{1}{2} \times (3\frac{1}{2} + 8\frac{1}{2}) \times \frac{1}{3} \}$  sq. ft.

$$= (1\frac{1}{2} \times 2 + 2\frac{1}{2} \times 2 + \frac{1}{2} \times 2)$$
 sq. ft. =  $1\frac{1}{2} \times 2$  sq. ft. = 30 sq. ft. 84 in.

23. The perpendicular distances of the parallel sides of the trapezoidal faces (found as in 21) = respectively  $\frac{1}{3}$  ft. and 2 ft. (observe only the *opposite* faces in this case are equal).  $\therefore$  whole surface of prismoid

$$= \{ 7 \times 6 + 4\frac{1}{2} \times 4\frac{1}{2} + 2 \times \frac{1}{2} \times (7 + 4\frac{1}{2}) \times \frac{1}{3} + 2 \times \frac{1}{2} \times (6 + 4\frac{1}{2}) \times 2 \}$$
 sq. ft.  

$$= (42 + 1\frac{1}{2} + 1\frac{1}{2} + 1\frac{1}{2})$$
 sq. ft. =  $1\frac{1}{2} \times 1\frac{1}{2}$  sq. ft. = 104 sq. ft. 84 in.

24. Whole surface =  $4 \times \frac{10^2}{4} \sqrt{3}$  sq. ft. =  $100 \times 1.732$  sq. ft. = 173.2 sq. ft.

25. Areas of each of the two unequal triangular faces (found as in Chap. XIII.) = respectively 12.51483 and 16.62958 sq. ft.  $\therefore$  whole surface of pyramid =  $\{ (4\frac{1}{2} \times 6\frac{1}{2}) + 2 \times 12.51483 + 2 \times 16.62958 \}$  sq. ft.

$$= (30 + 25.02966 + 33.25916)$$
 sq. ft. = 88.2888 sq. ft.

26. Whole surface of cube =  $6 \times \text{length}^2$  = (by hyp.)  $7\frac{1}{2}$  sq. ft. =  $1\frac{1}{2} \times 1\frac{1}{2}$  sq. ft.;

$$\therefore \text{length}^2 = \{ \frac{1}{2} \times 6 \}$$
 sq. ft.  $\therefore$  length =  $1\frac{1}{2}$  ft.;

$$\therefore \text{vol. of cube} = (\frac{1}{2} \text{ ft.})^3 = \frac{1}{8} \text{ cub. ft.} = 2197 \text{ cub. in.}$$

## 27. Whole surface of parallelepiped

$$= (2 \times 8 \times 7 + 2 \times 3 \times 9 + 2 \times 7 \times 9) \text{ sq. ft.} = 222 \text{ sq. ft.};$$

$$\therefore 6 \times \text{edge}^2 \text{ of cube} = 222 \text{ sq. ft.};$$

$$\therefore \text{edge of cube} = \sqrt{(1\frac{1}{2} \text{ sq. ft.})} = \sqrt{(37 \text{ sq. ft.})} = 6.083 \text{ ft.}$$

28. Area of base of wedge =  $(10 \text{ in.} \times 2 \text{ in.}) = 20 \text{ sq. ft.}$  The height of each of the trapezoidal faces (found as in 21) =  $24.98 \text{ in.}$ , which is likewise the height of the triangular ends.  $\therefore$  areas of the two trapezoids

$$= 2 \times \frac{12 + 10}{2} \times 24.98 \text{ sq. in.} = 22 \times 24.98 \text{ sq. in.}$$

$$\text{And areas of the triangular ends} = 2 \times \frac{2 \times 24.98}{2} \text{ sq. in.} = 2 \times 24.98 \text{ sq. in.};$$

$\therefore$  whole surface of wedge

$$= \{20 + 24.98 (22 + 2)\} \text{ sq. in.} = (20 + 599.52) \text{ sq. in.} = 619.52 \text{ sq. in.}$$

30. Whole surface of prism =  $\left(2 \times 6 \times \frac{2^3}{4} \sqrt{3} + 6 \times 2 \times 2\right) \text{ sq. ft.}$  (vide Chap. XV. 6) =  $(12\sqrt{3} + 24) \text{ sq. ft.} = 44.7846 \text{ sq. ft.}$

31. Area of one of the triangular faces (found as in Chap. XIII.) =  $1\frac{1}{2} \text{ sq. ft.}$ ; surface required =  $8 \times 1\frac{1}{2} \text{ sq. ft.} = 190 \text{ sq. ft.}$  96 in.

32. Vol. of parallelepiped =  $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \text{ cub. ft.} = 1 \text{ cub. foot} = (\text{by hyp.}) \text{ vol. of cube}$ ;  $\therefore$  edge of cube = 1 foot;  $\therefore$  whole surface of cube

$$= 6 \times 1^2 \text{ sq. ft.} = 6 \text{ sq. ft.}$$

33. Whole surface of 1<sup>st</sup> parallelepiped

$$= (2 \times 8 \times 18 + 2 \times 8 \times 21 + 2 \times 18 \times 21) \text{ sq. in.} = 1380 \text{ sq. in.}$$

And its vol. =  $8 \times 18 \times 21 \text{ cub. in.} = (\text{by hyp.}) \text{ vol. of 2<sup>nd</sup> parallelepiped}$ ;  $\therefore$  area of base of 2<sup>nd</sup> parallelepiped =  $\frac{8 \times 18 \times 21}{21} \text{ sq. in.} = 144 \text{ sq. in.}$ ;  $\therefore$  side of square base of 2<sup>nd</sup> parallelepiped =  $\sqrt{(144 \text{ sq. in.})} = 12 \text{ in.}$ ;  $\therefore$  whole surface of 2<sup>nd</sup> parallelepiped =  $(2 \times 12 \times 12 + 4 \times 12 \times 21) \text{ sq. in.} = 1296 \text{ sq. in.}$

34. Whole surface of parallelepiped *not* on a square base

$$= (2 \times 3 \times 4 + 2 \times 3 \times 5 + 2 \times 4 \times 5) \text{ sq. ft.} = 94 \text{ sq. ft.};$$

and side of square base of the other parallelepiped = (as in preceding)

$$\sqrt{(3 \times 4 \text{ sq. ft.})} = 2\sqrt{3} \text{ ft.} = 2 \times 1.73205 \text{ ft.};$$

$$\therefore \text{whole surface of latter} = (2 \times 2\sqrt{3} \times 2\sqrt{3} + 4 \times 2\sqrt{3} \times 5) \text{ sq. ft.}$$

$$= (24 + 40 \times 1.73205) \text{ sq. ft.} = 93.282 \text{ sq. ft.};$$

$\therefore$  whole surface of latter is less than whole surface of former.

37. (Observe that in the following examples, the thickness of the material is to be disregarded as in Art. 316 (3), of the text.)

Length<sup>3</sup> × height, i. e. length<sup>3</sup> = 1000 cub. in.;

∴ length =  $\sqrt[3]{(1000 \text{ cub. in.})} = 10 \text{ in.}$ ;

∴ whole surface =  $5 \times 10^2 \text{ sq. in.} = 500 \text{ sq. in.}$

38. Here  $2 \times \text{length}^2 = 1000 \text{ cub. in.}$ ; ∴ length =  $\sqrt[3]{(500 \text{ cub. in.})} = 7.937 \text{ in.}$ ;

∴ whole surface =  $\{(7.937)^2 + 4 \times 7.937 \times 2 \times 7.937\} \text{ sq. in.}$

$= (7.937)^2 (1 + 8) \text{ sq. in.} = 566.96 \text{ sq. in.}$

39. Length<sup>3</sup> ×  $\frac{1}{2}$  length, i. e.  $\frac{1}{2}$  length<sup>3</sup> = 1000 cub. in.;

∴ length =  $\sqrt[3]{(2000 \text{ cub. in.})}$ ;

∴ whole surface =  $[\{\sqrt[3]{(2000 \text{ cub. in.})}\}^2 + 4 \times \frac{1}{2} \{\sqrt[3]{(2000 \text{ cub. in.})}\}^2]$

$= (\sqrt[3]{2000})^2 \times 3 \text{ sq. in.} = 476.22 \text{ sq. in.}$

40. Here (working as above), whole surface

$= (\sqrt[3]{1296})^2 (1 + 12) \text{ sq. in.} = 624.97 \text{ sq. in.}$

41. Here whole surface =  $(\sqrt[3]{3000})^2 (1 + \frac{1}{2}) \text{ sq. in.} = 485.35 \text{ sq. in.}$

42. (As in 37), length = 10 in.;

∴ whole surface =  $6 \times (10)^2 \text{ sq. in.} = 600 \text{ sq. in.}$

43. (As in 38), length = 7.937 in.; ∴ whole surface

$= \{2 (7.937)^2 + 4 (7.937) \times 2 \times (7.937)\} \text{ sq. in.}$

$= (7.937)^2 (2 + 8) \text{ sq. in.} = 629.96 \text{ sq. in.}$

44. Whole surface =  $\{2 (\sqrt[3]{2000})^2 + 4 \times \frac{1}{2} (\sqrt[3]{2000})^2\} \text{ sq. in.}$

$= (\sqrt[3]{2000})^2 (2 + 2) \text{ sq. in.} = 634.96 \text{ sq. in.}$

45. Whole surface =  $\{2 (\sqrt[3]{1296})^2 + 4 \times 3 (\sqrt[3]{1296})^2\} \text{ sq. in.}$

$= (\sqrt[3]{1296})^2 (2 + 12) \text{ sq. in.} = 673.05 \text{ sq. in.}$

46. Whole surface =  $\{2 (\sqrt[3]{3000})^2 + 4 \times \frac{1}{2} (\sqrt[3]{3000})^2\} \text{ sq. in.}$

$= (\sqrt[3]{3000})^2 (2 + \frac{1}{2}) \text{ sq. in.} = 693.36 \text{ sq. in.}$

#### CHAPTER XXXIV.

1. Curved surface =  $48 \text{ in.} \times 26 \text{ in.} = 1248 \text{ sq. in.}$

3. Curved surface =  $2 \times 3.1416 \times 17 \text{ in.} \times 22 \text{ in.}$

$= 2349.9168 \text{ sq. in.} = 2349.92 \text{ sq. in. nearly.}$

6. Whole surface =  $\{2 \times 3.1416 \times 2^2 + 2 \times 3.1416 \times 2 \times 3\} \text{ sq. ft.}$

$= 2 \times 3.1416 (2^2 + 6) \text{ sq. ft.} = 62.832 \text{ sq. ft.}$

$$9. \text{ Radius of base} = \frac{20}{2 \times 3.1416} \text{ ft.} = \frac{10}{3.1416} \text{ ft.};$$

$$\therefore \text{ whole surface} = \left\{ 2 \times 3.1416 \times \left( \frac{10}{3.1416} \right)^2 + 20 \times \frac{11}{2} \right\} \text{ sq. ft.}$$

$$= \left( 2 \times \frac{100}{3.1416} + 110 \right) \text{ sq. ft.} = (63.662 + 110) \text{ sq. ft.} = 173.662 \text{ sq. ft.}$$

$$11. \text{ Circumference} \times \text{height} = \text{curved surface};$$

$$\therefore \text{ height} = \frac{\text{curved surface}}{\text{circumference}} = \frac{6 \text{ sq. ft.}}{3 \text{ ft. } 9 \text{ in.}} = \frac{6 \times 144 \text{ sq. in.}}{45 \text{ in.}} = 19.2 \text{ in.}$$

$$12. \text{ Height} = 5\frac{1}{2} \text{ sq. ft.} + (2 \times 3.1416 \times 2\frac{1}{2} \text{ ft.})$$

$$= \frac{45 \times 18}{2 \times 3.1416 \times 27} \text{ in.} = 4.775 \text{ in. nearly.}$$

$$13. \text{ Let } a = \text{radius of base, and height of cylinder. Then whole surface}$$

$$= 2 \times 3.1416 \times a^2 + 2 \times 3.1416 \times a \times a = 4 \times 3.1416 \times a^2$$

$$= (\text{by hyp.}) 14 \text{ sq. ft.} = 14 \times 144 \text{ sq. in.};$$

$$\therefore a^2 = (14 \times 144 \text{ sq. in.}) \div (4 \times 3.1416) = 3 \text{ sq. in.} \div .0187 = 160.427807 \text{ sq. in.};$$

$$\therefore a = \sqrt{(160.427807 \text{ sq. in.})} = 12.666 \text{ in.}$$

$$14. \text{ Let } a = \text{radius; then } 2a = \text{height.}$$

$$\therefore \text{ whole surface} = 2 \times 3.1416 \times a^2 + 2 \times 3.1416 \times a \times 2a = 6 \times 3.1416 \times a^2$$

$$= (\text{by hyp.}) 24 \times 144 \text{ sq. in.};$$

$$\therefore a^2 = (24 \times 144 \text{ sq. in.}) \div (6 \times 3.1416) = 24 \text{ sq. in.} \div .1309 = 183.346066 \text{ sq. in.};$$

$$\therefore a = \sqrt{(183.346066 \text{ sq. in.})} = 13.541 \text{ in.}$$

$$15. \text{ Let } a = \text{radius; then } \frac{1}{2} a = \text{height; } \therefore \text{ whole surface}$$

$$= 2 \times 3.1416 \times a^2 + 2 \times 3.1416 \times a \times \frac{1}{2} a = 3 \times 3.1416 \times a^2 = (\text{by hyp.}) 30 \times 144 \text{ sq. in.};$$

$$\therefore a^2 = (30 \times 144 \text{ sq. in.}) \div (3 \times 3.1416) = 60 \text{ sq. in.} \div .1309 = 458.365165 \text{ sq. in.};$$

$$\therefore a = \sqrt{(458.365165 \text{ sq. in.})} = 21.409 \text{ in.}$$

$$16. \text{ Let } a = \text{radius, and } b = \text{height. Then vol.} = 3.1416 a^2 b = (\text{by hyp.})$$

$$\frac{1}{4} \text{ cub. ft.}; \text{ and curved surface} = 2 \times 3.1416 ab = (\text{by hyp.}) \frac{1}{2} \text{ sq. ft. Now divid-}$$

$$\text{ing vol. by curved surface we get } \frac{3.1416 a^2 b}{2 \times 3.1416 ab} = \frac{\frac{1}{4}}{\frac{1}{2}} \text{ ft.}; \therefore a = \frac{1}{2} \text{ ft.} = 2.6 \text{ ft.}$$

Note. Radius of base of cylinder =  $2 \times$  volume of cylinder  $\div$  its curved surface.

$$17. (\text{As in preceding}) \text{ radius} = \frac{2 \times 5}{4} \text{ ft.} = \frac{5}{2} \text{ ft.}; \therefore \text{ area of one end}$$

$$= 3.1416 \times \left( \frac{5}{2} \text{ ft.} \right)^2 = 19.635 \text{ sq. ft.}$$

$$18. \text{ Let } a = \text{radius, and } b = \text{height. Then}$$

$$\text{vol.} = 3.1416 a^2 b = (\text{by hyp.}) \frac{1}{2} \text{ cub. ft.}$$

$$\therefore a^2 = \frac{9}{4 \times 3.1416 \times b} \text{ sq. ft. Again, curved surface}$$

$$= 2 \times 3.1416 ab = (\text{by hyp.}) 3 \text{ sq. ft. } \therefore a^2 = \left( \frac{3}{2 \times 3.1416 \times b} \right)^2 \text{ sq. ft.};$$

$$\therefore \left( \frac{3}{2 \times 3 \cdot 1416 \times b} \right)^3 = \frac{9}{4 \times 3 \cdot 1416 \times b}; \therefore 3 \cdot 1416 \times b = 1 \text{ ft.};$$

$$\therefore b = \frac{1}{3 \cdot 1416} \text{ ft.} = \frac{1}{2618} \text{ in.} = 3 \cdot 82 \text{ in. nearly.}$$

19. Height =  $\frac{3141 \cdot 6}{314 \cdot 16} \text{ in.} = 10 \text{ in.};$

$$\text{and radius} = \sqrt{\left( \frac{314 \cdot 16}{3 \cdot 1416} \text{ sq. in.} \right)} = \sqrt{(100 \text{ sq. in.})} = 10 \text{ in.}$$

$$\therefore \text{curved surface} = 2 \times 3 \cdot 1416 \times 10 \times 10 \text{ sq. in.} = 628 \cdot 32 \text{ sq. in.}$$

21. (Observe that in the following examples the thickness of the material is to be disregarded, as in Art. 316 (3) of the text.)

Let  $a$  = radius of base. Then vol. =  $3 \cdot 1416 \times a^2 \times a = 3 \cdot 1416 a^3 =$  (by hyp.)  $31416 \text{ cub. in.}; \therefore a^3 = 31416 \text{ cub. in.} \div 3 \cdot 1416 = 10,000 \text{ cub. in.};$

$$\therefore a = \sqrt[3]{(10,000 \text{ cub. in.})};$$

$$\therefore \text{whole internal surface} = 3 \cdot 1416 \times a^2 + 2 \times 3 \cdot 1416 \times a \times a \\ = 3 \cdot 1416 \times \{ \sqrt[3]{(10,000 \text{ cub. in.})} \}^2 \times 3 \text{ sq. in.} = 4375 \text{ sq. in.}$$

22. (Working as in preceding), whole internal surface  
 $= 3 \cdot 1416 \times (\sqrt[3]{5000})^2 \times 5 \text{ sq. in.} = 4593 \text{ sq. in.}$

23. (As above), whole internal surface  
 $= 3 \cdot 1416 \times (\sqrt[3]{20,000})^2 \times 2 \text{ sq. in.} = 4630 \text{ sq. in.}$

24. (As above), whole internal surface  
 $= 3 \cdot 1416 \times (\sqrt[3]{12800})^2 \times 7 \text{ sq. in.} = 4907 \text{ sq. in.}$

25. (As above), whole internal surface  
 $= 3 \cdot 1416 \times (\sqrt[3]{30000})^2 \times \frac{5}{3} \text{ sq. in.} = 5055 \text{ sq. in.}$

26. Whole surface =  $\{ 2 \times 3 \cdot 1416 (\sqrt[3]{10000})^2 + 2 \times 3 \cdot 1416 (\sqrt[3]{10000}) \} \text{ sq. in.}$   
 $= 3 \cdot 1416 \times (\sqrt[3]{10000})^2 \times 4 \text{ sq. in.} = 5833 \text{ sq. in.}$

27. Whole internal surface =  $3 \cdot 1416 \times (\sqrt[3]{5000})^2 \times 6 \text{ sq. in.} = 5512 \text{ sq. in.}$

28. Whole internal surface =  $3 \cdot 1416 (\sqrt[3]{20000})^2 \times 3 \text{ sq. in.} = 6944 \text{ sq. in.}$

29. Whole internal surface =  $3 \cdot 1416 (\sqrt[3]{128000})^2 \times 8 \text{ sq. in.} = 5608 \text{ sq. in.}$

30. Whole internal surface =  $3 \cdot 1416 (\sqrt[3]{300000})^2 \times \frac{5}{3} \text{ sq. in.} = 8088 \text{ sq. in.}$

81. Whole surface of cube =  $6 \times 10^2$  sq. in. = 600 sq. in. And vol. of cube =  $(10 \text{ in.})^3 = 1000$  cub. in. = (by hyp.) vol. of cylinder. But vol. of cylinder =  $3 \cdot 1416 \times \text{radius}^2 \times \text{height}$  = (in this case)  $3 \cdot 1416 \times \text{radius}^2$ ;

$$\therefore \text{radius}^2 \text{ of cylinder} = (1000 \div 3 \cdot 1416) \text{ cub. in.};$$

$\therefore$  radius of cylinder =  $\sqrt[3]{(1000 \text{ cub. in.} \div 3 \cdot 1416)}$ .  $\therefore$  whole surface of cylinder

$$\begin{aligned} &= \left\{ 2 \times 3 \cdot 1416 \left( \sqrt[3]{\frac{1000}{3 \cdot 1416}} \right)^2 + 2 \times 3 \cdot 1416 \times \left( \sqrt[3]{\frac{1000}{3 \cdot 1416}} \right)^2 \right\} \text{ sq. in.} \\ &= 3 \cdot 1416 \times \left( \sqrt[3]{\frac{1000}{3 \cdot 1416}} \right)^2 \times 4 \text{ sq. in.} = 400 \sqrt[3]{3 \cdot 1416} \text{ sq. in.} = 586 \text{ sq. in.} \end{aligned}$$

82. (Note. A cylindrical vessel *without* a lid is of the most advantageous shape, when its height = the radius of its base. See Art. 325.) Now the volume of 1000 gallons =  $1000 \times 277 \cdot 274$  cub. in. = 277274 cub. in.

$\therefore$  (as in preceding) radius =  $\sqrt[3]{\left( \frac{277274}{3 \cdot 1416} \right)}$  in.;  $\therefore$  whole internal surface

$$\begin{aligned} &= \left\{ 3 \cdot 1416 \times \left( \sqrt[3]{\frac{277274}{3 \cdot 1416}} \right)^2 + 2 \times 3 \cdot 1416 \times \left( \sqrt[3]{\frac{277274}{3 \cdot 1416}} \right)^2 \right\} \text{ sq. in.} \\ &= 3 \cdot 1416 \times \left( \sqrt[3]{\frac{277274}{3 \cdot 1416}} \right)^2 \times 3 \text{ sq. in.} \\ &= 3 \times \sqrt[3]{(277274^2 \times 3 \cdot 1416)} \text{ sq. in.} = 18683 \text{ sq. in.} \end{aligned}$$

### CHAPTER XXXV.

1. The angle subtended at the centre of the circle by the chord =  $60^\circ$ , because the triangle formed by the chord and the two radii is equilateral.  $\therefore$  (by Euc. VI. 33), length of arc : circum. of circle ::  $60^\circ : 360^\circ$ ;  $\therefore$  length of arc =  $\frac{1}{6}$  of circum. =  $\frac{1}{6} \times 2 \times 3 \cdot 1416 \times 15$  in.  $\therefore$  area of curved surface =  $\frac{1}{6} \times 2 \times 3 \cdot 1416 \times 15 \times 48$  sq. in. = 753·984 sq. in.

2. Length of arc : circum. of circle ::  $90^\circ : 360^\circ$ ;  $\therefore$  length of arc =  $\frac{1}{4} \times 2 \times 3 \cdot 1416 \times 8$  in.  $\therefore$  area of curved surface =  $\frac{1}{4} \times 2 \times 3 \cdot 1416 \times 8 \times 50$  sq. in. = 628·32 sq. in.

3. The circumference of the cylinder =  $2 \times 3 \cdot 1416 \times 16$  inches;  $\therefore$  curved surface cut off =  $2 \times 3 \cdot 1416 \times 16 \times 35$  sq. in. = 3518·592 sq. in.

4. Surface of ring =  $4 \times 20$  sq. in. = 80 sq. in.

5. Surface of ring =  $2 \times 3 \cdot 1416 \times \frac{3}{4} \times 25$  sq. in. = 117·81 sq. in.

6. Diam. of cross section =  $\left( \frac{4 \cdot 7}{2} - \frac{4 \cdot 1}{2} \right)$  in. (vide Chap. XXIV. 5) = 3 in.;  $\therefore$  circum. of cross section =  $3 \cdot 1416 \times 3$  in. = 94248 in. And length of ring =  $2 \times 3 \cdot 1416 \times \left( \frac{4 \cdot 1}{2} + \frac{3}{2} \right)$  in. (vide ibid.) =  $2 \times 3 \cdot 1416 \times 2 \cdot 2$  in. = 13·823 in.  $\therefore$  surface of ring = 94248  $\times$  13·823 sq. in. = 13·0279 sq. in. = 13·03 sq. in. nearly.

7. Circum. of cross section =  $2 \times 3.1416 \times 1$  in. = 6.2832 in. And length of ring (as in preceding) =  $2 \times 3.1416 (\frac{1}{2} + 1)$  in. = 40.8408 in.;  $\therefore$  surface of ring = 6.2832  $\times$  40.8408 sq. in. = 256.6109 sq. in.

8. Circum. of cross section =  $2 \times 3.1416 \times 2$  in. = 12.5664 in. And length of ring =  $2 \times 3.1416 \times (13 - 2)$  in. = 69.1152 in.;  $\therefore$  surface of ring = 12.5664  $\times$  69.1152 sq. in. = 868.52925 sq. in. = 868.53 in. nearly.

9. Length of ring (vide Chap. XXIV. 1)  
 =  $2 \times 3.1416 \times 2\frac{1}{2}$  in. = 10 in. = 68.54 in.;  
 $\therefore$  surface of ring = 10  $\times$  68.54 sq. in. = 685.4 sq. in.

10. Length of ring =  $2 \times 3.1416 \times 10$  in. + 12 in. (vide Chap. XXIV. 2)  
 = 74.832 in.;  $\therefore$  surface of ring = 12  $\times$  74.832 sq. in. = 897.984 sq. in.

11. Length of ring = surface  $\div$  circum. of cross section  
 =  $(100 \div 2 \times 3.1416 \times 1)$  in. =  $(25 \div 1.5708)$  in. = 15.92 in. nearly.

12. Circum. of cross section = surface  $\div$  length =  $(120 \div 20)$  in. = 6 in.  
 Now inner boundary + circum. of cross section = length of ring (vide Chap. XXIV. 2);  $\therefore$  inner diam. =  $\frac{20 - 6}{3.1416}$  in. =  $\frac{1}{.2244}$  in. = 4.46 in. nearly.

# CHAPTER XXXVI.

*Note.* If, as in Art. 334, the cone be made of cardboard, and cut up from any point in the circumference of its base, in a straight line to its vertex, and then spread out, its curved surface will then form a sector of a circle, the length of the arc of which is the circum. of the base of the original cone, and the radius of the sector, the slant height of the cone. The original height of the cone will, of course,

$$= \sqrt{\{(\text{slant height})^2 - (\text{radius of base})^2\}}.$$

1. Area of curved surface  
 =  $\frac{1}{2}$  (2 ft. 3 in.  $\times$  4 ft. 5 in.) =  $\frac{1}{2} \times 27 \times 53$  sq. in. = 14<sup>31</sup> sq. in. = 715.5 sq. in.

3. Curved surface required =  $\frac{2 \times 3.1416 \times 1 \text{ ft. } 9 \text{ in. } \times 2 \text{ ft.}}{2}$   
 =  $\frac{2 \times 3.1416 \times 21 \times 24}{2}$  sq. in. = 1583.3664 sq. in. = 1583.37 sq. in. nearly.

6. (By Euc. I. 47), slant height  
 =  $\sqrt{\{24^2 + 7^2\}}$  sq. in. =  $\sqrt{625}$  sq. in. = 25 in.;  
 $\therefore$  curved surface required =  $\frac{1}{2} \times 2 \times 3.1416 \times 7 \times 25$  sq. in. = 549.78 sq. in.

11. (By hyp.)  $2 \times 3.1416 \times \text{radius of base of cone} = 6.2832 \text{ ft.}$ ;  $\therefore$  radius of base of cone = 1 foot.  $\therefore$  slant height of cone

$$= \sqrt{\{(5^2 + 1^2) \text{ sq. ft.}\}} = 12\sqrt{26} \text{ in.} = 61.188 \text{ in.};$$

$\therefore$  curved surface required =  $\frac{1}{2} (6.2832 \times 12 \times 61.188) \text{ sq. in.}$

$$= 2306.7386 \text{ sq. in.} = 2306.74 \text{ sq. in. nearly.}$$

13. Whole surface = area of base + area of curved surface

$$= (3.1416 \times 2^2 + \frac{1}{2} \times 2 \times 3.1416 \times 2 \times 4) \text{ sq. ft.} = 4 \times 3.1416 \times 3 \text{ sq. ft.} = 37.6992 \text{ sq. ft.}$$

15. (As in 11), radius of base of cone =  $\frac{8}{2 \times 3.1416} \text{ ft.} = \frac{4}{3.1416} \text{ ft.};$

$\therefore$  whole surface =  $\left\{ 3.1416 \times \left( \frac{4}{3.1416} \right)^2 + \frac{1}{2} \times 6 \times 8 \right\} \text{ sq. ft.}$

$$= \left( \frac{2}{.3927} + 24 \right) \text{ sq. ft.} = (5.093 \text{ nearly} + 24) \text{ sq. ft.} = 29.093 \text{ sq. ft.}$$

17. (As in 6), slant height =  $\sqrt{\{1^2 + (\frac{5}{12})^2\}} \text{ sq. ft.} = \sqrt{\{1\frac{25}{144}\}} \text{ sq. ft.} = 1\frac{5}{12} \text{ ft.}$

$\therefore$  whole surface =  $\{3.1416 \times (\frac{5}{12})^2 + \frac{1}{2} \times 2 \times 3.1416 \times \frac{5}{12} \times 1\frac{5}{12}\} \text{ sq. ft.}$

$$= \frac{5}{12} \times 3.1416 \times 18 \text{ sq. ft.} = .1309 \times 15 \text{ sq. ft.} = 1.9635 \text{ sq. ft.}$$

19. (As in 11), radius of base =  $(27 \div 2 \times 3.1416) \text{ in.} = .36 \text{ ft. nearly};$  then (as in 6), slant height =  $\sqrt{\{(\frac{3}{4})^2 + (.36)^2\}} \text{ sq. ft.} = \sqrt{(2.3796)} \text{ sq. ft.} = 1.54 \text{ ft.};$

$\therefore$  whole surface =  $3.1416 \times [27 \div (2 \times 3.1416 \times 12)]^2 + \frac{1}{2} \times \frac{3}{4} \times 1.54 \} \text{ sq. ft.}$

$$= \frac{9}{8 \times 3.1416} + 1.54 \} \text{ sq. ft.} = \frac{9}{8} (.36 + 1.54) \text{ sq. ft.} = 2.137 \text{ sq. ft.}$$

21. Slant height = 2 curved surface  $\div$  circum. of base

$$= (2 \times 750 \div 50) \text{ in.} = 30 \text{ in.}$$

22. Slant height =  $(2 \times 800 \div 64) \text{ in.} = 25 \text{ in.};$

$$\text{and radius of base} = \frac{64}{2 \times 3.1416} \text{ in.} = \frac{4}{.3927} \text{ in.}$$

Now, height of cone =  $\sqrt{\left[ 25^2 - \left( \frac{4}{.3927} \right)^2 \right] \text{ sq. in.}}$

$$= \sqrt{\left( \frac{13.8175 \times 5.8175}{.3927 \times .3927} \text{ sq. in.} \right)} = \frac{8.9657}{.3927} \text{ in.} = 22.831 \text{ in.}$$

23. Slant height

$$= \{(2 \times 12) + (2 \times 3.1416 \times 1\frac{1}{2})\} \text{ ft.} = (1 \div .3927) \text{ ft.} = 2.5465 \text{ ft.}$$

24. Slant height

$$= \{(2 \times 25) + (2 \times 3.1416 \times 2\frac{1}{4})\} \text{ ft.} = \{100 + (9 \times 3.1416)\} \text{ ft.};$$

$\therefore$  height (as in 22) =  $\sqrt{\{100 + (9 \times 3.1416)\}^2 - (\frac{9}{2})^2} \text{ sq. ft.}$

$$= \sqrt{\left( \frac{163.6174 \times 36.3826}{9 \times 3.1416 \times 9 \times 3.1416} \text{ sq. ft.} \right)} = \frac{77.1546}{28.2744} \text{ ft.} = 2.729 \text{ ft. nearly.}$$

25. Circum. of base = 2 curved surface + slant height  
 $= (2 \times 650 + 25) \text{ in.} = 52 \text{ in.}$

26. Circum. of base =  $(2 \times 18 \div 3\frac{1}{2}) \text{ ft.} = (2 \times 18 \times 8 \div 25) \text{ ft.}; \therefore \text{radius of base} = \{(18 \times 8) \div (25 \times 3 \cdot 1416)\} \text{ ft.} = \frac{6}{3 \cdot 2726} \text{ ft.} = 1 \cdot 8335 \text{ ft.}$

27. Area of whole surface  
 $= 3 \cdot 1416 \times \text{radius}^2 \text{ of base} + \frac{1}{2} \times 2 \times 3 \cdot 1416 \times \text{radius of base} \times 3 \text{ radius of base}$   
 $= 4 \times 3 \cdot 1416 \times \text{radius}^2 \text{ of base} = (\text{by hyp.}) 15 \text{ sq. ft.};$   
 $\therefore \text{radius}^2 \text{ of base} = 1 \cdot 9365928 \text{ sq. ft.}$   
 $\therefore \text{radius of base} = \sqrt{1 \cdot 9365928 \text{ sq. ft.}} = 1 \cdot 0925 \text{ ft.}$

29. (As in 6), slant height =  $\sqrt{[(\frac{1}{2})^2 + 2^2] \text{ sq. yds.}} = 2\frac{1}{2} \text{ yds.}; \therefore \text{curved surface of tent} = \frac{1}{2} \times 2 \times 3 \cdot 1416 \times 2 \times 2\frac{1}{2} \text{ sq. yds.} = 3 \cdot 1416 \times 5 \text{ sq. yds.},$  which is also the quantity of canvas required for the tent.  $\therefore \text{length of canvas required} = 3 \cdot 1416 \times 2\frac{1}{2} \times \frac{1}{2} \text{ yds.} = 27 \cdot 9253 \text{ yds.} = 28 \text{ yds. nearly.}$

31. (In the following examples we shall represent the radius of the base by  $a$ .) Area of whole surface

$= (3 \cdot 1416 \times a^2 + 3 \cdot 1416 \times a \times 2a) = 3 \times 3 \cdot 1416 \times a^2 = (\text{by hyp.}) 100 \text{ sq. ft.};$   
 $\therefore a^2 = (100 \div 3 \times 3 \cdot 1416) \text{ sq. ft.}$  Now, height of cone (as in 22)  
 $= \sqrt{(4a^2 - a^2)} = \sqrt{3a^2} = \sqrt{\{(100 \div 3 \cdot 1416) \text{ sq. ft.}\}} = \sqrt{(31 \cdot 830914 \text{ sq. ft.})} = 5 \cdot 642 \text{ ft.};$   
 $\therefore \text{vol. of cone} = \frac{1}{3} \times 3 \cdot 1416 \times \frac{100}{3 \times 3 \cdot 1416} \times 5 \cdot 642 \text{ cub. ft.} = 62 \cdot 688 \text{ cub. ft.}$

32. (Working as in preceding),  $a^2 = \{100 \text{ sq. ft.} \div (4 \times 3 \cdot 1416)\}; \therefore \text{height}$   
 $= \sqrt{\{(3a)^2 - a^2\}} = \sqrt{8a^2} = \sqrt{\{(200 \div 3 \cdot 1416) \text{ sq. ft.}\}}$   
 $= \sqrt{(63 \cdot 661828 \text{ sq. ft.})} = 7 \cdot 9788 \text{ ft.}$

Hence vol. of cone

$$= \frac{1}{3} \times 3 \cdot 1416 \times (100 \div 4 \times 3 \cdot 1416) \times 7 \cdot 9788 \text{ cub. ft.} = 66 \cdot 490 \text{ cub. ft.}$$

33. Area of whole surface

$$= 3 \cdot 1416a^2 + \frac{1}{2} \times 2 \times 3 \cdot 1416a \times 4a = 5 \times 3 \cdot 1416a^2 = 100 \text{ sq. ft. (by hyp.);}$$

$$\therefore a^2 = 100 \div (5 \times 3 \cdot 1416) \text{ sq. ft.};$$

$$\text{and the height} = \sqrt{(4a)^2 - a^2} = \sqrt{15a^2}$$

$$= \sqrt{300 \div 3 \cdot 1416} = 9 \cdot 77204 \text{ ft.}$$

The solution may be completed as before.

34. Area of whole surface

$$= 3 \cdot 1416a^2 + \frac{1}{2} \times 2 \times 3 \cdot 1416a \times 5a = 6 \times 3 \cdot 1416a^2 = 100 \text{ sq. ft. (by hyp.);}$$

$$\therefore a^2 = 100 \div (6 \times 3 \cdot 1416) \text{ sq. ft.};$$

$$\text{and the height} = \sqrt{(5a)^2 - a^2} = \sqrt{24a^2}$$

$$= \sqrt{400 \div 3 \cdot 1416} = 11 \cdot 28378 \text{ ft.}$$

The solution may be completed as before.

35. Area of whole surface

$$= 3 \cdot 1416 a^2 + \frac{1}{2} \times 2 \times 3 \cdot 1416 a \times 6a = 7 \times 3 \cdot 1416 a^2 = 100 \text{ sq. ft. (by hyp.)};$$

$$\therefore a^2 = 100 \div (7 \times 3 \cdot 1416) \text{ sq. ft.};$$

$$\begin{aligned} \text{and the height} &= \sqrt{(6a)^2 - a^2} = \sqrt{35a^2} \\ &= \sqrt{500 \div 3 \cdot 1416} = 12 \cdot 6156 \text{ ft.} \end{aligned}$$

The solution may be completed as before.

$$36. \text{ Vol. of cone} = \frac{1}{3} \times 3 \cdot 1416 \times a^2 \times a = \frac{3 \cdot 1416}{3} a^3 = (\text{by hyp.}) 314160 \text{ cub. in.};$$

$$\therefore a^3 = 300000 \text{ cub. in.}; \therefore a = \sqrt[3]{(300000 \text{ cub. in.})}. \text{ Now slant height (as in 6)}$$

$$= \sqrt{(a^2 + a^2)} = a\sqrt{2} = \sqrt[3]{(300000 \text{ cub. in.})} \times \sqrt{2}; \therefore \text{ whole surface}$$

$$\begin{aligned} &= \{3 \cdot 1416 (\sqrt[3]{300000})^2 + \frac{1}{2} \times 2 \times 3 \cdot 1416 (\sqrt[3]{300000})^2 \times \sqrt{2}\} \text{ sq. in.} \\ &= (\sqrt[3]{300000})^2 \times 3 \cdot 1416 (1 + 1 \cdot 4142) \text{ sq. in.} = 33989 \text{ sq. in.} \end{aligned}$$

$$37. \text{ Vol. of cone} = \frac{1}{3} \times 3 \cdot 1416 \times a^2 \times 2a = \frac{2}{3} \times 3 \cdot 1416 a^3$$

$$= (\text{by hyp.}) 314160 \text{ cub. in.};$$

$$\therefore a^3 = 150000 \text{ and } a = \sqrt[3]{150000} \text{ in.}$$

$$\text{And the slant height} = \sqrt{(2a)^2 + a^2} = a\sqrt{5} = \sqrt{150000} \times \sqrt{5};$$

$$\begin{aligned} \therefore \text{ whole surface} &= 3 \cdot 1416 \sqrt[3]{(150000)}^2 + \frac{1}{2} \times 2 \times 3 \cdot 1416 \sqrt[3]{(150000)}^2 \times \sqrt{5} \\ &= \sqrt[3]{(150000)}^2 \times 3 \cdot 1416 (1 + 2 \cdot 2361) \\ &= 2823 \cdot 1 \times 3 \cdot 1416 \times 3 \cdot 2361 \\ &= 28701 \text{ sq. in.} \end{aligned}$$

$$38. \text{ Vol. of cone} = \frac{1}{3} \times 3 \cdot 1416 \times a^2 \times 3a = 3 \cdot 1416 a^3 = (\text{by hyp.}) 314160 \text{ cub. in.}$$

$$\therefore a^3 = 100000 \text{ and } a^2 = \sqrt[3]{(100000)}^2 = 2154 \cdot 435 \text{ in.}$$

Also the slant height  $= \sqrt{(3a)^2 + a^2} = a\sqrt{10}$ ; and the solution may be completed as before.

$$39. \text{ Vol. of cone} = \frac{1}{3} \times 3 \cdot 1416 \times a^2 \times \frac{1}{2}a = \frac{1}{6} \times 3 \cdot 1416 \times a^3 = 314160 (\text{by hyp.});$$

$$\therefore a^3 = 600000 \text{ and } a^2 = \sqrt[3]{(600000)}^2 = 7113 \cdot 79 \text{ in.}$$

Also the slant height  $= \sqrt{(\frac{1}{2}a)^2 + a^2} = \frac{1}{2}a\sqrt{5}$ ; and the solution may be completed as before.

$$40. \text{ Vol. of cone} = \frac{1}{3} \times 3 \cdot 1416 \times a^2 \times \frac{1}{4}a = \frac{1}{12} \times 3 \cdot 1416 \times a^3 = 814160 (\text{by hyp.});$$

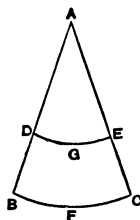
$$\therefore a^3 = 900000 \text{ and } a^2 = \sqrt[3]{(900000)}^2 = 9321 \cdot 7 \text{ in.}$$

Also the slant height  $= \sqrt{(\frac{1}{4}a)^2 + a^2} = \frac{1}{4}a\sqrt{10}$ ; and the solution may be completed as before.

## CHAPTER XXXVII.

*Note.* With respect to the solutions in this chapter, the student may consult the note at the head of Chapter XXVI. and also that at the head of the preceding chapter. The curved surface of the frustum of a cone is equal to the difference between the curved surface of the whole cone of which the frustum is a part, and the curved surface of the smaller cone cut off from the whole cone by the frustum.

In the following solutions when the expressions, "the cone  $ABC$ ", or "the cone  $ADE$ ", etc., are employed, we mean to signify the cone which is now represented by the sector  $ABC$ , or the sector  $ADE$ , etc.



1. Diam. of  $DGE=15$  in.  $\div 3.1416$ ; and diam. of  $BFC=17$  in.  $\div 3.1416$ . Now (by similar triangles, as may be seen from a diagram like that in Chap. XXVI. 2),

$$AD : 15 \text{ in.} \div 3.1416 :: AB : 17 \text{ in.} \div 3.1416;$$

$$\therefore AD = \frac{15AB}{17} = \frac{15(AD+11)}{17} \text{ in.};$$

$$\therefore AD = 1\frac{11}{16} \text{ in.}, \text{ and } AB = (1\frac{11}{16} + 11) \text{ in.} = 12\frac{11}{16} \text{ in.};$$

$\therefore$  curved surface of frustum  $DBCE$  = curved surface of sector  $ABFC$  - curved surface of sector  $ADGE = \frac{1}{2} (17 \times 12\frac{11}{16} - 15 \times 1\frac{11}{16})$  sq. in.  $= \frac{1}{2} (17^2 - 15^2)$  sq. in.  $= \frac{1}{2} \times 32 \times 2$  sq. in.  $= 176$  sq. in.

$$3. \text{ (As in 1), } AD = \frac{14AB}{18} = \frac{14(AD+5 \text{ in.})}{18}; \therefore AD = 2\frac{5}{9} \text{ in.};$$

and  $AB = 4\frac{1}{9}$  in.;  $\therefore$  curved surface of frustum  $DBCE$

$$= (\frac{1}{2} \times 8.1416 \times 18 \times 4\frac{1}{9} - \frac{1}{2} \times 3.1416 \times 14 \times 2\frac{5}{9}) \text{ sq. in.} = 8.1416 \times \frac{1}{2} (9^2 - 7^2) \text{ sq. in.} \\ = 7.854 \times 16 \times 2 \text{ sq. in.} = 251.328 \text{ sq. in.}$$

$$5. \text{ Height of cone } ADE = \frac{11 \times 12}{16 - 11} \text{ in. (vide Chap. XXVI. 5, note)} = 2\frac{2}{5} \text{ in.}$$

$$\therefore AD \text{ (by Eucl. I. 47)} = \sqrt{\{(\frac{11}{5})^2 + 11^2\}} \text{ sq. in.}]$$

$$= \sqrt{\{11^2 (\frac{12^2}{5^2} + 1) \text{ sq. in.}\}} = \sqrt{\{11^2 \times \frac{13^2}{5^2} \text{ sq. in.}\}} = 12\frac{1}{5} \text{ in.};$$

and (as in Example 1)  $AB : 16 \text{ in.} :: AD : 11 \text{ in.};$

$$\therefore AB = 16 \times 12\frac{1}{5} \times \frac{1}{11} \text{ in.} = 20\frac{2}{5} \text{ in.};$$

$\therefore$  curved surface of frustum  $DBCE$

$$= (\frac{1}{2} \times 2 \times 3.1416 \times 16 \times 20\frac{2}{5} - \frac{1}{2} \times 2 \times 3.1416 \times 11 \times 12\frac{1}{5}) \text{ sq. in.} \\ = 3.1416 \times \frac{1}{2} (16^2 - 11^2) \text{ sq. in.} = 62832 \times 13 \times 27 \times 5 \text{ sq. in.} = 1102.7016 \text{ sq. in.}$$

9.  $AD$  (found as in 1)  $= 70$  in., and  $AB = 80$  in.;  $\therefore$  whole surface of frustum  $DBCE$  = areas of 2 circles  $DGE$  and  $BFC$  + area of curved surface of  $DBCE = \{8.1416 (16 \div 2 \times 3.1416)^2 + 8.1416 (14 \div 2 \times 3.1416)^2$

$$+ \frac{1}{2} (16 \times 80 - 14 \times 70) \} \text{ sq. in.} = \{ (1 \div 4 \times 8.1416) (16^2 + 14^2) + 5 (128 - 98) \} \text{ sq. in.} \\ = (35.97 \text{ nearly} + 150) \text{ sq. in.} = 185.97 \text{ sq. in.}$$

11.  $AD$  (found as in 3) = 5 ft., and  $AB = 7\frac{1}{2}$  ft.;  $\therefore$  whole surface of frustum  $DBCE = \{3 \cdot 1416 (2^2 + 5^2) + 3 \cdot 1416 (3 \times 7\frac{1}{2} - 2 \times 5)\}$  sq. ft.  
 $= 3 \cdot 1416 (4 + 9 + \frac{15}{2})$  sq. ft. = 80.1108 sq. ft.

13.  $AD$  (found as in 5) = 20 in., and  $AB = 30$  in.;  $\therefore$  whole surface of frustum  $DBCE = (\text{as in 11}) 3 \cdot 1416 \{(12^2 + 18^2) + (18 \times 30 - 12 \times 20)\}$  sq. in.  
 $= 3 \cdot 1416 \times 768$  sq. in. = 2412.7488 sq. in. = 2412.75 sq. in. nearly.

15. Height of frustum  $DBCE = (\text{by hyp.}) \frac{6 \times 4}{6 + 4}$  ft. =  $1\frac{2}{5}$  ft.  $AD$  (found as in 5) =  $2\frac{2}{5}$  ft., and  $AB = 3\frac{2}{5}$  ft.;  $\therefore$  area of 2 ends of frustum  
 $= 3 \cdot 1416 (3^2 + 2^2)$  sq. ft. = 40.8408 sq. ft.;

and curved surface of frustum  $DBCE = 3 \cdot 1416 (3 \times 2\frac{2}{5} - 2 \times 1\frac{2}{5})$  sq. ft.  
 $= 3 \cdot 1416 \times 13 (\frac{2}{5} - \frac{1}{5})$  sq. ft. =  $3 \cdot 1416 \times 13$  sq. ft. = 40.8408 sq. ft.

19. Here  $AD$  (found as in 1) =  $3\frac{5}{8}$  ft.; and  $AB = 5\frac{1}{8}$  ft.;  
 $\therefore$  curved surface of frustum  $DBCE$   
 $= 3 \cdot 1416 \times (8 \times 3\frac{5}{8} - 5 \times 3\frac{5}{8})$  sq. ft. =  $3 \cdot 1416 \times \frac{1}{8} (8^2 - 5^2)$  sq. ft.  
 $= 3 \cdot 1416 \times 52$  sq. ft.

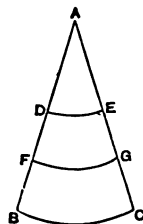
$\therefore$  (by hyp.) curved surface of frustum  $DFGE$   
 $= 3 \cdot 1416 \times 26$  sq. ft.

Consequently curved surface of sector  $AFG$   
 $= (3 \cdot 1416 \times 5 \times 3\frac{5}{8} + 3 \cdot 1416 \times 26)$  sq. ft. =  $3 \cdot 1416 \times 17\frac{3}{8}$  sq. ft.  
 Now sector  $AFG$  : sector  $ADE :: AF^2 : AD^2$ ;

$$\therefore AF^2 = \frac{3 \cdot 1416 \times 17\frac{3}{8} \times (\frac{5}{8})^2}{3 \cdot 1416 \times 5 \times 3\frac{5}{8}} \text{ sq. ft.} = 1\frac{1}{8} \text{ sq. ft.};$$

$$\therefore AF = 8.894 \text{ ft.} \quad \therefore DF = AF - AD = (8.894 - 6.666) \text{ ft.} = 2.228 \text{ ft.}$$

$$\text{And } FB = DB - DF = (4 - 2.228) \text{ ft.} = 1.772 \text{ ft.}$$



20. Slant height of conical part (by *Eucl. I. 47*)

$$= \sqrt{\{(6^2 + 8^2) \text{ sq. ft.}\}} = \sqrt{100 \text{ sq. ft.}} = 10 \text{ ft.};$$

$\therefore$  curved surface of conical part =  $3 \cdot 1416 \times 8 \times 10$  sq. ft. =  $3 \cdot 1416 \times 80$  sq. ft.  
 Again, slant height of the cone, of which the lower portion of the tent is the frustum, (found as in 5), and of the cone left after the frustum has been cut off = respectively  $1\frac{2}{5}$  ft., and  $2\frac{2}{5}$  ft.  $\therefore$  curved surface of this frustum

$$= 3 \cdot 1416 (14 \times 1\frac{2}{5} - 8 \times \frac{2}{5}) \text{ sq. ft.} = 3 \cdot 1416 \times 220 \text{ sq. ft.}$$

$\therefore$  curved surface of whole tent =  $(3 \cdot 1416 \times 80 + 3 \cdot 1416 \times 220)$  sq. ft.

=  $3 \cdot 1416 \times 300$  sq. ft. = 942.48 sq. ft. = 104.72 sq. yds. = 105 sq. yds. nearly,  
 and this is the no. of square yards of canvas required.

## CHAPTER XXXVIII.

*Note.* Surface of a sphere is equal to curved surface of circumscribing cylinder, i.e. of a cylinder, the diameter of the base, and the height of which, is each equal to the diameter of the sphere. Vide Art. 348.

1. Surface of sphere =  $8 \cdot 1416 \times 10 \times 10$  sq. in. =  $814 \cdot 16$  sq. in.

4. Surface of sphere =  $20 \times (20 \div 8 \cdot 1416)$  sq. in. =  $127 \cdot 82$  sq. in.

7. Surface of sphere =  $8 \cdot 1416 \times \text{diam.}^2 = (\text{by hyp.}) 400$  sq. in. ;  
 $\therefore$  diam. =  $\sqrt{(400 \text{ sq. in.} \div 8 \cdot 1416)} = 20 \text{ in.} + 1 \cdot 7724 = 11 \cdot 284 \text{ in.}$

10. (As in 7), diam. =  $\sqrt{(20 \text{ sq. ft.} \div 8 \cdot 1416)}$ ;  $\therefore$  vol. of sphere  
 $= \frac{1}{2} \times 8 \cdot 1416 \times \text{diam.}^2 = \{\frac{1}{2} \times 8 \cdot 1416 \times (20 \div 8 \cdot 1416) (\sqrt{20 \div 8 \cdot 1416})\}$  cub. ft.  
 $= \frac{1}{2} \times \sqrt{(6 \cdot 866182)}$  cub. ft. =  $\frac{1}{2} \times 2 \cdot 52313$  cub. ft. =  $8 \cdot 4104$  cub. ft.

13. Surface of sphere =  $8 \cdot 1416 \times 2^2$  sq. ft. =  $8 \cdot 1416 \times 4$  sq. ft.  $\therefore$  (as in 7)  
 diam. of sphere =  $\sqrt{\{(8 \cdot 1416 \times 4 \text{ sq. ft.}) \div 8 \cdot 1416\}} = 2 \text{ ft.}$ ;  
 $\therefore$  vol. of sphere =  $\frac{1}{2} \times 8 \cdot 1416 \times 8$  cub. ft. =  $4 \cdot 1888$  cub. ft.

15. Whole surface of solid = curved surface of cylinder + curved surface  
 of the two hemispheres =  $(8 \cdot 1416 \times 3 \times 5 + 2 \times 8 \cdot 1416 \times 3 \times \frac{1}{2})$  sq. ft.  
 $= 8 \cdot 1416 \times 3 \times 8$  sq. ft. =  $75 \cdot 3984$  sq. ft.

16. Whole surface of cylinder  
 $= (2 \times 8 \cdot 1416 \times 10^2 + 2 \times 8 \cdot 1416 \times 10 \times 10)$  sq. in.  
 $= 4 \times 8 \cdot 1416 \times 10^2$  sq. in. = (by hyp.) surface of sphere;  
 $\therefore$  (as in 7) diam. of sphere =  $\sqrt{\{(4 \times 8 \cdot 1416 \times 10^2 \text{ sq. in.}) \div 8 \cdot 1416\}} = 20 \text{ in.}$ ;  
 $\therefore$  vol. of sphere =  $\frac{1}{2} \times 8 \cdot 1416 \times 20^2$  cub. in. =  $4188 \cdot 8$  cub. in. ;  
 and vol. of cylinder =  $8 \cdot 1416 \times 10^2 \times 10$  cub. in. =  $8141 \cdot 6$  cub. in.

17. Whole surface of cube =  $6 \times 1^2$  sq. ft. =  $6$  sq. ft. = (by hyp.) surface of  
 sphere.  $\therefore$  (as in 7), diam. of sphere  
 $= \sqrt{(6 \text{ sq. ft.} \div 8 \cdot 1416)} = \sqrt{(1 \cdot 909855 \text{ sq. ft.})} = 1 \cdot 382 \text{ ft.}$ ;  
 $\therefore$  vol. of sphere =  $\frac{1}{2} \times 8 \cdot 1416 \times (6 \div 8 \cdot 1416) \times 1 \cdot 382$  cub. ft. =  $1 \cdot 382$  cub. ft.  
 And vol. of cube =  $1$  cub. foot.

18. Proceed as in 16.

19. Vol. of cube =  $1$  cub. ft. = (by hyp.) vol. of sphere =  $\frac{1}{2} \times 8 \cdot 1416 \times \text{diam.}^2$   
 of sphere;  $\therefore$  diam.<sup>2</sup> of sphere =  $\frac{6 \times 1}{8 \cdot 1416}$  cub. ft. =  $1 \cdot 909855$  cub. ft.;  $\therefore$  diam.  
 of sphere =  $\sqrt{(1 \cdot 909855 \text{ cub. ft.})} = 1 \cdot 2407 \text{ ft.}$ ;  $\therefore$  whole surface of sphere  
 $= 8 \cdot 1416 \times (1 \cdot 2407)^2$  sq. ft. =  $4 \cdot 836$  sq. ft.  
 And whole surface of cube =  $6 \times 1^2$  sq. ft. =  $6$  sq. ft.

20. Vol. of cylinder =  $8 \cdot 1416 \times 1^2 \times 2$  cub. ft. = (by hyp.) vol. of sphere;  
 $\therefore$  (as in preceding), diam. of sphere  
 $= \sqrt{\{(6 \times 8 \cdot 1416 \times 2 \text{ cub. ft.}) \div 8 \cdot 1416\}} = \sqrt{(12 \text{ cub. ft.})}$ .

∴ surface of sphere

$$= 8 \cdot 1416 (\sqrt[3]{12})^2 \text{ sq. ft.} = 8 \cdot 1416 \times 5 \cdot 2415 \text{ sq. ft.} = 16 \cdot 4666 \text{ sq. ft.}$$

And whole surface of cylinder

$$= (2 \times 8 \cdot 1416 \times 1^2 + 2 \times 8 \cdot 1416 \times 1 \times 2) \text{ sq. ft.} = 6 \times 8 \cdot 1416 \text{ sq. ft.} = 18 \cdot 8496 \text{ sq. ft.}$$

### CHAPTER XXXIX.

*Note.* The curved surface of a zone of a sphere, or of a segment of a sphere is equal to the curved surface of a cylinder, the height of which is equal to that of the zone or segment, and the diameter of the base of which is equal to the diameter of the sphere. See Art. 356.

1. Curved surface of segment =  $85 \times 10 \text{ sq. in.} = 850 \text{ sq. in.}$

3. Curved surface of segment =  $2 \times 3 \cdot 1416 \times 16 \times 9 \text{ sq. in.} = 904 \cdot 7808 \text{ sq. in.}$

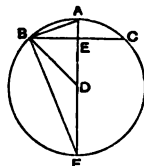
5. Here  $AE = 2 \text{ ft.}$ ,  $BD = DA = 7 \text{ ft.}$ ; ∴  $ED = 5 \text{ ft.}$   
 ∴  $BE^2$  (or radius<sup>2</sup> of base of segment  $BAC$ )

$$= BD^2 - DE^2 = (7^2 - 5^2) \text{ sq. ft.} = 24 \text{ sq. ft.}$$

∴ whole surface of segment  $BAC$

$$= (3 \cdot 1416 \times 24 + 2 \times 3 \cdot 1416 \times 7 \times 2) \text{ sq. ft.}$$

$$= 4 \times 3 \cdot 1416 \times 13 \text{ sq. ft.} = 52 \times 3 \cdot 1416 \text{ sq. ft.} = 163 \cdot 8632 \text{ sq. ft.}$$



7. Here  $AF = (27 \text{ ft.} \div 3 \cdot 1416)$ ;

∴  $EF = (27 \text{ ft.} \div 3 \cdot 1416) - 3 \text{ ft.}$  Now (by *Euc. VI. 8. Cor.*),  $AE : EB :: EB : EF$ ;

$$\therefore EB^2 = AE \times EF = 3 \left( \frac{27}{3 \cdot 1416} - 3 \right) \text{ sq. ft.} = \frac{81 - 9 \times 3 \cdot 1416}{3 \cdot 1416} \text{ sq. ft.}$$

∴ area of base of segment  $BAC$

$$= \left( 3 \cdot 1416 \times \frac{81 - 9 \times 3 \cdot 1416}{3 \cdot 1416} \right) \text{ sq. ft.} = (81 - 28 \cdot 2744) \text{ sq. ft.}$$

And curved surface of segment =  $27 \times 3 \text{ sq. ft.} = 81 \text{ sq. ft.}$  ∴ whole surface of segment  $BAC = (81 + 81 - 28 \cdot 2744) \text{ sq. ft.} = 133 \cdot 7256 \text{ sq. ft.}$

9. Here  $AH = 10 \text{ ft.}$ ,  $AG = 3 \text{ ft.}$ ,  $HO = 1 \text{ ft.}$ ,

∴  $HG = 7 \text{ ft.}$  ∴ (By *Euc. VI. 8. Cor.*)

$AG : GB :: GB : GF$ ; ∴  $GB^2 = 3 \times 19 \text{ sq. ft.} = 57 \text{ sq. ft.}$

And  $AH : HD :: HD : HF$ ;

$$\therefore HD^2 = 10 \times 12 \text{ sq. ft.} = 120 \text{ sq. ft.}$$

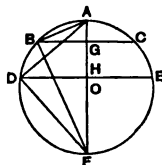
∴ areas of two ends of zone  $BDEC$

$$= 3 \cdot 1416 (57 + 120) \text{ sq. ft.} = 3 \cdot 1416 \times 177 \text{ sq. ft.}$$

And curved surface of zone  $BDEC = 2 \times 3 \cdot 1416 \times 11 \times HG = 3 \cdot 1416 \times 154 \text{ sq. ft.}$

∴ whole surface of zone  $BDEC$

$$= (3 \cdot 1416 \times 177 + 3 \cdot 1416 \times 154) \text{ sq. ft.} = 3 \cdot 1416 \times 331 \text{ sq. ft.} = 1039 \cdot 8696 \text{ sq. ft.}$$



11. (Vide diag. to 9.) Here  $AG=2$  ft.,  $AH=5$  ft.,

$$AO = \frac{40}{2 \times 3 \cdot 1416} \text{ ft.} = \frac{20}{3 \cdot 1416} \text{ ft.};$$

$$\therefore GF = \left( \frac{40}{3 \cdot 1416} - 2 \right) \text{ ft.} = \frac{40 - 2 \times 3 \cdot 1416}{3 \cdot 1416} \text{ ft.} \quad \text{And } HF = \frac{40 - 5 \times 3 \cdot 1416}{3 \cdot 1416} \text{ ft.}$$

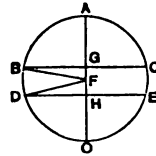
$$\therefore (\text{as in preceding}), GB^2 = GA \times GF = \frac{80 - 4 \times 3 \cdot 1416}{3 \cdot 1416} \text{ sq. ft.}$$

$$\text{And } HD^2 = AH \times HF = \frac{200 - 25 \times 3 \cdot 1416}{3 \cdot 1416} \text{ sq. ft.}; \therefore \text{area of two ends of zone}$$

$$BDEC = (80 - 4 \times 3 \cdot 1416 + 200 - 25 \times 3 \cdot 1416) \text{ sq. ft.} \\ = (280 - 29 \times 3 \cdot 1416) \text{ sq. ft.} = 188 \cdot 8936 \text{ sq. ft.}$$

$$\text{And curved surface of zone } BDEC = 40 \times 3 \text{ sq. ft.} = 120 \text{ sq. ft.} \quad \therefore \text{whole surface of zone } BDEC = (120 + 188 \cdot 8936) \text{ sq. ft.} = 308 \cdot 8936 \text{ sq. ft.}$$

13. Here  $FB=FD=9$  ft.,  $FG=2$  ft.,  $FH=3$  ft.  
 $\therefore$  (by Euc. I. 47)  $BG^2 = (9^2 - 2^2) \text{ sq. ft.} = 77 \text{ sq. ft.}$  And  
 $DH^2 = (9^2 - 3^2) \text{ sq. ft.} = 72 \text{ sq. ft.}$   $\therefore$  area of two ends of  
 zone  $BDEC = (3 \cdot 1416 \times 77 + 3 \cdot 1416 \times 72) \text{ sq. ft.}$   
 $= 3 \cdot 1416 \times 149 \text{ sq. ft.}$



And curved surface of zone  $BDEC$

$$= 2 \times 3 \cdot 1416 \times 9 \times GH = 3 \cdot 1416 \times 90 \text{ sq. ft.}$$

$\therefore$  whole surface of zone  $BDEC$

$$= 3 \cdot 1416 (149 + 90) \text{ sq. ft.} = 750 \cdot 8424 \text{ sq. ft.}$$

14. Proceed as in 9.

15. (Vide diag. to 13.) Here  $GF=3$  ft.,  $FH=4$  ft.,  $FB=FD=\frac{16}{3 \cdot 1416} \text{ ft.}$

$$\therefore (\text{by Euc. I. 47}), BG^2 = BF^2 - FG^2 = \frac{256 - 9 (3 \cdot 1416)^2}{(3 \cdot 1416)^2} \text{ sq. ft.}$$

$$\text{And } DH^2 = FD^2 - FH^2 = \frac{256 - 16 (3 \cdot 1416)^2}{(3 \cdot 1416)^2} \text{ sq. ft.} \quad \therefore \text{area of two ends of}$$

$$\text{zone } BDEC = 3 \cdot 1416 \left\{ \frac{256 - 9 (3 \cdot 1416)^2}{(3 \cdot 1416)^2} + \frac{256 - 16 (3 \cdot 1416)^2}{(3 \cdot 1416)^2} \right\} \text{ sq. ft.}$$

$$= \left( \frac{512}{3 \cdot 1416} - 25 \times 3 \cdot 1416 \right) \text{ sq. ft.} = (162 \cdot 9743 - 78 \cdot 54) \text{ sq. ft.} = 84 \cdot 4343 \text{ sq. ft.}$$

$$\text{And curved surface of zone } BDEC = 32 \times 7 \text{ sq. ft.} = 224 \text{ sq. ft.} \quad \therefore \text{whole surface of zone} = (224 + 84 \cdot 4343) \text{ sq. ft.} = 308 \cdot 4343 \text{ sq. ft.}$$

16. (Vide diag. to 9.) Here  $OH=6$  in.,  $OG=10$  in., join  $B, O$ ; and  $D, O$ .  $\therefore BO=DO=\frac{45}{3 \cdot 1416} \text{ in.}$

$$\therefore (\text{by Euc. I. 47}) BG^2 = BO^2 - OG^2 = \frac{2025 - 100 (3 \cdot 1416)^2}{(3 \cdot 1416)^2} \text{ sq. in.};$$

$$\text{and } DH^2 = DO^2 - OH^2 = \frac{2025 - 36 (3 \cdot 1416)^2}{(3 \cdot 1416)^2} \text{ sq. in.};$$

∴ (as in preceding), area of two ends of zone  $BDEC$

$$= \left( \frac{4050}{3 \cdot 1416} - 136 \times 3 \cdot 1416 \right) \text{ sq. in.}$$

$$= (1289 \cdot 1520 - 427 \cdot 2576) \text{ sq. in.} = 861 \cdot 8944 \text{ sq. in.}$$

And curved surface of this zone  $= 90 \times 4 \text{ sq. in.} = 360 \text{ sq. in.}$ ; ∴ whole surface of zone  $BDEC = (861 \cdot 8944 + 360) \text{ sq. in.} = 1221 \cdot 8944 \text{ sq. in.}$

17. Here  $AC = 41 \text{ ft.}$ ,  $BC = CO = 40 \text{ ft.}$  ∴ (by Euc.

VI. 8. Cor.)  $AC : CB :: CB : CD$ ; ∴  $CD = \frac{40^2}{41} \text{ ft.}$

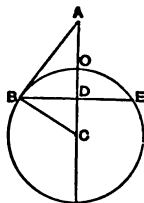
$$\therefore OD = \left( 40 - \frac{40^2}{41} \right) \text{ ft.} = 4\frac{1}{41} \text{ ft.};$$

∴ curved surface of segment  $BOE = 3 \cdot 1416 \times 80 \times 4\frac{1}{41} \text{ sq. ft.}$

And surface of whole sphere  $= 3 \cdot 1416 \times 80 \times 80 \text{ sq. ft.}$

∴ fraction required

$$= (3 \cdot 1416 \times 80 \times 4\frac{1}{41}) \div (3 \cdot 1416 \times 80 \times 80) = \frac{1}{88}.$$



18. (Vide diag. to preceding.) Here  $AO = 8 \text{ ft.}$ ,  $BC = CO = 45 \text{ ft.}$ ;

$$\therefore AC = 53 \text{ ft.} \quad \therefore \text{(as above), } OD = \left( 45 - \frac{45^2}{53} \right) \text{ ft.} = 3\frac{2}{53} \text{ ft.}$$

∴ curved surface of segment  $BOE = 3 \cdot 1416 \times 90 \times 3\frac{2}{53} \text{ sq. ft.}$  And surface of whole sphere  $= 3 \cdot 1416 \times 90 \times 90 \text{ sq. ft.}$  ∴ fraction required

$$= \frac{90 \times 360}{90 \times 90 \times 53} = \frac{4}{53}.$$

19. Surface of whole sphere  $= 2 \times 3 \cdot 1416 \times BC \times 2BC = 4 \times 3 \cdot 1416 \times BC^2$ .  
∴ curved surface of segment  $BOE = \frac{1}{2} \times 3 \cdot 1416 \times BC^2$ . But curved surface of segment also  $= 2 \times 3 \cdot 1416 \times BC \times OD$ ;

$$\therefore 2 \times 3 \cdot 1416 \times BC \times OD = \frac{1}{2} \times 3 \cdot 1416 \times BC^2; \therefore OD = \frac{1}{4} BC.$$

$$\therefore DC = OC - OD = BC - \frac{1}{4} BC = \frac{3}{4} BC.$$

Now, (by Euc. VI. 8. Cor.),  $AC : BC :: BC : CD$ ; ∴  $AC = \frac{BC^2}{CD} = \frac{4}{3} BC$ .

$$\therefore AO = AC - OC = \frac{4}{3} BC - BC = \frac{1}{3} BC.$$

20. (Working as in preceding),  $OD = \frac{1}{4} BC$ ; ∴  $CD = \frac{3}{4} BC$ ;

∴ (as in preceding),  $AC = BC^2 \div \frac{3}{4} BC = \frac{4}{3} BC$ ; ∴  $AO = \frac{4}{3} BC - BC = \frac{1}{3} BC$ .

## CHAPTER XL.

1. Area of 4 walls of room  $= 2 \times 11\frac{1}{2} \times (24\frac{1}{2} + 11\frac{1}{2}) \text{ sq. ft.} = 23 \times 36 \text{ sq. ft.}$   
∴ cost of painting  $= 23 \times 86 \times 1s. 6d. = \text{£}2\frac{2}{3} = \text{£}2. 2s.$

5. Area of 4 walls  $= 2 \times 4\frac{1}{2} \times (7\frac{5}{8} + 5\frac{1}{8}) \text{ sq. yds.} = 2 \times 2\frac{1}{2} \times 4\frac{1}{2} \text{ sq. yds.}$   
∴ length of paper required  $= 2 \times 2\frac{1}{2} \times 4\frac{1}{2} \text{ yds.}$

$$\therefore \text{cost} = 2 \times 2\frac{1}{2} \times 4\frac{1}{2} \times \text{£}\frac{8}{100} = \text{£}2\frac{2}{5} = \text{£}1. 3s. 4d.$$

6. Surface of box =  $(294 \div 4)$  sq. ft. =  $147$  sq. ft. But surface of box, which is a cube, =  $6 \times \text{length}^2$ ;

$\therefore$  length<sup>2</sup> of box =  $147$  sq. ft.  $\times \frac{1}{8} = \frac{49}{2}$  sq. ft.;

$\therefore$  length of box  $= \sqrt{\left(\frac{49}{2} \text{ sq. ft.}\right)} = \frac{7}{\sqrt{2}} \text{ ft.} = 3\frac{1}{2} \text{ ft.}$

### 7. Surface to be lined

$$= (2 \times 12\frac{3}{4} \times 6\frac{1}{2} + 2 \times 8\frac{1}{4} \times 6\frac{1}{2} + 12\frac{3}{4} \times 8\frac{1}{4}) \text{ sq. ft.}$$

$$= 2 \times \frac{1}{2} \left( \frac{51}{2} + \frac{33}{2} \right) \text{ sq. ft.} + \frac{51}{2} \times \frac{33}{2} \text{ sq. ft.} = (273 + 1683) \text{ sq. ft.} = 1956 \text{ sq. ft.};$$

$$\therefore \text{weight of lead required} = \frac{6051}{8} \times 8 \text{ lbs.} = \frac{6051}{2 \times 112} \text{ lbs.} = \frac{6051}{2 \times 112} \text{ cwt.};$$

$$\therefore \text{cost} = \frac{6051}{2 \times 112} \times £\frac{7}{8} = £\frac{6051}{160} = £37.16s.4\frac{1}{2}d.$$

8. Vol. of cistern = depth  $\times 4\frac{1}{2}$  ft.  $\times 2\frac{1}{2}$  ft. = (by hyp.) 42 cub. ft.

$\therefore$  depth of cistern  $= 42 \times \frac{2}{3} \times \frac{1}{4}$  ft.  $= \frac{7}{2}$  ft.  $\therefore$  surface to be lined

$$= 2 \times \frac{7}{8} (4\frac{1}{2} + 2\frac{3}{4}) \text{ sq. ft.} + 4\frac{1}{2} \times 2\frac{3}{4} \text{ sq. ft.} = 37\frac{3}{8} \text{ sq. ft.}$$

$$\therefore \text{weight of lead required} = \frac{37.3}{8} \times 6 \text{ lbs.} = 373 \text{ lbs.}$$

9. Whole exterior surface of box =  $2 \times 3\frac{1}{2} \times 2\frac{1}{2}$  sq. ft.

$$+ 2 \times 3\frac{1}{2} \text{ ft.} \times (1\frac{3}{4} \text{ ft.} - 2 \times 1\frac{1}{2} \text{ in.}) + 2 (2\frac{1}{2} \text{ ft.} - 2 \times 1\frac{1}{2} \text{ in.}) \times (1\frac{3}{4} \text{ ft.} - 2 \times 1\frac{1}{2} \text{ in.})$$

$$= \left( \frac{35}{9} + \frac{21}{9} + \frac{27}{9} \right) \text{ sq. ft.} = \frac{83}{3} \text{ sq. ft.} = 27\frac{2}{3} \text{ sq. ft.}$$

10. Vol. of lead required

$$= 17 \text{ ft. } 4 \text{ in.} \times 13 \text{ ft. } 4 \text{ in.} \times \frac{1}{16} \text{ in.} = 208 \times 160 \times \frac{1}{16} \text{ cub. in.} = 2080 \text{ cub. in.};$$

$$\therefore \text{weight of this lead} = 2080 \times \frac{1}{16} \text{ oz.} = 65 \times 13 \text{ lbs.}$$

$$\therefore \text{cost} = 65 \times 13 \times 3\frac{1}{4}d. = 65 \times 13 \times \frac{13}{4}d. = \frac{11025}{4}d. = \text{£}2. 6s. 5\frac{1}{4}d.$$

11. Area of wall =  $3.1416 \times 6 \times 10$  sq. yds.;  $\therefore$  cost =  $3.1416 \times 82 \times 7\frac{1}{2}$  d.

$$= \text{£}3.1416 = \text{£}3. 2s. 9.984d. = \text{£}3. 2s. 10d. \text{ nearly.}$$

12. Curved surface of spire =  $\frac{1}{2} \times \frac{64}{1} \times \frac{108}{1} \text{ sq. yds.} = 64 \times 6 \text{ sq. yds.}$

$\therefore$  cost of painting  $= 64 \times 6 \times 7\frac{1}{2}d. = \text{£}12.$

13. Inner surface of bowl =  $\frac{1}{2} \times 3.1416 \times 28 \times 28$  sq. in. ;  $\therefore$  cost of gilding

$$= 8.1416 \times 14 \times 28 \times 11d. = \pounds 7.69692 = \pounds 7. 13.9384s. = \pounds 7. 14s. \text{ nearly.}$$

**14. Portion of room to be plastered**

$$= (3.1416 \times 28 \times 15 + \frac{1}{2} \times 3.1416 \times 28 \times 28) \text{ sq. ft.} = 28 \times 3.1416 \times 29 \text{ sq. ft.}$$

$$\therefore \text{cost} = 28 \times 3.1416 \times 29 \times 9d. = \pounds 95.66172 = \pounds 95. 13s. 2.8128d.$$

**=£95. 13s. 3d. nearly.**

15. Circum. of moulding =  $3.1416 \times 28$  ft.;

$$\therefore \text{cost} = 8.1416 \times 28 \times 15d. = £5.4978 = £5. 9.956s. = £5. 10s. \text{ nearly.}$$

16. Area of footway =  $100 \times \frac{1}{2}$  sq. yds. =  $50$  sq. yds.  $\therefore$  area of portion turfed =  $(100 \times \frac{1}{2} - 50)$  sq. yds. =  $0$  sq. yds.  $\therefore$  whole cost

$$= \frac{200}{100} \times 4s. 6d. + \frac{200}{100} \times 9d. = 200 (18d. + 42d.) = 200 \times £\frac{1}{2} = £50.$$

## 17. Area of all the windows

$$= (3 \times 8 \times 4 + 3 \times 7 \times 4 + 3 \times 5 \times 4) \text{ sq. ft.} = 240 \text{ sq. ft.}$$

$$\therefore \text{cost of glazing} = 240 \times 1s. = £12.$$

## 18. Quantity of flooring

$$= (3 \times 58 \times 34 - 3 \times 15\frac{1}{2} \times 8) \text{ sq. ft.} = (5916 - 386) \text{ sq. ft.} = 5550 \text{ sq. ft.}$$

19. Area of 4 walls =  $2 \times 14\frac{1}{2} (22 + 20) \text{ sq. ft.} = 4\frac{1}{2} \text{ sq. yds.}$  And area of doors, fireplace, and window

$$= (2 \times 8 \times 5\frac{1}{2} + 6\frac{1}{2} \times 6 + 12 \times 5\frac{1}{2}) \text{ sq. ft.} = (84 + 39 + 67) \text{ sq. ft.} = 1\frac{1}{2} \text{ sq. yds.}$$

$\therefore$  portion to be papered =  $(4\frac{1}{2} - 1\frac{1}{2}) \text{ sq. yds.} = 3 \text{ sq. yds.}$   $\therefore$  length of paper required =  $3 \text{ sq. yds.} \div 30 \text{ in.} = 1\frac{1}{2} \text{ sq. yds.} \times \frac{1}{2} \text{ yds.}$

$$\therefore \text{cost} = 1\frac{1}{2} \times \frac{1}{2} \times 11\frac{1}{2}d. = £\frac{1}{2} 7s. = £6. 8s. 6d.$$

20. Area of basin =  $3 \cdot 1416 \times (\frac{1}{2} \text{ ft.})^2 = 132 \cdot 7326 \text{ sq. ft.}$  And area of flower bed =  $2 \times 36 \times 4 \text{ sq. ft.} + (36 - 2 \times 4) \text{ ft.} \times 4 \text{ ft.} = (288 + 112) \text{ sq. ft.} = 400 \text{ sq. ft.}$

$\therefore$  portion to be paved

$$= \{36 \times 36 - (400 + 132 \cdot 7326)\} \text{ sq. ft.} = 763 \cdot 2674 \text{ sq. ft.} = \frac{763 \cdot 2674}{9} \text{ sq. yds.}$$

$$\therefore \text{cost} = \frac{763 \cdot 2674}{9} \times 8s. = £33. 18s. 5 \cdot 518d. = £33. 18s. 6d. \text{ nearly.}$$

## CHAPTER XLI.

*Note.* (1) 272 sq. ft. of wall  $1\frac{1}{2}$  bricks thick = 1 standard rod of brick work.

(2) To find the no. of *standard yards* in a wall. Find the area of the surface of the wall in square yards; then multiply by the number of half-bricks in the thickness of the wall, and divide by 3.

(3) A *square* = 100 square feet.

$$1. \text{ Area of surface} = 62\frac{1}{2} \times 14\frac{1}{2} \text{ sq. ft.} = \frac{22 \times 125}{8} \text{ sq. ft.} \therefore \text{no. of standard}$$

$$\text{rods in wall} = \frac{5}{3} \times \frac{22 \times 125}{8} \div 272 = \frac{13750}{9 \times 272} = 5 \cdot 62 \text{ nearly.}$$

$$2. \text{ Area of gable end} = \left\{ 22 \times 27 + \frac{22 (36 - 27)}{2} \right\} \text{ sq. ft.} = 11 \times 63 \text{ sq. ft.}$$

$$\therefore \text{no. of standard rods in gable end} = \frac{1}{3} \times 11 \times 63 \div 272 = 3 \cdot 4 \text{ nearly.}$$

$$3. \text{ Area of gable end} = \left( \frac{24 \times 10}{2} + 36 \times 24 \right) \text{ sq. ft.} = 984 \text{ sq. ft.} \therefore \text{no. of standard rods} = \frac{1}{3} \times 984 \div 272 = 3\frac{1}{4};$$

$$\therefore \text{cost} = 3\frac{1}{4} \times 84s. = 205s. = £10. 5s.$$

4. Areas of the respective portions =  $80 \times 20$  sq. ft.,  $80 \times 20$  sq. ft., and  $\frac{80 \times 10}{2}$  sq. ft. i.e.  $2\frac{1}{2}$  sq. yds.,  $2\frac{1}{2}$  sq. yds., and  $1\frac{1}{2}$  sq. yds.  $\therefore$  nos. of standard yards in these areas are respectively,  $2\frac{1}{2} \times \frac{3}{2} = 3\frac{3}{4}$ ,  $2\frac{1}{2} \times \frac{3}{2} = 3\frac{3}{4}$ , i.e.  $1\frac{1}{2}$ ,  $1\frac{1}{2}$ ,  $1\frac{1}{2}$ ;  $\therefore$  no. of standard yards in the wall =  $1\frac{1}{2} + 1\frac{1}{2} + 1\frac{1}{2} = 2\frac{1}{2}$

5. No. of standard rods in wall (found as in 1) = 12.  $\therefore$  no. of bricks required =  $12 \times 4500$ ;  $\therefore$  whole cost

$$= \frac{12 \times 4500 \times £2}{1000} + 12 \times 22s. + 12 \times £2 = £108 + £13. 4s. + £24 = £145. 4s.$$

6. Area of roof =  $\frac{3}{4} \times 40 \times 35$  sq. ft. =  $\frac{3 \times 40 \times 35}{2 \times 100}$  squares;

$$\therefore \text{cost} = \frac{3 \times 40 \times 35}{2 \times 100} \times 15s. = £2\frac{1}{4} = £15. 15s.$$

7. Area of roof =  $2 \times 120 \times 40$  sq. ft. =  $\frac{2 \times 120 \times 40}{100}$  squares = 96 squares.

$$\therefore \text{cost} = 96 \times 25s. = £120.$$

8. Area of roof =  $1\frac{1}{2} \times 80 \times 20$  sq. ft. =  $11 \times 60$  sq. ft.;  $\therefore$  quantity of lead required =  $11 \times 60 \times 8$  lbs. =  $1\frac{1}{2}$  cwt.  $\therefore$  cost =  $1\frac{1}{2} \times 21s. = £2\frac{1}{4} = £49. 10s.$

9. Area of partition =  $45\frac{1}{2} \times 8\frac{1}{2}$  sq. ft. =  $\frac{109 \times 49}{12 \times 120}$  squares;

$$\therefore \text{cost} = \frac{109 \times 49}{12 \times 120} \times £6\frac{1}{2} = £2\frac{1}{4}\frac{1}{2}\frac{1}{2} = £25. 0s. 8\frac{1}{2}d.$$

10. Area of flooring =  $(44 \times 24 - 2 \times 7 \times 4)$  sq. ft. = 1000 sq. ft. = 10 squares.  $\therefore$  cost =  $10 \times £6\frac{1}{2} = £62. 10s.$

11. Wainscoting =  $2 \times 12 (34 + 18\frac{1}{2})$  sq. ft. =  $12 \times 105$  sq. ft. =  $2\frac{1}{2}$  squares.  $\therefore$  cost =  $2\frac{1}{2} \times £10 = £126.$

12. The roof, therefore, consists of two rectangles, the length of each of which = 50 ft. Let  $a$  and  $b$  equal respectively the other sides of the rectangles. Then (by hyp.)  $a + b = 60$  ft. Now area of roof

$$= 50 \text{ ft.} \times a + 50 \text{ ft.} \times b = 50 \text{ ft.} \times (a + b) = 50 \text{ ft.} \times 60 \text{ ft.} = 30 \text{ squares.}$$

$$\therefore \text{cost} = 30 \times £2\frac{1}{2} = £71. 5s.$$

13. No. of standard yards in wall

$$= \left( \frac{180 \times 7}{9} \times \frac{2}{3} \right) + \left\{ \frac{18 \times 7 \times 3}{9 \times 2} \times \frac{1}{3} \right\} = \frac{43}{3} \times 7 = 100\frac{1}{3}.$$

14. Area of 4 walls =  $2 \times 12 (36 + 18)$  sq. ft. = 144 sq. yds.  $\therefore$  cost of plastering =  $144 \times 1s. = £7. 4s.$  Again, area of ceiling =  $36 \times 18$  sq. ft. = 72 sq. yds.;  $\therefore$  cost of plastering ceiling =  $72 \times 18d. = £5. 8s.$   $\therefore$  whole cost =  $£7. 4s. + £5. 8s. = £12. 12s.$

15. Area of the two floors  
 $= (28 \times 16 + 24 \times 15\frac{1}{2})$  sq. ft.  $= 4 \times 205$  sq. ft.  $= 820$  squares;  
 $\therefore$  cost  $= 820 \times 5 = £410$ .
16. Area of 4 walls  $= 2 \times 12 (25 + 20)$  sq. ft.  $= 840$  squares;  
 $\therefore$  cost  $= 840 \times 3 \times 10s. = £252 = £252$  4s.

## CHAPTER XLII.

*Note.* (1) Vol. of squared timber  
 $=$  mean breadth  $\times$  mean thickness  $\times$  length.

(2) Vol. of round or unsquared timber  $= (\frac{1}{2} \text{ mean girt})^2 \times \text{length}$ .

(3) Rule of Art. 283 (alluded to in the examples); Vol.  $= \frac{\text{height}}{6}$  (sum of areas of two ends  $+ 4 \times$  area of section parallel to the two ends, and midway between them). Observe, the length and breadth of this section are equal respectively, to half the sum of the lengths of the two ends, and half the sum of their two breadths (vide Chap. XIV. 15.).

(4) Rule of Art. 268 (referred to in the examples); Vol.  $= \frac{\text{height}}{8}$  (sum of areas of two ends  $+ \text{the square root of their product}$ ).

(5) Rule of Art. 376 (also referred to in the examples);  
 Vol.  $= 2 \times \text{length} \times (\frac{1}{2} \text{ mean girt})^2$ .

- Mean breadth  $= \frac{1}{2} (2\frac{3}{4} + 2\frac{1}{4})$  ft.  $= \frac{5}{2}$  ft., and mean thickness  $= \frac{1}{2} (1\frac{1}{2} + 1\frac{1}{2})$  ft.  $= \frac{5}{4}$  ft.  
 $\therefore$  vol. of timber  $= \frac{5}{2} \times \frac{5}{4} \times 22\frac{1}{2}$  cub. ft.  $= 140\frac{3}{4}$  cub. ft.
- Vol. of timber  $= 3\frac{1}{2} \times 1\frac{1}{2} \times 27$  cub. ft.  $= 137\frac{1}{2}$  cub. ft.
- Vol. of timber  $= (\frac{3}{2} \text{ ft.})^2 \times 56$  ft.  $= 126$  cub. ft.
- Mean girt  $= \frac{1}{2} (25 + 35)$  in.  $= \frac{30}{2}$  in.;  
 $\therefore$  vol. of timber  $= (\frac{30}{2} \text{ ft.})^2 \times 32$  ft.  $= 1440$  cub. ft.
- Worked like 4.
- Mean breadth  $= \frac{1}{2} (30 + 24)$  in.  $= \frac{27}{2}$  in., and mean thickness  $= \frac{1}{2} (20 + 18)$  in.  $= \frac{19}{2}$  in.  
 $\therefore$  vol. of timber  $= \frac{27}{2} \times \frac{19}{2} \times 36$  cub. ft.  $= 1821\frac{3}{4}$  cub. ft.
- Vol.  $= \{ (\frac{5}{2} \times \frac{5}{2}) \text{ sq. ft.} + (2 \times \frac{5}{2}) \text{ sq. ft.} + 4 \times \frac{1}{2} (\frac{5}{2} + 2) \times \frac{1}{2} (\frac{5}{2} + \frac{5}{2}) \text{ sq. ft.} \}$   
 $\times \frac{25}{8}$  ft.  $= (\frac{25}{2} + 5 + \frac{25}{2}) \times 6$  cub. ft.  $= 120$  cub. ft.
- Mean girt  $= \frac{1}{2} (7 + 4)$  ft.  $= \frac{11}{2}$  ft.,  $\therefore$  vol. of timber  $= (\frac{11}{2} \text{ ft.})^2 \times 40$  ft.  $= 2420$  cub. ft.
- Vol. of timber  $= 2 \times 40 \times (\frac{11}{2})^2$  cub. ft.  $= 2420$  cub. ft.

11. Vol. of timber

$$\begin{aligned}
 &= \frac{1}{2} \left[ 8.1416 \left\{ \left( \frac{7}{2 \times 8.1416} \right)^2 + \left( \frac{4}{2 \times 8.1416} \right)^2 \right\} + \sqrt{\frac{49}{(8.1416)^2}} \right] \text{ cub. ft.} \\
 &= \frac{1}{2} \left( \frac{49}{4 \times 8.1416} + \frac{4}{8.1416} + \frac{7}{8.1416} \right) \text{ cub. ft.} = \frac{1}{2} \times \frac{98}{4 \times 8.1416} \text{ cub. ft.} \\
 &= \frac{310}{8.1416} \text{ cub. ft.} = 98.676 \text{ cub. ft.}
 \end{aligned}$$

12. Side of square inscribed in one of the circular ends=(by Euc. IV. 6, and I. 47)= $\sqrt{(2 \text{ radius})^2}$ =radius  $\sqrt{2} = \frac{7\sqrt{2}}{2 \times 8.1416}$  ft.; and side of square inscribed in the other end= $\frac{4\sqrt{2}}{2 \times 8.1416}$  ft. Now the timber has become a prismoid, and hence its volume (applying the rule of Art. 288)

$$\begin{aligned}
 &= \frac{1}{2} \left\{ \left( \frac{7\sqrt{2}}{2 \times 8.1416} \right)^2 + \left( \frac{4\sqrt{2}}{2 \times 8.1416} \right)^2 + 4 \times \left( \frac{1}{2} \times \frac{7\sqrt{2} + 4\sqrt{2}}{2 \times 8.1416} \right)^2 \right\} \text{ cub. ft.} \\
 &= \frac{1}{2} \times \left\{ \frac{98}{4 \times (8.1416)^2} + \frac{82}{4 \times (8.1416)^2} + \frac{242}{4 \times (8.1416)^2} \right\} \text{ cub. ft.} \\
 &= \frac{1}{2} \times \frac{98}{(8.1416)^2} \text{ cub. ft.} = \frac{620}{9.87 \text{ nearly}} \text{ cub. ft.} = 62.8 \text{ cub. ft. nearly.}
 \end{aligned}$$

### CHAPTER XLIII.

Note. (1) Vol. of cask in gallons=(39 × bung diam.<sup>2</sup> + 25 × head diam.<sup>2</sup> + 26 × bung diam. × head diam.) × length of cask × .000081473.

(2) Wet ullage (in gallons) of cask not half full

$$\begin{aligned}
 &= \left\{ \text{bung diam.} - \frac{(\text{dry inches})^2 \times (\text{bung diam.} - \text{head diam.})}{\text{length}^2} \right\}^2 \\
 &\quad \times \text{wet inches} \times .0028326.
 \end{aligned}$$

(3) Dry inches are obtained by subtracting wet inches from the length of the cask.

(4) If wet inches are less than half the length of the cask, the cask is less than half full.

1. Vol. of cask

$$\begin{aligned}
 &= \{39 (31.5)^2 + 25 (22.7)^2 + 26 (31.5 \times 22.7)\} \times 50.2 \times .000081473 \text{ gallons} \\
 &= (38697.75 + 12832.25 + 18591.3) \times .0015799446 \text{ gallons} \\
 &= (70171.3 \times .0015799446) \text{ gallons} = 110.87 \text{ gallons nearly.}
 \end{aligned}$$

8. Dry inches=60 - 12=48; ∴ mean diam. of occupied portion

$$= \left\{ 36 - \frac{(48)^2 \times (36 - 30)}{60^2} \right\} \text{ in.} = \left( 36 - \frac{48 \times 48 \times 6}{60 \times 60} \right) \text{ in.} = (36 - \frac{144}{15}) \text{ in.} = \frac{396}{15} \text{ in.}$$

∴ no. of gallons in wet ullage

$$= \left( \frac{396}{15} \right)^2 \times 12 \times .0028326 = 1084.2656 \times .0039912 = 35.1559.$$

T. M. K.

## 11. Vol. of cask

$$\begin{aligned}
 &= \{39 (81.5)^2 + 25 (22.7)^2 + 26 (81.5 \times 22.7)\} \times 50.8 \times .000081473 \text{ gallons} \\
 &= 70171.8 \times .000081473 (50.2 + 1) \text{ gallons} = 70171.8 \times .0015830919 \text{ gallons} \\
 &= 111.0876 \dots \text{gallons} = 111.09 \text{ gallons nearly.}
 \end{aligned}$$

$\therefore$  increase of vol. =  $(111.09 - 110.87)$  gallons = .22 gallons.

## CHAPTER XLVI.

N.B.—The diagrams to some of the examples (3, 4, 5, 6, 7, 8, 11, 12,) have been erroneously drawn by inadvertence; the right and left of the base-line in the field-book have been put to the left and right of the base-line in the diagrams. The calculations and areas are unaffected. See Art. 403.

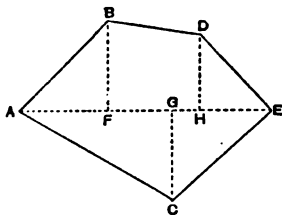
1. Here  $AF=180$ ,  $FH=220$ ,  $HD=100$ ,

$HE=150$ ,  $AE=550$ ,  $GC=110$ ,  $FB=155$ .

(All the measurements in this and the following examples are in links).  $\therefore$  area of  $ABDEC$  = area of  $\Delta AFB$ ,  $DHE$ ,  $ACE$ , and trapezoid  $BFHD$

$$\begin{aligned}
 &= \left( \frac{180 \times 155}{2} + \frac{150 \times 100}{2} + \frac{550 \times 110}{2} \right. \\
 &\quad \left. + \frac{155 + 100}{2} \times 220 \right) \text{ sq. lks.}
 \end{aligned}$$

$= (18950 + 7500 + 30250 + 28050) \text{ sq. lks.} = 79750 \text{ sq. lks.} = .7975 \text{ of an acre.}$



2. Here  $AF=100$ ,  $FB=160$ ,

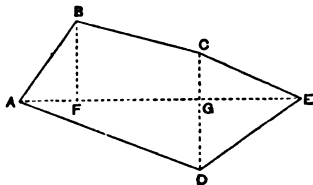
$FG=120$ ,  $GC=50$ ,  $GE=280$ ,  $AE=500$ ,

$GD=140$ .

$\therefore$  area of  $ABCED$  = areas of  $\Delta AFB$ ,  $CGE$ ,  $ADE$ , and trapezoid  $BFGC$

$$\begin{aligned}
 &= \left( \frac{100 \times 160}{2} + \frac{280 \times 50}{2} + \frac{500 \times 140}{2} \right. \\
 &\quad \left. + \frac{160 + 50}{2} \times 120 \right) \text{ sq. lks.}
 \end{aligned}$$

$= (8000 + 7000 + 35000 + 12600) \text{ sq. lks.} = 62600 \text{ sq. lks.} = .626 \text{ of an acre.}$

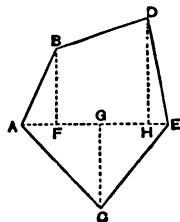


3. Here  $AF=66$ ,  $FB=122$ ,  $FH=177$ ,  $HD=180$ ,

$HE=57$ ,  $AE=300$ ,  $GC=136$ .  $\therefore$  area of  $ABDEC$  = areas of  $\Delta AFB$ ,  $DHE$ ,  $ACE$ , and trapezoid  $BFHD$

$$\begin{aligned}
 &\left( \frac{66 \times 122}{2} + \frac{57 \times 180}{2} + \frac{300 \times 136}{2} \right. \\
 &\quad \left. + \frac{122 + 180}{2} \times 177 \right) \text{ sq. lks.}
 \end{aligned}$$

$= (4026 + 5130 + 20400 + 26727) \text{ sq. lks.} = 56283 \text{ sq. lks.}$   
 $= .56283 \text{ of an acre.}$

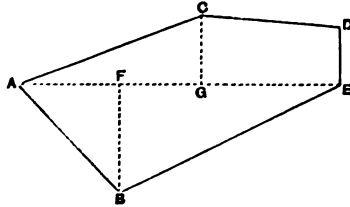


4. Here  $AF=150$ ,  $FB=200$ ,  
 $AG=290$ ,  $GC=90$ ,  $GE=160$ ,  
 $ED=80$ ,  $AE=450$ .

$\therefore$  area of  $ABEDC$  = areas of  
 $\Delta ABE$ ,  $\Delta AGC$ , and trapezoid  $CGED$

$$= \left( \frac{450 \times 200}{2} + \frac{290 \times 90}{2} \right. \\ \left. + \frac{90+80}{2} \times 160 \right) \text{ sq. lks.}$$

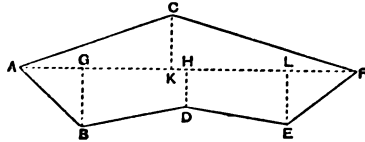
$$= (45000 + 13050 + 13600) \text{ sq. lks.} = 71650 \text{ sq. lks.} = 7165 \text{ of an acre.}$$



5. Here  $AG=180$ ,  $GB=150$ ,  
 $GH=220$ ,  $HL=250$ ,  $LF=150$ ,  
 $HD=70$ ,  $LE=120$ ,  $AF=800$ , and  
 $KC=110$ .  $\therefore$  area of  $ABDEFC$   
 = areas of  $\Delta ACF$ ,  $\Delta AGB$ ,  $\Delta LEF$ ,  
 and trapezoids  $GBHD$  and  $HDEL$

$$= \left( \frac{800 \times 110}{2} + \frac{180 \times 150}{2} \right. \\ \left. + \frac{150 \times 120}{2} + \frac{150+70}{2} \times 220 + \frac{70+120}{2} \times 250 \right) \text{ sq. lks.}$$

$$= (44000 + 13500 + 9000 + 24200 + 23750) \text{ sq. lks.} = 114450 \text{ sq. lks.} = 11445 \text{ acres.}$$

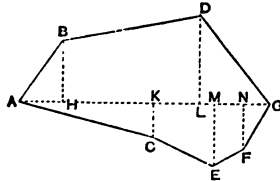


6. Here  $AH=100$ ,  $HB=150$ ,  $HL=370$ ,  
 $LD=170$ ,  $LG=130$ ,  $AK=880$ ,  $KC=50$ ,  
 $KM=100$ ,  $ME=150$ ,  $MN=80$ ,  $NF=140$ ,  
 $NG=40$ .  $\therefore$  area of  $ABDGFEC$  = areas  
 of  $\Delta AHB$ ,  $\Delta DLG$ ,  $\Delta AKC$ ,  $\Delta NGF$ , and trape-  
 zoids  $BHLD$ ,  $KCEM$ ,  $MEFN$

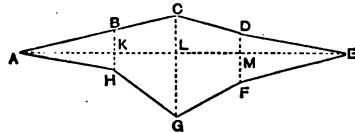
$$= \left( \frac{100 \times 150}{2} + \frac{130 \times 170}{2} + \frac{880 \times 50}{2} \right. \\ \left. + \frac{40 \times 140}{2} + \frac{150+170}{2} \times 370 + \frac{50+150}{2} \times 100 + \frac{150+140}{2} \times 80 \right) \text{ sq. lks.}$$

$$= (7500 + 11050 + 9500 + 2800 + 59200 + 10000 + 11600) \text{ sq. lks.}$$

$$= 111650 \text{ sq. lks.} = 11165 \text{ acres.}$$

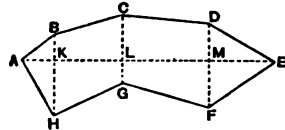


7. Here  $AK=21$ ,  $KB=5$ ,  
 $KL=15$ ,  $LC=8$ ,  $LM=17$ ,  $MD=4$ ,  
 $ME=25$ ,  $KH=4$ ,  $LG=14$ ,  $MF=8$ .  
 $\therefore$  area of  $ABCDEFGH$  = areas of  
 $\Delta ABK$ ,  $\Delta DME$ ,  $\Delta AHK$ ,  $\Delta MEF$ , and  
 trapezoids  $BKLC$ ,  $CLMD$ ,  $KHLG$ ,  
 $LGFM$



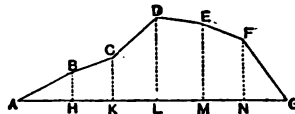
$$\begin{aligned}
 &= \left( \frac{21 \times 5}{2} + \frac{25 \times 4}{2} + \frac{21 \times 4}{2} + \frac{25 \times 8}{2} + \frac{5+8}{2} \times 15 + \frac{8+4}{2} \times 17 + \frac{4+14}{2} \times 15 \right. \\
 &\quad \left. + \frac{14+8}{2} \times 17 \right) \text{ sq. lks.} \\
 &= (1\frac{1}{2}^2 + 50 + 42 + 100 + 1\frac{1}{2}^2 + 102 + 135 + 187) \text{ sq. lks.} = 766 \text{ sq. lks.}
 \end{aligned}$$

8. Here  $AK=12$ ,  $KB=7$ ,  $KL=28$ ,  
 $LC=12$ ,  $LM=35$ ,  $MD=8$ ,  $ME=27$ ,  
 $KH=17$ ,  $LG=4$ ,  $MF=9$ .  
 $\therefore$  area of  $ABCDEFGH$  = areas of  $\Delta^s AKB$ ,  
 $DME$ ,  $AKH$ ,  $MFE$ , and trapezoids  $BKLC$ ,  
 $CLMD$ ,  $HKLG$ ,  $LGFM$



$$\begin{aligned}
 &= \left( \frac{12 \times 7}{2} + \frac{27 \times 8}{2} + \frac{12 \times 17}{2} + \frac{27 \times 9}{2} + \frac{7+12}{2} \times 28 + \frac{12+8}{2} \times 35 \right. \\
 &\quad \left. + \frac{17+4}{2} \times 28 + \frac{4+9}{2} \times 35 \right) \text{ sq. lks.} \\
 &= (42 + 108 + 102 + 2\frac{1}{2}^2 + 266 + 350 + 294 + 4\frac{1}{2}^2) \text{ sq. lks.} = 1511 \text{ sq. lks.}
 \end{aligned}$$

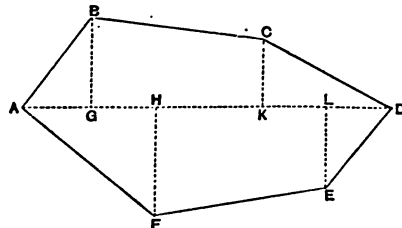
9. Here  
 $AH=HK=KL=LM=MN=NG=20$ ,  
 $HB=18$ ,  $KC=25$ ,  $LD=27$ ,  $ME=26$ ,  
 $NF=19$ .  $\therefore$  area of  $ABCDEFGH$  = areas  
of  $\Delta^s ABH$  and  $FNG$ , and trapezoids  
 $BHKC$ ,  $CKLD$ ,  $DLME$  and  $EMNF$



$$\begin{aligned}
 &= \left( \frac{20 \times 18}{2} + \frac{20 \times 19}{2} + \frac{18+25}{2} \times 20 + \frac{25+27}{2} \times 20 + \frac{27+26}{2} \times 20 \right. \\
 &\quad \left. + \frac{26+19}{2} \times 20 \right) \text{ sq. lks.} \\
 &= (180 + 190 + 430 + 520 + 530 + 450) \text{ sq. lks.} = 2300 \text{ sq. lks.}
 \end{aligned}$$

10. Here  $AG=30$ ,  $GB=28$ ,  
 $GK=60$ ,  $KC=26$ ,  $KD=40$ ,  
 $AH=50$ ,  $HF=40$ ,  $HL=60$ ,  
 $LE=22$ ,  $LD=20$ .

$\therefore$  area of  $ABCDEF$  = areas of  
 $\Delta^s ABG$ ,  $CKD$ ,  $AHF$ , and  $LED$ ,  
and trapezoids  $BGKC$  and  
 $HFEL$



$$\begin{aligned}
 HFEL &= \left( \frac{30 \times 28}{2} + \frac{26 \times 40}{2} \right. \\
 &\quad \left. + \frac{50 \times 40}{2} + \frac{20 \times 22}{2} \right. \\
 &\quad \left. + \frac{28+26}{2} \times 60 + \frac{40+22}{2} \times 60 \right) \text{ sq. lks.} \\
 &= (420 + 520 + 1000 + 220 + 1620 + 1860) \text{ sq. lks.} = 5640 \text{ sq. lks.}
 \end{aligned}$$

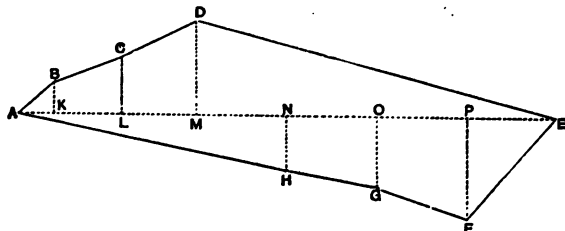
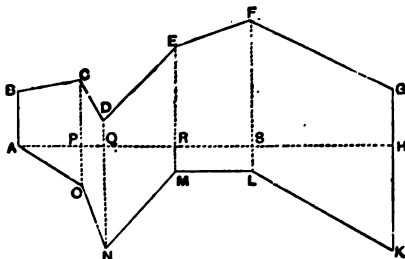
11. Here  $AB=50$ ,  $AP=80$ ,  
 $CO=80$ ,  $PQ=20$ ,  $DN=70$ ,  
 $QR=80$ ,  $EM=80$ ,  $RS=80$ ,  
 $FL=104$ ,  $SH=120$ ,  $GK=80$ .

$\therefore$  area of figure = areas of  
 trapezoids  $ABCO$ ,  $OCDN$ ,

$$\begin{aligned} & DNME, EMLF, FLKG, \\ &= \left( \frac{50+80}{2} \times 80 + \frac{80+70}{2} \times 20 \right. \\ & \quad \left. + \frac{70+80}{2} \times 80 + \frac{80+104}{2} \times 80 \right. \end{aligned}$$

$$\left. + \frac{104+80}{2} \times 120 \right) \text{ sq. lks.}$$

$$= (5200 + 1500 + 6000 + 7860 + 11040) \text{ sq. lks.} = 31100 \text{ sq. lks.} = 311 \text{ of an acre.}$$



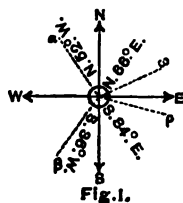
12. Here  $AK=38$ ,  $KB=42$ ,  $KL=122$ ,  $LC=62$ ,  $LM=208$ ,  $MD=235$ ,  
 $ME=1026$ ,  $AN=614$ ,  $NH=184$ ,  $NO=326$ ,  $OG=220$ ,  $OP=172$ ,  $PF=270$ ,  
 $PE=282$ .  $\therefore$  area of figure = areas of  $\triangle ABK$ ,  $DME$ ,  $ANH$ ,  $PEF$ , and  
 trapezoids  $BKLC$ ,  $CLMD$ ,  $NHGO$ , and  $OGFP$

$$\begin{aligned} &= \left( \frac{38 \times 42}{2} + \frac{1026 \times 235}{2} + \frac{614 \times 184}{2} + \frac{282 \times 270}{2} + \frac{42 \times 62}{2} \times 122 \right. \\ & \quad \left. + \frac{62 \times 235}{2} \times 208 + \frac{184 \times 220}{2} \times 326 + \frac{220 \times 270}{2} \times 172 \right) \text{ sq. lks.} \end{aligned}$$

$$= (798 + 120555 + 56488 + 38070 + 6344 + 30888 + 65852 + 42140) \text{ sq. lks.}$$

$$= 361135 \text{ sq. lks.} = 361135 \text{ acres.}$$

13. Let (Fig. 1) represent the four cardinal points of the compass. Make angle  $NO\alpha=52^\circ$ . Now (vide accompanying diag.) Draw  $AB$  parallel to  $O\alpha$  and we thus have the chain-line  $AB$  running N.  $52^\circ$  W. Here  $AB=560$ ,  $LD=30$ ,  $BE=10$ ,  $BM=100$ ,  $MF=30$ ,  $MN=100$ ,  $NG=40$ ,  $NC=190$ ,  $CO=200$ ,  $OH=50$ ,  $OA=50$ ,  $AK=10$ ,  $BC=390$ ,  $AC=250$ .  $\therefore$  area of figure = areas of trapezoids  $EBMF$ ,  $FMNG$ ,  $HOAK$ , and  $\triangle GNC$ ,  $COH$ ,  $ADB$ , and  $ABC = \left( \frac{10+80}{2} \times 100 \right.$



$$\begin{aligned}
 & + \frac{80+40}{2} \times 100 + \frac{50+10}{2} \times 50 + \frac{190 \times 40}{2} \\
 & \quad + \frac{200 \times 50}{2} + \frac{560 \times 30}{2} \\
 & + \frac{1}{2} \sqrt{[(390+250)^2 - 560^2]} \\
 & \quad \{560^2 - (390-250)^2\} \text{ sq. lks.} \\
 & = (2000 + 3500 + 1500 + 3800 + 5000 \\
 & + 8400 + 42000) \text{ sq. lks.} = 66200 \text{ sq. lks.} \\
 & = .662 \text{ of an acre.}
 \end{aligned}$$

14. (Vide Fig. 1 in preceding).

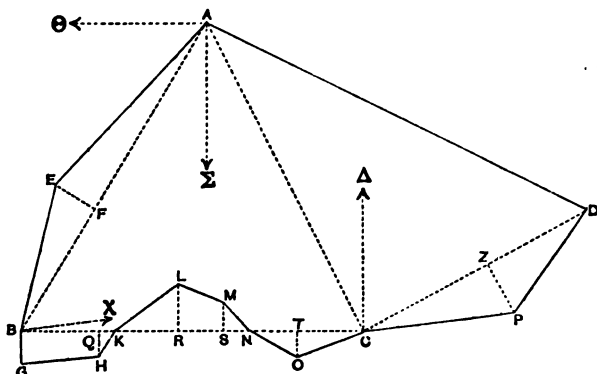
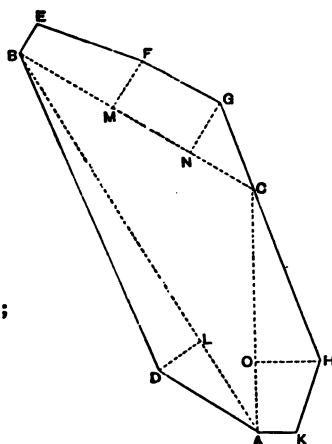
Make angle  $\beta OS = 36^\circ$ ;  $\therefore \angle \beta OW = 54^\circ$ ;

make angle  $SO\rho = 84^\circ$ ;  $\therefore \angle \rho OE = 6^\circ$ ;

make  $\angle NO\omega = 66^\circ$ ;  $\therefore \angle \omega OE = 24^\circ$ .

(Now vide accompanying diag.) draw  $AB$  parallel to  $O\beta$ , and make it = 1000 lks. Draw  $BC$  parallel to  $O\rho$ , and make it = 1000 lks., i.e. =  $AB$ .

Draw  $CD$  parallel to  $O\omega$ , and make it = 750 lks. Join  $A, D$ ; and hence the base lines are all laid down. Complete the rest of the figure in the ordinary

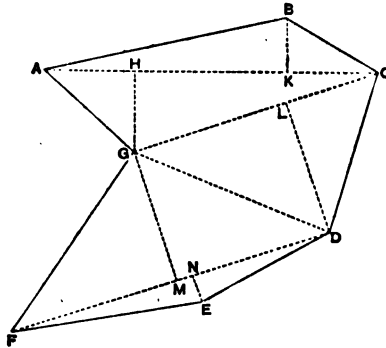


way. Now if we draw  $A\Theta$  and  $B\chi$  each parallel to  $WE$  (in Fig. 1), and  $AZ$  and  $CA$  parallel to  $NS$  (also in Fig. 1), we have  $\angle \Theta AB = 54^\circ$ , and  $\angle AB\chi$  also =  $54^\circ$  (by Euc. I. 29). And  $\angle \chi BC = 6^\circ$ , because it =  $\angle EO\rho$  (in Fig. 1);  $\therefore$  whole angle  $ABC = 60^\circ$ ; and  $AB = BC$  (by hyp.);  $\therefore$  triangle  $ABC$  is equilateral (by Euc. I. 5, 32, and 6);  $\therefore AC$  also = 1000 lks. Again, since  $\angle \Theta A\Sigma = 90^\circ$ , and  $\angle \Theta AB = 54^\circ$ ,  $\therefore \angle B A\Sigma = 36^\circ$ . But  $\angle BAC$  was just proved to be  $60^\circ$ ;  $\therefore \Sigma AC = 24^\circ = \angle A\Delta A$  (Euc. I. 29). And  $\angle ACD = 66^\circ$ , because it =  $\angle NO\omega$  (in Fig. 1).  $\therefore$  whole  $\angle ACD = 90^\circ$ ;  $\therefore$  the triangle  $ACD$  is right angled. Now  $AB = 1000$ ,  $FE = 80$ ,  $BQ = 200$ ,  $BG = 40$ ,  $QH = 10$ ,  $QK = 40$ ,

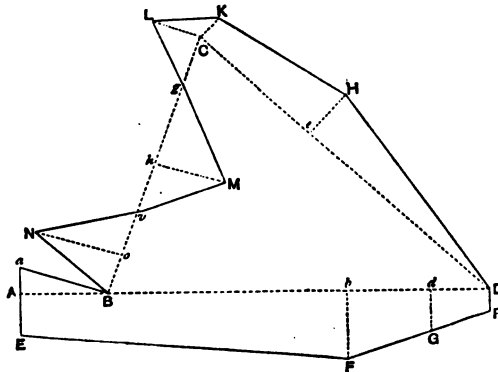
$KR=180, RL=80, SM=20, RS=180, SN=60, NC=340, TO=10, CD=750, ZP=60.$   $\therefore$  area of figure = (areas of triangles  $ABE, ABC, QHK, NOC, CPD, ACD$  and trapezoid  $BGHQ$ ) - (areas of triangles  $KLR, MSN$ , and trapezoid  $LRS M$ ) =  $\left\{ \left( \frac{1000 \times 80}{2} + \frac{(1000)^2}{4} \sqrt{8} + \frac{40 \times 10}{2} + \frac{840 \times 10}{2} + \frac{750 \times 60}{2} + \frac{750 \times 1000}{2} + \frac{40 + 10}{2} \times 200 \right) - \left( \frac{180 \times 80}{2} + \frac{60 \times 20}{2} + \frac{80 + 20}{2} \times 180 \right) \right\}$  sq. lks.  
 $= \left\{ \left( 40000 + \frac{1000000 \times 1.7320508}{4} + 200 + 1700 + 22500 + 375000 + 5000 \right) - (7200 + 600 + 9000) \right\}$  sq. lks. =  $(877412.7 - 16800)$  sq. lks.  
 $= 860612.7$  sq. lks. = 8.606127 acres.

15. Draw  $AC$  parallel to  $WE$  (in Fig. 1), then  $AC$  runs due east. Here  $AC=1650$ ,  $KB=362$ ,  $HG=390$ ,  $GC=1305$ ,  $LD=690$ ,  $DF=1560$ ,  $GM=690$ ,  $EN=20$ .

$\therefore$  area of figure = areas of  $\triangle ABC, AGC, GCD, DGF$ , and  $DEF$ , =  $\left( \frac{1650 \times 362}{2} + \frac{1650 \times 390}{2} + \frac{1305 \times 690}{2} + \frac{1560 \times 690}{2} + \frac{1560 \times 20}{2} \right)$  sq. lks.  
 $= (298650 + 821750 + 450225 + 538200 + 15600)$  sq. lks. = 1624425 sq. lks. = 16.24425 acres.



16. Here  $Aa=10, AE=40, AB=312, Ab=680, Fb=80, bd=360, dG=20, DR=10, Dd=252, De=640, He=80, KC=10, Ce=884, \triangle LCK=300$  sq. lks.;



$Cg=120$ ,  $CL=20$ ,  $gv=390$ ,  $Mh=50$ ,  $Bv=434$ ,  $cN=60$ ,  $\triangle BCD=416732$  sq. lks.;  
 $\therefore$  area of figure = (areas of  $\triangle AaB$ ,  $BCD$ ,  $DEH$ ,  $KCL$ ,  $LCG$ ,  $NBv$ , and  
 trapezoids  $AEFb$ ,  $bFGd$ ,  $dGRD$ ,  $HeCK$ ) - area of  $\triangle gvM$

$$= \left( \frac{312 \times 10}{2} + 416732 + \frac{640 \times 30}{2} + 300 + \frac{120 \times 20}{2} + \frac{434 \times 60}{2} + \frac{40 + 80}{2} \times 680 \right. \\
\left. + \frac{80 + 20}{2} \times 860 + \frac{20 + 10}{2} \times 252 + \frac{30 + 10}{2} \times 384 - \frac{390 \times 50}{2} \right) \text{ sq. lks.} \\
= (1560 + 416732 + 9600 + 300 + 1200 + 13020 + 40800 + 18000 + 3780 \\
+ 7680 - 9750) \text{ sq. lks.} = 502922 \text{ sq. lks.} = 5.02922 \text{ acres.}$$

## CHAPTER XLVIII.

|                                                                                                                        |                                                                                                                                                                                     |                                                                                                                                                                                                                                              |
|------------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 1. $\begin{array}{r} 2 \text{ ft. } 3 \text{ in.} \\ 4 \text{ ft.} \\ \hline 9 \text{ ft. } 0 \text{ pr.} \end{array}$ | 3. $\begin{array}{r} 3 \text{ ft. } 8 \text{ in.} \\ 2 \text{ ft. } 6 \text{ in.} \\ \hline 7 \quad 4 \\ 1 \quad 10 \quad 0 \\ \hline 9 \text{ sq. ft. } 2 \text{ pr.} \end{array}$ | 7. $\begin{array}{r} 4 \text{ ft. } 3 \text{ in. } 4 \text{ twelfths.} \\ 3 \text{ ft. } 3 \text{ in.} \\ \hline 12 \quad 10 \quad 0 \\ 1 \quad 0 \quad 10 \quad 0 \\ \hline 13 \text{ sq. ft. } 10 \text{ pr. } 10 \text{ in.} \end{array}$ |
|------------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|

|                                                                                                                                                                                                                                                                                                                |                                                                                                                                   |
|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------|
| 9. $\begin{array}{r} 5 \text{ ft. } 4 \text{ in. } 8 \text{ twelfths.} \\ 2 \text{ ft. } 7 \text{ in. } 3 \text{ twelfths.} \\ \hline 10 \quad 9 \quad 4 \\ 3 \quad 1 \quad 8 \quad 8 \\ 1 \quad 4 \quad 2 \quad 0 \\ \hline 14 \text{ sq. ft. } 0 \text{ pr. } 4 \text{ in. } 10 \text{ thirds.} \end{array}$ | 11. $3 \text{ ft.} \times 3 \text{ ft.} \times 1 \text{ ft. } 6 \text{ in.}$<br>$= 13 \text{ cub. ft. } 6 \text{ pr. (as in 1).}$ |
|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------|

13. Area of base =  $3 \text{ ft. } 4 \text{ in.} \times 3 \text{ ft. } 8 \text{ in.} = 10 \text{ sq. ft. } 10 \text{ pr. (as in 2); } \therefore \text{ vol.} = 10 \text{ sq. ft. } 10 \text{ pr.} \times 4 \text{ ft.} = 43 \text{ cub. ft. } 4 \text{ pr. (as in 1).}$

15. Area of base =  $6 \text{ ft. } 3 \text{ in.} \times 5 \text{ ft. } 8 \text{ in.} = 32 \text{ sq. ft. } 9 \text{ pr. } 9 \text{ in. (as in 3.)}$   
 $\therefore \text{ vol.} =$

|                                                                                                                                                                                                                                                         |
|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| $\begin{array}{r} 32 \text{ sq. ft. } 9 \text{ pr. } 9 \text{ in.} \\ 3 \text{ ft. } 9 \text{ in.} \\ \hline 98 \quad 5 \quad 3 \\ 24 \quad 7 \quad 8 \quad 9 \\ \hline 123 \text{ cub. ft. } 0 \text{ pr. } 6 \text{ sec. } 9 \text{ in.} \end{array}$ |
|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|

## CHAPTER XLIX.

1. Circum. =  $3.1416 \times 15 \text{ metres} = 47.124 \text{ metres.}$
2. Area of rectangle =  $407.75 \times 304 \text{ sq. metres} = 123956 \text{ sq. metres}$   
 $= 12 \text{ hectares } 39.56 \text{ ares.}$
3. Area of trapezoid =  $\frac{157.6 + 94}{2} \times 72 \text{ sq. metres} = 9057.6 \text{ sq. metres}$   
 $= 90.576 \text{ ares.}$
4. Vol. =  $48 \times 3.4 \times .45 \text{ cub. metres} = 73.44 \text{ cub. metres.}$

5. Vol. of cone =  $\frac{1}{3} \times 8 \cdot 1416 \times (.4)^2 \times 2 \cdot 4$  cub. metres =  $\cdot 4021248$  cub. metres  
=  $402 \cdot 1248$  cub. decimetres.
6. Weight of water =  $(6 \cdot 9 - 1 \cdot 67)$  kilogrammes =  $5 \cdot 23$  kilogrammes  
 $\therefore$  Capacity of vessel =  $5 \cdot 23 \times 1000$  cub. centimetres  
=  $\frac{5 \cdot 23 \times 1000}{1000}$  cub. decimetres =  $5 \cdot 23$  cub. decimetres.
7. Vol. of cistern =  $(8 \cdot 5 \times 10) (6 \times 10) (9 \cdot 2 \times 10)$  cub. decimetres  
=  $469200$  cub. decimetres =  $469200$  litres =  $469 \frac{2}{3}$  hectolitres  
=  $4692$  hectolitres.
8.  $36$  kilogrammes = weight of  $36000$  cub. centimetres of water;  
 $\therefore$  height of cylinder =  $\frac{36000}{8 \cdot 1416 \times 15^2}$  centimetres =  $\frac{20}{\cdot 3927}$  centimetres  
=  $50 \cdot 9$  centimetres =  $51$  centimetres nearly.
9. Vol. of wire =  $8 \cdot 1416 \times \left(\frac{5}{4 \times 100}\right)^2 \times 850000$  cub. decimetres  
=  $\cdot 19685 \times 25 \times 85$  cub. decimetres =  $171 \cdot 80625$  cub. decimetres  
=  $172$  cub. decimetres nearly.
10. Vol. of iron ball =  $\frac{4}{3} \times 8 \cdot 1416 \times (.1)^3 \times 2$  cub. metres  
=  $8 \cdot 1416 \times \frac{\cdot 004}{3}$  cub. metres =  $8141 \cdot 6 \times \frac{4}{3}$  cub. centimetres;  $\therefore$  its weight  
=  $8141 \cdot 6 \times \frac{4}{3} \times 7 \cdot 5 \times \frac{1}{1000}$  kilogrammes =  $81 \cdot 416$  kilogrammes.
11. Whole surface of cone =  $(8 \cdot 1416 \times 8^2 + 8 \cdot 1416 \times 8 \times 8)$  sq. metres  
=  $8 \cdot 1416 (9 + 24)$  sq. metres =  $103 \cdot 6728$  sq. metres =  $104$  sq. metres nearly.
12. Surface of sphere =  $8 \cdot 1416 \times 9 \times 9$  sq. metres =  $254 \cdot 4696$  sq. metres.
13. 1 acre =  $[4840 \times 9 \times 144 \div \{100 \times (89 \cdot 87079)^2\}]$  ares =  $40 \cdot 47$  ares.
14. 1 cub. yard =  $\{1728 \times 27 \div \frac{1}{1000} (89 \cdot 87079)^3\}$  cub. decimetres  
=  $764 \cdot 5$  cub. decimetres.
15. 1 gallon =  $\{277 \cdot 274 \div \frac{1}{1000} (89 \cdot 87079)^3\}$  cub. decimetres  
=  $4 \cdot 54$  cub. litres.
16. Content of sphere =  $\frac{4}{3} \times 8 \cdot 1416 \times \text{radius}^3 \times 2$  radius  
=  $\frac{4}{3} \times 8 \cdot 1416 \times \text{radius}^3$  = (by hyp.) 1 litre =  $\frac{1}{1000}$  of a cub. metre;  
 $\therefore$  radius =  $\frac{8}{4 \times 8 \cdot 1416 \times 1000}$  of a cub. metre =  $\cdot 000238732$  of a cub. metre;  
 $\therefore$  radius =  $\sqrt[3]{(\cdot 000238732 \text{ of a cub. metre})}$  =  $\cdot 062$  of a metre.

17. Content of cylinder =  $3 \cdot 1416 \times \text{radius}^2 \times 2 \text{ radius}$   
 $= 2 \times 3 \cdot 1416 \times \text{radius}^3 = (\text{by hyp.}) \frac{1}{1000}$  of a cub. metre;  
 $\therefore \text{radius}^3 = \frac{1}{2 \times 3 \cdot 1416 \times 1000}$  of a cub. metre =  $\cdot 000159155$  of a cub. metre;  
 $\therefore \text{radius} = \sqrt[3]{\cdot 000159155}$  of a cub. metre =  $\cdot 054$  of a metre.  
 And height =  $2 \times \cdot 054$  of a metre =  $\cdot 108$  of a metre.
18. Vol. of cone =  $\frac{1}{3} \times 3 \cdot 1416 \times \text{radius}^2 \times \text{radius}$   
 $= \frac{1}{3} \times 3 \cdot 1416 \times \text{radius}^3 = (\text{by hyp.}) \frac{1}{1000}$  of a cub. metre;  
 $\therefore \text{radius}^3 = \frac{3}{3 \cdot 1416 \times 1000}$  of a cub. metre =  $\cdot 000954927$  of a cub. metre;  
 $\therefore \text{radius} = \sqrt[3]{\cdot 000954927}$  of a cub. metre =  $\cdot 098$  of a metre = height, likewise.

## MISCELLANEOUS EXAMPLES.

1. (Vide diag. to Chap. V. 19.) Let  $CD$  represent the broken piece. Then  $CD = 39$  ft., and  $BD = 15$  ft.  $\therefore$  (by Euc. I. 47),  
 $BC = \sqrt{(CD^2 - DB^2)} = \sqrt{\{(39^2 - 15^2) \text{ sq. ft.}\}} = \sqrt{\{(39 + 15)(39 - 15) \text{ sq. ft.}\}}$   
 $= \sqrt{\{54 \times 24 \text{ sq. ft.}\}} = \sqrt{\{9 \times 6 \times 6 \times 4 \text{ sq. ft.}\}} = 3 \times 6 \times 2 \text{ ft.} = 36 \text{ ft.}$   
 $\therefore$  height of whole may-pole =  $BC + CD = (36 + 39) \text{ ft.} = 75 \text{ ft.}$
2. No. of square feet required =  $1200 + 144 = 25 \div 3 = 8\frac{1}{3}$ .
3. Let  $a$  be the side of one of the squares, then  $3a$  is the side of the other.  $\therefore$  area of both squares together =  $a^2 + 9a^2 = 10a^2 = (\text{by hyp.}) 100$  acres;  $\therefore a^2$  (i.e. the area of one of the squares) = 10 acres; and  $9a^2$  (i.e. the area of the other square) =  $9 \times 10$  acres = 90 acres.
4. Side of square =  $\sqrt{(2533 \text{ sq. ft. } 64 \text{ in.})} = \sqrt{(2533\frac{1}{3} \text{ sq. ft.})}$   
 $= \sqrt{(22\frac{1}{3} \times 11 \text{ sq. ft.})} = 15\frac{1}{3} \text{ ft.} = 50 \text{ ft. } 4 \text{ in.}$
5. Let  $a$  = breadth of rectangle. Then  $3a$  = its length.  $\therefore$  perimeter of rectangle =  $2(a + 3a) = 8a = (\text{by hyp.}) 144 \text{ yds.}$ ;  $\therefore a = 18 \text{ yds.}$ ; and  $3a = 54 \text{ yds.}$   
 $\therefore$  area of rectangle =  $18 \times 54 \text{ sq. yds.} = 972 \text{ sq. yds.}$
6. Area of floor =  $21 \times 20 \text{ sq. ft.} = 420 \text{ sq. yds.}$   $\therefore$  length of carpet required =  $420 \text{ sq. yds.} \div \frac{27}{12 \times 3} \text{ yds.} = \frac{140}{3} \times \frac{12 \times 3}{27} \text{ yds.} = \frac{560}{9} \text{ yds.}$   
 $\therefore$  cost =  $\frac{560}{9} \times 4s. 6d. = \frac{560}{9} \times \frac{54}{4} = \pounds 14.$
7. Area of 4 walls  
 $= 2 \times 12 \text{ ft. } (16 \text{ ft. } 2\frac{1}{2} \text{ in.} + 15 \text{ ft. } 8\frac{1}{2} \text{ in.}) = 24 \times 31\frac{1}{2} \text{ sq. ft.} = 84 \text{ sq. yds.}$   
 $\therefore$  length of paper required =  $84 \text{ sq. yds.} \div \frac{9}{12 \times 3} \text{ yds.} = 84 \times 4 \text{ yds.} = 336 \text{ yds.}$   
 And cost =  $336 \times 2\frac{1}{2}d. = 336 \times \frac{5}{2}d. = \pounds 83. 10s.$
8. Area of glass in all the windows =  $40 \times 12 \times 20 \times 16 \text{ sq. in.}$   
 $+ 23 \times 9 \times 16 \times 16 \text{ sq. in.} = (42304 + 368) \text{ sq. ft.} = 42672 \text{ sq. ft.}$   
 $\therefore$  cost of glazing =  $42672 \times 2s. 3d. = 42672 \times \frac{45}{4} = \pounds 161. 8s.$

$$\begin{aligned}
 9. \text{ Area} &= \frac{1}{2} \sqrt{\{(1000 + 990)^2 - 890^2\} \{890^2 - (1000 - 990)^2\}} \text{ sq. lks.} \\
 &= \frac{1}{2} \sqrt{\{(1990^2 - 890^2)(890^2 - 10^2)\}} \text{ sq. lks.} = \frac{1}{2} \sqrt{(2880 \times 1100 \times 900 \times 880)} \text{ sq. lks.} \\
 &= \frac{1}{2} \sqrt{(36 \times 8 \times 10 \times 11 \times 100 \times 30^2 \times 8 \times 11 \times 10)} \text{ sq. lks.} \\
 &= \frac{1}{2} \sqrt{(6^2 \times 8^2 \times 10^2 \times 11^2 \times 10^2 \times 30^2)} \text{ sq. lks.} = \frac{1}{2} \times 6 \times 8 \times 10 \times 11 \times 10 \times 30 \text{ sq. lks.} \\
 &= 396000 \text{ sq. lks.} = 3.96 \text{ acres.}
 \end{aligned}$$

10. Let  $a$  and  $b$  be the parallel sides. Then area of trapezoid

$$= \frac{a+b}{2} \times 19 \text{ ft.} = (\text{by hyp.}) 475 \text{ sq. ft.}; \therefore a+b = \frac{475 \times 2}{19} \text{ ft.} = 50 \text{ ft.}$$

Hence we have two equations viz. (1)  $a+b=50$  ft. and (2) (by hyp.)  $a-b=4$  ft. Adding these two equations together, we get  $2a=54$  ft.;  $\therefore a=27$  ft. And subtracting the second equation from the first, we get  $2b=46$  ft.;  $\therefore b=23$  ft.

$$11. \text{ Distance gone by one man in 10 min.} = \frac{4 \times 10}{60} \text{ miles} = \frac{2}{3} \text{ of a mile.}$$

And distance gone by the other in the same time  $= \frac{3 \times 10}{60} \text{ miles} = \frac{1}{2} \text{ a mile.}$

Now (vide diag. to Chap. V. 19), let  $BC = \frac{2}{3}$  of a mile, and  $BD = \frac{1}{2}$  of a mile. Then (by Euc. I. 47),  $CD = \sqrt{(BC^2 + BD^2)} = \sqrt{(\frac{4}{9} + \frac{1}{4})}$  of a sq. mile;

$$= \sqrt{(\frac{17}{36})} \text{ of a sq. mile} = \frac{\sqrt{17}}{6} \text{ of a mile} = \text{the required distance.}$$

$$12. \frac{1}{16} \text{ of a sq. mile} = \frac{1}{16} \times 1760 \times 1760 \text{ sq. yds.}$$

$$= 110 \times 1760 \text{ sq. yds.} = 193600 \text{ sq. yds.}$$

13. Let  $a$  and  $b$  be the sides of the two squares respectively. Then perimeter of 1<sup>st</sup> square  $= 4a = (\text{by hyp.}) 748$  in.;  $\therefore a = 187$  in. In the same way  $4b = 336$  in.;  $\therefore b = 84$  in.  $\therefore$  area of square equal in area to these two squares together  $= (187^2 + 84^2) \text{ sq. in.} = \{(200 - 13)^2 + (90 - 6)^2\} \text{ sq. in.}$

$$= \{(40000 - 5200 + 169) + (8100 - 1080 + 36)\} \text{ sq. in.}$$

$$= (48305 - 6280) \text{ sq. in.} = 42025 \text{ sq. in.} = 205^2 \text{ sq. in.}$$

$$\therefore \text{ side of this square} = \sqrt{(205^2 \text{ sq. in.})} = 205 \text{ in.}$$

$$\therefore \text{ its perimeter} = 4 \times 205 \text{ in.} = 820 \text{ in.}$$

$$14. \text{ Side of square} = \sqrt{(3690 \text{ sq. ft.} \cdot 81 \text{ in.})}$$

$$= \sqrt{(3690 \cdot \frac{3}{4} \text{ sq. ft.})} = \sqrt{(\frac{5223}{4} \text{ sq. ft.})} = 24 \frac{3}{4} \text{ ft.} = 60 \text{ ft. } 9 \text{ in.}$$

$$15. \text{ Side of square} = \sqrt{(13 \text{ ac.} \cdot 1089 \text{ yds.})} = \sqrt{(64009 \text{ sq. yds.})} = 253 \text{ yds.};$$

$$\therefore \text{ perimeter of square} = 4 \times 253 \text{ yds.} = 1012 \text{ yds.};$$

$$\therefore \text{ time required} = \frac{1012}{2 \frac{1}{2} \times 1760} \times 60 \text{ min.} = 2 \frac{1}{2} \text{ min.} = 13.8 \text{ min.}$$

$$16. \text{ Area of floor} = 16 \text{ ft. } 8 \text{ in.} \times 12 \text{ ft.} = 212 \frac{2}{3} \text{ sq. yds.}; \therefore \text{ length of carpet required} = 21 \frac{2}{3} \text{ yds.}; \therefore \text{ cost} = 21 \frac{2}{3} \times 6s. = 21 \frac{2}{3} \times 2 \frac{1}{2} \text{ }^s = \text{£}6. 13s. 4d.$$

17. Area of 4 walls =  $2 \times 14 \text{ ft. } 8 \text{ in. } (24 \text{ ft.} + 20 \text{ ft.}) = 2 \times \frac{11}{2} \times 44 \text{ sq. ft.} = \frac{11}{2} \times 44 \text{ sq. yds.}$ ; and areas of doors, windows, and fireplace =  $4 \times 8 \text{ ft.} \times 5 \text{ ft. } 8 \text{ in.} + 8 \times 10 \text{ ft.} \times 6 \text{ ft. } 8 \text{ in.} + 6 \text{ ft. } 6 \text{ in.} \times 4 \text{ ft.} = (\frac{11}{2} \times 8 + \frac{11}{2} \times 20 + \frac{11}{2}) \text{ sq. yds.} = \frac{11}{2} \times 28 \text{ sq. yds.}$ ;  $\therefore$  portion to be papered =  $(\frac{11}{2} \times 44 - \frac{11}{2} \times 28) \text{ sq. yds.} = \frac{11}{2} \times 8 \text{ sq. yds.}$   $\therefore$  length of paper required =  $\frac{11}{2} \times 8 \text{ sq. yds.} \div \frac{80}{12 \times 3} \text{ yds.} = \frac{11}{2} \times 3 \text{ yds.}$ ;

$$\therefore \text{cost} = \frac{11}{2} \times 11 \frac{1}{2} d. = \frac{11}{2} \times 2 \frac{1}{4} s. = 2 \frac{1}{4} s. = 25. 7s. 6d.$$

18. Let  $a$  = breadth, and  $b$  = height of room. Then  $2a$  = length;  $\therefore$  area of ceiling =  $a \times 2a = 2a^2$ ; but area of ceiling in square yards  $\times \frac{1}{4} d. =$  (by hyp.)  $\pounds 2. 12s. 1d. = 625d.$ ;  $\therefore$  area of ceiling in square yards =  $625 \div \frac{1}{4} = 2500$ ;  $\therefore 2a^2 = 2500 \text{ sq. yds.}$ ;  $\therefore a^2 = 1250 \text{ sq. yds.}$ ;  $\therefore a = \sqrt{1250} \text{ sq. yds.} = 35.35 \text{ yds.}$  Again, area of 4 walls =  $2b(a + 2a) = 6ab = 6 \times 35.35 \text{ yds.} \times b = 212.1 \text{ yds.} \times b$ . But area of 4 walls in square yards  $\times 2s. 4d. =$  (i. e.  $\times 28d.$ )

$$= (\text{by hyp.}) \pounds 35 = 85 \times 20 \times 12d.$$

$$\therefore \text{area of 4 walls in square yards} = \frac{85 \times 20 \times 12}{28} = 800;$$

$$\therefore 50 \text{ yds.} \times b = 800 \text{ sq. yds.}; \therefore b = 16 \text{ yds.}$$

19. Area of triangle

$$\begin{aligned} &= \frac{1}{2} \sqrt{[(988 + 900)^2 - 848^2] \text{ sq. lks.} \times \{848^2 - (988 - 900)^2\} \text{ sq. lks.}} \\ &= \frac{1}{2} \sqrt{\{(1888^2 - 848^2) \text{ sq. lks.} \times (848^2 - 88^2) \text{ sq. lks.}\}} \\ &= \frac{1}{2} \sqrt{(2736 \times 1040 \text{ sq. lks.} \times 936 \times 760 \text{ sq. lks.})} \\ &= \frac{1}{2} \sqrt{\{(16 \times 9 \times 19)(10 \times 104) \text{ sq. lks.} \times (104 \times 9)(10 \times 4 \times 19) \text{ sq. lks.}\}} \\ &= \frac{1}{2} \sqrt{(16 \times 4 \times 19^2 \text{ sq. lks.} \times 9^2 \times 10^2 \times 104^2 \text{ sq. lks.})} \\ &= \frac{1}{2} \times 4 \times 2 \times 19 \times 9 \times 10 \times 104 \text{ sq. lks.} = 855680 \text{ sq. lks.} = 2.5568 \text{ acres.} \end{aligned}$$

20. Area of rectangle =  $41 \text{ yds. } 1 \text{ ft. } 8 \text{ in.} \times 10 \text{ yds. } 10 \text{ in.} = \frac{41}{2} \times \frac{11}{2} \text{ sq. yds.} = 497 \times 185 \times 6 \text{ sq. in.}$  And area of circle of 1 inch radius =  $\frac{1}{4} \pi \text{ sq. in.}$  (by hyp.);  $\therefore$  no. of circles required =  $497 \times 185 \times 6 \times \frac{1}{4} \pi = 175602$ .

21. (For the construction vide Art. 50 of text-book.) To measure off 1.87, place one foot of a pair of compasses on the point where the seventh line (from the bottom), parallel to  $AD$  intersects  $Cc$ , and the other foot on the point where the diagonal line beginning with 8 intersects the above-mentioned seventh line.

22.  $15 \text{ ft. square} = 15 \times 15 \text{ square feet} = 225 \text{ square feet.}$   $\therefore$  excess required =  $(225 - 15) \text{ sq. ft.} = 210 \text{ sq. ft.}$

23. Area of one field =  $945 \times 1344 \text{ sq. yds.}$ ; and area of the other =  $1134 \text{ yds.} \times \text{shorter side}$ ;  $\therefore 1134 \text{ yds.} \times \text{shorter side of 2nd field} =$  (by hyp.)  $945 \times 1344 \text{ sq. yds.}$ ;  $\therefore$  this shorter side

$$= \frac{945 \times 1344}{1134} \text{ yds.} = 85 \times 82 \text{ yds.} = 1120 \text{ yds.}$$

$$\therefore \text{area of 2nd field} = 1134 \times 1120 \text{ sq. yds.} = \frac{1134 \times 1120}{4840} \text{ acres} = \frac{81752}{121} \text{ acres} \\ = 262 \text{ ac. 1 r. 26 p. } 3\frac{1}{2} \text{ sq. yds.} = \text{also area of 1st field.}$$

$$24. \text{ Side of square} = \sqrt{(7867 \text{ sq. ft. } 52 \text{ in.})} = \sqrt{(7867\frac{1}{3} \text{ sq. ft.})} \\ = \sqrt{(12\frac{1}{3} \times 12\frac{1}{3} \text{ sq. ft.})} = 12\frac{1}{3} \text{ ft.} = 85 \text{ ft. } 10 \text{ in.}$$

$$25. \text{ Let } a = \text{breadth of rectangle; then } 3a = \text{length. } \therefore \text{area of rectangle} \\ = a \times 3a = 3a^2 = (\text{by hyp.}) 6 \text{ ac. } 960 \text{ yds.} = 80000 \text{ sq. yds.; } \therefore a^2 = 10000 \text{ sq. yds.;} \\ \therefore a = 100 \text{ yds. } \therefore \text{perimeter of rectangle} = 2(a + 3a) = 8a = 800 \text{ yds. And} \\ \text{distance from corner to corner (in other words, the diagonal of the rectangle)} \\ = (\text{by Euc. I. 47, } \sqrt{(a^2 + (3a)^2)}) = \sqrt{(10a^2)} = \sqrt{(100000 \text{ sq. yds.})} \\ = 316.228 \text{ yds. nearly.}$$

$$26. \text{ Area of floor} = 18 \text{ ft. } 9 \text{ in.} \times 17 \text{ ft. } 6 \text{ in.} = 1\frac{3}{4} \times 1\frac{1}{2} \text{ sq. ft.} = \frac{25 \times 35}{24} \text{ sq. yds.;} \\ \therefore \text{length of carpet required} = \frac{25 \times 35}{24} \times \frac{12 \times 8}{27} \text{ yds.} = \frac{25 \times 35}{18} \text{ yds.;} \\ \therefore \text{cost} = \frac{25 \times 35}{18} \times 5s. 3d. = \frac{25 \times 35}{18} \times 2\frac{1}{2}d. = 2\frac{1}{2}d. = 2\frac{1}{2}d. = 2\frac{1}{2}d. = 2\frac{1}{2}d. = 2\frac{1}{2}d. = 2\frac{1}{2}d. = 2\frac{1}{2}d.$$

$$27. \text{ Area of 4 walls} \\ = 2 \times 9 \text{ ft. } 9 \text{ in.} (15 \text{ ft.} + 10 \text{ ft.}) = 2 \times 1\frac{3}{4} \times 25 \text{ sq. ft.} = 1\frac{3}{4} \text{ sq. yds.} \\ \text{And area of ceiling} = 15 \times 10 \text{ sq. ft.} = 1\frac{3}{4} \text{ sq. yds. } \therefore \text{portion to be painted} \\ = (1\frac{3}{4} + 1\frac{3}{4}) \text{ sq. yds.} = 1\frac{3}{4} \text{ sq. yds.;} \\ \therefore \text{cost} = 1\frac{3}{4} \times 1s. 9d. = 1\frac{3}{4} \times 2\frac{1}{2}d. = 2\frac{1}{4}d. = 2\frac{1}{4}d. = 2\frac{1}{4}d. = 2\frac{1}{4}d. = 2\frac{1}{4}d. = 2\frac{1}{4}d.$$

$$28. \text{ Let } a = \text{breadth, and } b = \text{height of room. Then } 3a = \text{length of room.} \\ \therefore \text{area of floor} = a \times 3a = 3a^2; \text{ but area of floor in square yards } \times 4s. 6d. \\ (\text{i.e. } 54d.) = (\text{by hyp.}) 48s. 4\frac{1}{2}d. = 1\frac{1}{2} \text{ sq. yds.; } \therefore \text{area of floor in square yards} \\ = \frac{3969}{2 \times 54} = \frac{1828}{36}; \therefore 3a^2 = \frac{1828}{36} \text{ sq. yds.; } \therefore a^2 = \frac{441}{36} \text{ sq. yds.} \\ \therefore a = 1\frac{1}{2} \text{ yds.} = \frac{3}{2} \text{ yds.} = 1\frac{1}{2} \text{ yds.; } \therefore 3a = 1\frac{1}{2} \text{ yds.} = 10\frac{1}{2} \text{ yds.}$$

$$\text{Again, area of 4 walls} = 2b(a + 3a) = 8ab = 8 \times \frac{3}{2} \text{ yds.} \times b = 28 \text{ yds.} \times b. \text{ But} \\ \text{area of 4 walls in square yards } \times 9d. = (\text{by hyp.}) 4 \text{ guineas} = 1008d.; \therefore \text{area} \\ \text{of 4 walls in square yards} = 1\frac{1}{2} \times 112 = 112; \therefore 28 \text{ yds.} \times b = 112 \text{ sq. yds.;} \\ \therefore b = 4 \text{ yds.}$$

$$29. \text{ Let } a = \text{third side; then each equal side} = \frac{2}{3}a. \therefore \text{perimeter of tri-} \\ \text{angle} = \frac{2}{3}a + \frac{2}{3}a + a = \frac{4}{3}a = (\text{by hyp.}) 306 \text{ ft.; } \therefore a = 136 \text{ ft. } \therefore \text{height of triangle} \\ (\text{by Euc. I. 5, 26, and 47}) = \sqrt{(\frac{2}{3} \times 136 \text{ ft.})^2 - (\frac{1}{3} \times 136 \text{ ft.})^2} = \sqrt{(85^2 - 68^2) \text{ sq. ft.}} \\ = \sqrt{(153 \times 17 \text{ sq. ft.})} = \sqrt{(17 \times 9 \times 17 \text{ sq. ft.})} = 17 \times 3 \text{ ft.} = 51 \text{ ft.} \\ \therefore \text{area of triangle} = \frac{136 \times 51}{2} \text{ sq. ft.} = 3468 \text{ sq. ft.}$$

30. Here  $AB = 20$  ft.,  $BC = 18$  ft.,  $CD = 32$  ft.,  
 $DE = 13$  ft.

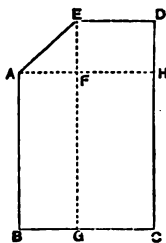
Draw  $EFG$  parallel to  $AB$  or  $DC$ , and  $AFH$  parallel to  $ED$  or  $BC$ .  $\therefore EF = EG - FG = DC - AB$  (by Euc. I. 34)  
 $= (32 - 20)$  ft. = 12 ft.

And  $AF = AH - FH = BC - ED = (18 - 13)$  ft. = 5 ft.

$\therefore$  (by Euc. I. 29, 47)  $AE = \sqrt{(AF^2 + FE^2)}$   
 $= \sqrt{\{(12 \text{ ft.})^2 + (5 \text{ ft.})^2\}} = \sqrt{169 \text{ sq. ft.}} = 13 \text{ ft.}$

$\therefore$  area of  $ABCDE$  = area of rectangle  $ABCH$  + area of  
 trapezoid  $EACH = BC \times BA + \frac{ED + AH}{2} \times EF$

$$= \left( 18 \times 20 + \frac{13 + 18}{2} \times 12 \right) \text{ sq. ft.} = (360 + 186) \text{ sq. ft.} = 546 \text{ sq. ft.}$$



31. (Vide diag. to Chap. VII. 1.) Here  $AB = 96$  ft.,  $CD = 12$  ft., required  $CE$ . Now (Euc. VI. 8. Cor.),  $CD : DA :: DA : DF$ ;

$$\therefore DF = \frac{DA^2}{CD} = \frac{48^2}{12} \text{ ft.} = 192 \text{ ft.}$$

$$\therefore CE = \frac{1}{2} CF = \frac{1}{2} (CD + DF) = \frac{1}{2} (12 + 192) \text{ ft.} = 102 \text{ ft.}$$

32. Area of one square =  $\frac{1}{4}$  of a sq. yard =  $\frac{9 \times 144}{576}$  sq. in. =  $\frac{1}{4}$  sq. in.  
 $\therefore$  side of square =  $\sqrt{(\frac{1}{4} \text{ sq. in.})} = \frac{1}{2}$  in. =  $1\frac{1}{2}$  in.

33. Area of base =  $764^2$  sq. ft. =  $\frac{764^2}{4840 \times 9}$  acres =  $\frac{72962}{5445}$  acres = 13.39 acres,  
 which is a little more than  $13\frac{3}{4}$  acres.

34. Area of rectangle =  $81 \text{ ft.} \times 60\frac{1}{2} \text{ in.} = 243 \times 243 \text{ sq. in.}$ ;  $\therefore$  side of square equal in area to this rectangle =  $\sqrt{(243^2 \text{ sq. in.})} = 243 \text{ in.}$

35. Diagonal (by Euc. I. 47)

$$= \sqrt{\{(300^2 + 200^2) \text{ sq. yds.}\}} = \sqrt{(130000 \text{ sq. yds.})} = 360.555 \text{ yds.}$$

Now sides of interior rectangle are  $300 - 2 \times 30$ ,  $200 - 2 \times 30$  yds.

$$\therefore \text{its area} = 240 \times 140 \text{ sq. yds.} = 33600 \text{ sq. yds.}$$

36. Area of court =  $96 \times 84 \text{ sq. ft.} = 896 \text{ sq. yds.}$ ; and area of grass plots  
 $= 4 \times 22\frac{1}{2} \times 18 \text{ sq. ft.} = 180 \text{ sq. yds.}$

$\therefore$  portion to be paved =  $(896 - 180) \text{ sq. yds.} = 716 \text{ sq. yds.}$

$$\therefore \text{cost} = 716 \times 8\frac{1}{2}d. = 716 \times \text{£} \frac{17}{400} = \text{£} \frac{12173}{100} = \text{£} 25. 7s. 2d.$$

37. Is worked like 7. The second room has four times the same area as the first room to be papered, while the cost of papering any given area is the same as in the first room.

38. Let  $a$  = breadth of room; then  $2a$  = length;  $\therefore$  area of 4 walls  
 $= 2 \times 11 \text{ ft.} (a + 2a) = 66 \text{ ft.} \times a$ ;

∴ length of paper required

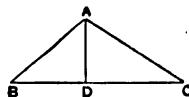
$$= \frac{66 \text{ ft.} \times a}{2 \text{ ft.}} = 33 \times a = (\text{by hyp.}) 165 \text{ yds.} = 165 \times 3 \text{ ft.}; \therefore a = 15 \text{ ft.}$$

∴ area of floor (or of carpet required)

$$= a \times 2a = 15 \times 30 \text{ sq. ft.} = 450 \text{ sq. ft.} = 50 \text{ sq. yds.}$$

39. Here  $AB = 25 \text{ ft.}$ ,  $AC = 39 \text{ ft.}$ ,  $BC = 56 \text{ ft.}$

$$\begin{aligned} \therefore \text{area of } \triangle ABC &= \frac{1}{2} \sqrt{\{(39+25)^2 - 56^2\}} \\ \{56^2 - (39-25)^2\} \text{ sq. ft.} &= \frac{1}{2} \sqrt{\{(64^2 - 56^2)(56^2 - 14^2)\}} \text{ sq. ft.} \\ &= \frac{1}{2} \sqrt{(120 \times 8 \times 70 \times 42)} \text{ sq. ft.} \\ &= \frac{1}{2} \sqrt{(4^2 \times 3^2 \times 10^2 \times 2^2 \times 7^2)} \text{ sq. ft.} = 420 \text{ sq. ft.} \end{aligned}$$



$$\text{But area of } \triangle ABC \text{ also} = \frac{BC \times AD}{2} = 28 \text{ ft.} \times AD; \therefore AD \times 28 \text{ ft.} = 420 \text{ sq. ft.};$$

$$\begin{aligned} \therefore AD &= 15 \text{ ft.} \text{ Again, (by Euc. I. 47) } BD = \sqrt{(AB^2 - AD^2)} \\ &= \sqrt{\{(25^2 - 15^2) \text{ sq. ft.}\}} = \sqrt{(40 \times 10 \text{ sq. ft.})} = \sqrt{(400 \text{ sq. ft.})} = 20 \text{ ft.} \end{aligned}$$

$$\therefore \text{area of } \triangle ABD = \frac{BD \times DA}{2} = \frac{20 \times 15}{2} \text{ sq. ft.} = 150 \text{ sq. ft.} \text{ And area of}$$

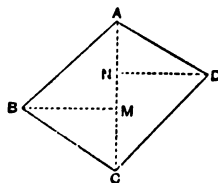
$$\triangle ADC = \text{area of } \triangle ABC - \text{area of } \triangle ABD = (420 - 150) \text{ sq. ft.} = 270 \text{ sq. ft.}$$

40. Here  $AC = 1220 \text{ lks.}$ ,  $BM = 740 \text{ lks.}$ ,

$$DN = 816 \text{ lks.},$$

∴ area of  $ABCD = \text{areas of triangles } ABC \text{ and } ADC$

$$\begin{aligned} &= \left( \frac{1220 \times 740}{2} + \frac{1220 \times 816}{2} \right) \text{ sq. lks.} \\ &= 11212 (740 + 816) \text{ sq. lks.} = 610 \times 1556 \text{ sq. lks.} \\ &= 949160 \text{ sq. lks.} = 9 \cdot 4916 \text{ acres.} \end{aligned}$$



41. (Vide diag. to Chap. VII. 1.) Here  $AB = 24 \text{ ft.}$ ,  $CD = 5 \text{ ft.}$  ∴ (by Euc. III. 3),  $AD = DB = \frac{1}{2} AB = 12 \text{ ft.}$ ; ∴ (by Euc. I. 47),

$$AC = \sqrt{(CD^2 + DA^2)} = \sqrt{\{(5^2 + 12^2) \text{ sq. ft.}\}} = \sqrt{(169 \text{ sq. ft.})} = 13 \text{ ft.}$$

$$\therefore \text{length of arc} = \frac{8 \times 13 - 24}{3} \text{ ft.} = 26\frac{1}{3} \text{ ft.}$$

42. Area of floor of room =  $15 \text{ ft. } 9 \text{ in.} \times \text{breadth} = \frac{2}{3} \text{ ft.} \times \text{breadth}$  (by hyp.)  $21 \text{ sq. yds.} = 21 \times 9 \text{ sq. ft.}$ ; ∴ breadth of room =  $21 \times 9 \times \frac{3}{2} \text{ ft.} = 12 \text{ ft.}$

43. Area of rectangle =  $20 \text{ ft. } 5\frac{1}{2} \text{ in.} \times \text{the other side} = \frac{4}{3} \text{ ft.} \times \text{the other side}$  (by hyp.)  $372 \text{ sq. ft. } 32\frac{1}{2} \text{ in.} = \frac{211 \times 66 \frac{2}{3}}{4} \text{ sq. ft.}$ ; ∴ the other side of the rectangle =  $\frac{211 \times 66 \frac{2}{3}}{4 \times \frac{4}{3}} \text{ ft.} = \frac{211 \times 66 \frac{2}{3}}{4} \text{ ft.} = 18 \text{ ft. } 2\frac{1}{2} \text{ in.}$

44. Is worked like 34.

45. Area of 1<sup>st</sup> land =  $375 \text{ ft. } 6 \text{ in.} \times 75 \text{ ft. } 6 \text{ in.} = \frac{751}{2} \times \frac{151}{2} \text{ sq. ft.}$  And area of 2<sup>nd</sup> land =  $278 \text{ ft. } 9 \text{ in.} \times 157 \text{ ft.} = \frac{1115}{2} \times 157 \text{ sq. ft.}$

∴ cost of latter =  $(\frac{1115}{2} \times 157) \div (\frac{751}{2} \times \frac{151}{2}) \times £118. 2s. 6\frac{1}{2}d.$

$$= \frac{1115 \times 157}{751 \times 151} \times £ \frac{118401}{960} = £ \frac{35011}{192} = £182. 6s. 11\frac{1}{2}d.$$

$$46. \text{ Area of street} = \frac{1}{2} \times 1760 \text{ yds.} \times 4\frac{1}{2} \text{ yds.} = \frac{880 \times 47}{8} \text{ sq. yds.};$$

$$\therefore \text{ cost} = \frac{880 \times 47}{8} \times 7\frac{1}{2}d. = \frac{880 \times 47}{8} \times \frac{1}{82} = \frac{2585}{6} = £430. 16s. 8d.$$

47. Area of 4 walls

$$= 2 \times 16 \times (20\frac{1}{2} + 15\frac{1}{2}) \text{ sq. ft.} = 32 \times 36 \text{ sq. ft.} = 128 \text{ sq. yds.}$$

And areas of doors, windows, and fireplace together

$$= (2 \times 8 \times 3\frac{1}{2} + 5 \times 7 + 2 \times 5 \times 4 + 4\frac{1}{2} \times 3) \text{ sq. ft.}$$

$$= (60 + 35 + 40 + 14) \text{ sq. ft.} = 149 \text{ sq. yds.};$$

$$\therefore \text{ portion of room to be papered} = (128 - 149) \text{ sq. yds.} = 149 \text{ sq. yds.}$$

$$\therefore \text{ length of paper required} = \frac{1008}{9} \times \frac{12 \times 8}{80} \text{ yds.} = \frac{2006}{15} \text{ yds.}$$

$$\therefore \text{ cost} = \frac{2006}{15} \times 7\frac{1}{2}d. = \frac{2006}{15} \times \frac{15}{80} = \frac{1003}{240} = £4. 8s. 7d.$$

48. Is worked like 28.

49. Here  $AB = BC = CD = 325 \text{ ft.};$

$$AD = 733 \text{ ft.},$$

and is parallel to  $BC$ . Draw  $CE$  parallel to  $AB$ , and  $CF$  perpendicular to  $AD$ ; then (by hyp., and Euc. I. 5, 26),

$$EF = FD = \frac{1}{2} ED = \frac{1}{2} (AD - AE)$$

$$= \frac{1}{2} (AD - BC) \text{ (by Euc. I. 34)} = 204 \text{ ft.}$$

$$\therefore FC = \sqrt{(EC^2 - EF^2)} = \sqrt{(AB^2 - EF^2)} = \sqrt{\{(325^2 - 204^2) \text{ sq. ft.}\}}$$

$$= \sqrt{\{(529 \times 121) \text{ sq. ft.}\}} = 23 \times 11 \text{ ft.} = 253 \text{ ft.}$$

$\therefore$  area of trapezoid  $ABCD$

$$= \frac{AD + BC}{2} \times FC = \frac{733 + 325}{2} \times 253 \text{ sq. ft.} = 529 \times 253 \text{ sq. ft.} = 133837 \text{ sq. ft.}$$

50. By Euc. IV. 6 and I. 47, side<sup>2</sup> of inscribed square = radius<sup>2</sup> + radius<sup>2</sup> = 2 radius<sup>2</sup> =  $2 \times 6^2 \text{ sq. ft.} = 72 \text{ sq. ft.}$  = area of inscribed square.

51. Here  $AB = 12 \text{ in.}, BC = 15 \text{ in.}, AC = 20 \text{ in.},$

$$DF = 25 \text{ in.},$$

required  $DE$  and  $EF$ . Now  $\triangle ABC$  and  $DEF$  are similar;

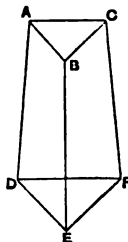
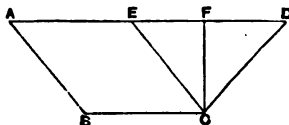
$\therefore$  (by Euc. VI. 4),  $AC : BC :: DF : FE$ ;

$$\therefore FE = \frac{BC \times DF}{AC} = \frac{15 \times 25}{20} \text{ in.} = \frac{75}{4} \text{ in.} = 18\frac{3}{4} \text{ in.}$$

And  $AC : AB :: DF : DE$ ;

$$\therefore DE = \frac{AB \times DF}{AC} = \frac{12 \times 25}{20} \text{ in.} = 15 \text{ in.}$$

52. Is worked like 45.



53. Is worked like 43.

54. 10000 sq. metres =  $(85881 + 3)$  sq. yds.;

$\therefore$  1 sq. metre =  $\frac{1}{10000} \times (85881 + 3)$  sq. yds. =  $1 \cdot 19603$  sq. yds.

$\therefore$  1 metre =  $\sqrt{1 \cdot 19603}$  sq. yds. =  $1 \cdot 0936$  yds.

55. Area of platform =  $54 \times 21$  sq. yds. =  $54 \times 21 \times 9$  sq. ft.; and area of 1 plank =  $13\frac{1}{2}$  ft.  $\times$   $10\frac{1}{2}$  in. =  $\frac{3}{4} \times \frac{1}{4}$  sq. ft.;  $\therefore$  no. of planks required

$$= (54 \times 21 \times 9) \div (\frac{3}{4} \times \frac{1}{4}) = 864;$$

and cost of wood

$$= 54 \times 21 \times 9 \times 5\frac{1}{2}d. = 54 \times 21 \times 9 \times \frac{11}{2} = \frac{54 \times 21 \times 9 \times 11}{2} = \pounds 233. 17s. 9d.$$

56. Area of floor = 17 ft. 9 in.  $\times$  12 ft. 6 in. =  $\frac{71 \times 25}{8 \times 9}$  sq. yds.  $\therefore$  length of Brussels carpet required =  $\frac{71 \times 25}{8 \times 9} \times \frac{1}{3}$  yds. =  $\frac{71 \times 25}{6 \times 9}$  yds., and length of

Kidderminster carpet required =  $\frac{71 \times 25}{8 \times 9} \times \frac{2}{3}$  yds. =  $\frac{71 \times 25}{7 \times 9}$  yds.  $\therefore$  cost of

former carpet =  $\frac{71 \times 25}{6 \times 9} \times 4s. 6d.$ ; and cost of latter =  $\frac{71 \times 25}{7 \times 9} \times 3s. 6d.$   $\therefore$  dif-

ference in cost =  $\frac{71 \times 25}{9} (\frac{1}{3} \times 4s. 6d. - \frac{1}{3} \times 3s. 6d.) = \frac{71 \times 25}{9} \times 8d.$

$$= \frac{71 \times 25}{9} \times \frac{8}{16} = \frac{71 \times 25}{18} = \pounds 2. 9s. 3\frac{1}{2}d.$$

57. Area of 4 walls =  $2 \times 10 (15\frac{3}{4} + 9\frac{1}{2})$  sq. ft. = 500 sq. ft.; and area of windows and doors together =  $(2 \times 5 \times 4 + 3 \times 6\frac{3}{4} \times 3)$  sq. ft. = 100 sq. ft.;  $\therefore$  portion of room to be covered with postage stamps

$$= (500 - 100) \text{ sq. ft.} = 400 \text{ sq. ft.} = 400 \times 144 \text{ sq. in.}$$

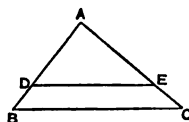
But area of 1 postage stamp =  $\frac{1}{4} \times \frac{1}{4}$  sq. in. =  $\frac{1}{16}$  sq. in.;  $\therefore$  no. of postage stamps required =  $400 \times 144 \times \frac{16}{1} = 81920$ .

58. Here  $\triangle ADE = \frac{1}{4} \triangle ABC$ ,  $DE$  is parallel to  $BC$ , which = 200 yards. Now  $\triangle ADE$  and  $\triangle ABC$  are similar,  $\therefore$  area of  $\triangle ABC$  : area of  $\triangle ADE :: BC^2 : DE^2$ ;  
 $\therefore$  area of  $\triangle ABC = \frac{1}{4}$  area of  $\triangle ABC$

$$\therefore 40000 \text{ sq. yds.} : DE^2;$$

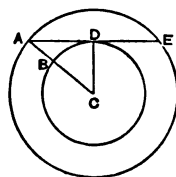
$$\therefore DE^2 = \frac{1}{4} \times 40000 \text{ sq. yds.} = 20000 \text{ sq. yds.}$$

$$\therefore DE = \sqrt{20000 \text{ sq. yds.}} = 141 \cdot 42 \text{ yds.}$$



59. Let  $a$  = radius  $AC$  of outer circle, and  $b$  = radius  $BC$  of inner circle.  $\therefore$  area of space between the circles =  $3 \cdot 1416 a^2 - 3 \cdot 1416 b^2 = 3 \cdot 1416 (a^2 - b^2)$ .

Now draw chord  $ADE$  touching inner circle at  $D$ . Join  $DC$ . Then angles at  $D$  are right angles (by Euc. III. 18).  $\therefore$  (by Euc. I. 47),  $AD^2 = (AC^2 - CD^2) = a^2 - b^2$ ;  $\therefore$  area of space between the circles =  $3 \cdot 1416 AD^2$ , i.e. = area of a circle whose radius is  $AD$ , or whose diam. is  $AE$  (for  $AD = DE$  by Euc. III. 3).



T. M. K.

60. Area of court =  $8 \cdot 1416 (15 \text{ ft.})^2 = 706 \cdot 86 \text{ sq. ft.}$ ; and area of hexagonal space =  $6 \times \frac{2^2}{4} \sqrt{3} \text{ sq. ft.}$  (vide Chap. XV. 6) =  $6 \times 1 \cdot 732 \text{ sq. ft.} = 10 \cdot 392 \text{ sq. ft.}$ ;  
 $\therefore$  area of portion of court to be paved =  $(706 \cdot 86 - 10 \cdot 392) \text{ sq. ft.} = 696 \cdot 468 \text{ sq. ft.}$   
 $\therefore$  cost =  $696 \cdot 468 \times 2s. 3d. = 696 \cdot 468 \times \mathcal{L} \frac{25}{8} = \mathcal{L} 78 \cdot 3526 = \mathcal{L} 78. 7s. \text{ nearly.}$

61. No. of bricks in wall =  $120 \times 15 \times 2 = 3600.$

62. Vol. of solid =  $2 \text{ yds.} \times 1 \frac{1}{4} \text{ yds.} \times \text{thickness} = 8 \text{ sq. yds.} \times \text{thickness}$   
 = (by hyp.) 1 cub. yd. 6 ft. 1296 in. =  $\frac{1}{2}$  cub. yds.;  
 $\therefore$  thickness =  $\frac{1}{2} \times \frac{1}{4}$  of a yard =  $\frac{1}{8}$  yds. = 15 in.

63. Vol. of water =  $18 \frac{3}{4} \times 13 \frac{1}{2} \times \frac{1}{8}$  cub. ft. =  $1 \frac{3}{8}$  cub. ft.;  $\therefore$  weight of water =  $1 \frac{3}{8} \times 62 \frac{1}{2}$  lbs. =  $1 \frac{1}{2} \times 62 \frac{1}{2}$  lbs. = 260  $\frac{1}{4}$  lbs.

64. Vol. of all the required no. of leaves together  
 =  $56 \times 1 \text{ cub. in.} = \frac{1}{8}$  cub. ft.;

$\therefore$  their weight =  $\frac{1}{8} \times 10 \text{ cwt. 95 lbs.} = \frac{1}{8} \times 1215 \text{ lbs. Av.}$   
 =  $\frac{1}{8} \times 1215 \times 7000 \text{ grs. Troy} = 275625 \text{ grs. Troy.}$

But 1 leaf (by hyp.) weighs 1 gr. Troy;

$\therefore$  no. of leaves that weigh 275625 grs. = 275625.

65. Edge of cube

=  $\sqrt[3]{(2 \cdot 370 \text{ cub. yds.})} = \sqrt[3]{(2 \frac{3}{8} \frac{7}{8} \text{ cub. yds.})} = \sqrt[3]{(64 \text{ cub. ft.})} = 4 \text{ ft.}$

66. Every internal dimension of figure = corresponding external dimension - 2 in.  $\therefore$  the internal dimensions are respectively, 7 ft. 1 in., 6 ft. 3 in., 4 ft. 1 in.  $\therefore$  vol. of box were it full = 7 ft. 3 in.  $\times$  6 ft. 5 in.  $\times$  4 ft. 3 in.

=  $2 \frac{1}{2} \times 1 \frac{1}{2} \times 1 \frac{1}{2}$  cub. ft. =  $2 \frac{1}{8} \frac{1}{8} \frac{1}{8}$  cub. ft.;

and vol. of empty portion = 7 ft. 1 in.  $\times$  6 ft. 3 in.  $\times$  4 ft. 1 in.

=  $1 \frac{5}{8} \times 2 \frac{3}{8} \times 1 \frac{1}{8}$  cub. ft. =  $1 \frac{0}{8} \frac{4}{8} \frac{1}{8} \frac{3}{8}$  cub. ft.

$\therefore$  vol. of material in box =  $(\frac{2}{8} \frac{9}{8} \frac{1}{8} - \frac{10}{8} \frac{4}{8} \frac{1}{8} \frac{3}{8})$  cub. ft. =  $\frac{4}{8} \frac{5}{8} \frac{7}{8}$  cub. ft., and this weighs 2 cwt. 2 qrs. 7 lbs., i.e.  $\frac{1}{4}$  cwt.;  $\therefore$  weight of  $\frac{2}{8} \frac{9}{8} \frac{1}{8}$  cub. ft.

=  $\frac{2}{8} \frac{9}{8} \frac{1}{8} \times \frac{2}{8} \frac{5}{8} \times \frac{1}{8}$  cwt. =  $\frac{2}{8} \frac{5}{8}$  cwt. =  $\frac{2}{8} \frac{5}{8}$  lbs. = 334  $\frac{1}{2}$  lbs.

67. Vol. of monolith =  $113 \times 115 \text{ cub. ft.} = \frac{113 \times 115}{27} \text{ cub. yds.}$ ;  $\therefore$  vol. of

obelisk =  $\frac{2}{3} \times \frac{113 \times 115}{27} \text{ cub. yds.} = \frac{25990}{81} \text{ cub. yds.} = 320 \frac{10}{81} \text{ cub. yds.}$  Now

since  $\frac{25990}{81} \text{ cub. yds.}$  or  $\frac{25990}{81} \text{ cub. ft.}$  of granite weigh 639 tons,  $\therefore$  1 cub.

foot of granite weighs  $\frac{3 \times 639}{25990} \text{ tons} = \frac{429408}{25990} \text{ lbs.} = 165 \text{ lbs. nearly.}$

68. Vol. of wood

=  $3 \cdot 1416 \times (\frac{1}{2} \text{ ft.})^2 \times \frac{1}{2} \text{ ft.} = 0 \cdot 6545 \times 49 \text{ cub. ft.} = 3 \cdot 20705 \text{ cub. ft.}$

Upper surface of table =  $3 \cdot 1416 \times (\frac{1}{2} \text{ ft.})^2 = 38 \cdot 4846 \text{ sq. ft.}$ ;  $\therefore$  cost of polishing =  $38 \cdot 4846 \times 6d. = 38 \cdot 4846 \times \frac{1}{2}s. = 19 \cdot 2423s. = 19s. 3d. \text{ nearly.}$

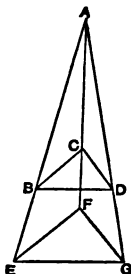
69. Content of hose =  $8.1416 \times (1 \text{ in.})^2 \times 480 \text{ cub. in.}$ ;  
 and 1 gallon contains 277.274 cub. in.;  
 $\therefore$  no. of gallons hose can contain =  $\frac{8.1416 \times 480}{277.274} = \frac{753.984}{138.637} = 5.44$  nearly.

70. Here  $BC = 2 \text{ ft.}$ ,  $CD = 3 \text{ ft.}$ ,  $EF = 8 \text{ ft.}$  Now from similar  $\triangle BCD$  and  $EFG$  (by Euc. VI. 4),

$$FG : FE :: CD : CB; \therefore FG = \frac{FE \times CD}{CB} = \frac{8 \times 3}{2} \text{ ft.} = 12 \text{ ft.}$$

Thus area of triangle  $BCD = \frac{3 \times 2}{2} = 3 \text{ sq. ft.}$ ; and area of  $EFG = \frac{8 \times 12}{2} = 48 \text{ sq. ft.}$ ; also the square root of their product =  $\sqrt{(3 \times 48)} = 12$ . Hence applying the rule of Art. 268 of the text,

$$\begin{aligned} \text{Vol. of frustum} &= \frac{1}{3} \times 7 \times (3 + 12 + 48) \\ &= 147 \text{ cub. ft.} \end{aligned}$$



71. Vol. of wall =  $72 \times 8 \times \frac{3}{4} \text{ cub. ft.} = 72 \times 12 \times 1728 \text{ cub. in.}$ ; and vol. of 1 brick =  $9 \times 4\frac{1}{2} \times 3 \text{ cub. in.} = 9 \times \frac{9}{2} \times 3 \text{ cub. in.}$ ;  $\therefore$  no. of bricks required

$$= 72 \times 12 \times 1728 \times \frac{2}{81 \times 3} = 12288.$$

72. Vol. of solid = length  $\times 1 \text{ ft.} \times 18 \text{ in.} = \frac{3}{4} \text{ sq. ft.} \times \text{length} = (\text{by hyp.}) 3 \text{ cub. ft.}$   $216 \text{ in.} = 2\frac{3}{4} \text{ cub. ft.}$   $\therefore$  length of solid =  $2\frac{3}{4} \times \frac{4}{3} \text{ ft.} = 2\frac{2}{3} \text{ ft.} = 25 \text{ in.}$

73. Vol. of box =  $4 \text{ ft.} \times 2\frac{1}{2} \text{ ft.} \times 1\frac{1}{2} \text{ ft.} = 15 \text{ cub. ft.} = 15 \times 1728 \text{ cub. in.}$ ; and vol. of the 252 books =  $252 \times 8 \times 5 \times \frac{3}{4} \text{ cub. in.} = 252 \times 60 \text{ cub. in.}$ ;  $\therefore$  vol. of portion of box still to be filled

$$= (15 \times 1728 - 60 \times 252) \text{ cub. in.} = 60 (432 - 252) \text{ cub. in.} = 10800 \text{ cub. in.}$$

Now vol. of one of the other books =  $6 \times 3 \times \frac{3}{4} \text{ cub. in.} = \frac{45}{2} \text{ cub. in.}$ ;  $\therefore$  no. of the latter books required to fill the remaining portion of the box

$$= 10800 \times \frac{2}{45} = 480.$$

74. Vol. of parallelepiped = base  $\times$  height =  $26 \text{ sq. ft.} \times 6 \text{ in.} \times \text{height}$   
 $= \frac{42\frac{1}{2}}{3} \text{ sq. ft.} \times \text{height} = (\text{by hyp.}) 659 \text{ cub. ft.}$   $1248 \text{ in.} = \frac{11\frac{3}{4}}{3} \text{ cub. ft.}$ ;  
 $\therefore$  height =  $\frac{11\frac{3}{4}}{3} \times \frac{3}{2} \text{ ft.} = 7\frac{3}{4} \text{ ft.} = 25 \text{ ft. } 4 \text{ in.}$

75. Edge of cube =  $\sqrt[3]{(10970.645048 \text{ cub. in.})} = 22.22 \text{ in.}$

76. Surface of cube =  $6 \times \text{edge}^2 = (\text{by hyp.}) 393.66 \text{ sq. ft.}$ ;  
 $\therefore$  edge =  $\sqrt{(65.61 \text{ sq. ft.})} = 8.1 \text{ ft.}$ ;  
 and cubical content =  $(8.1 \text{ ft.})^3 = 531.441 \text{ cub. ft.}$

77. Vol. of cone =  $\frac{1}{3} \times 8.1416 \times 1^2 \times 3 \text{ cub. ft.} = 8.1416 \text{ cub. ft.}$

78. Vol. of roller =  $8.1416 (10 + \frac{1}{2})^2 \times 30 \text{ cub. in.} - 8.1416 \times 10^2 \times 30 \text{ cub. in.}$   
 $= 8.1416 \times 30 \{ (\frac{21}{2})^2 - 10^2 \} \text{ cub. in.} = 8.1416 \times 30 \times \frac{41}{4} \times \frac{1}{2} \text{ cub. in.}$ ;  $\therefore$  its weight  
 $= 8.1416 \times 30 \times \frac{41}{4} \times 4.562 \text{ oz.} = 8927 \times 30 \times 10.25 \times 2.281 \text{ lbs.} = 275.4 \text{ lbs.}$

79. Area of flat roof =  $21^2$  sq. ft. = 441 sq. ft.

∴ cost of leaden covering =  $441 \times 6d. = £44\frac{1}{2} = £11. 0s. 6d.$

Now if diag. to Chap. XXV, 21 represent the pyramidal roof

$$AC = \sqrt{[10^2 + (\frac{1}{2} \times 21)^2]} \text{ sq. ft.} = \sqrt{(\frac{25}{4} \times 21 \text{ sq. ft.})} = \frac{1}{2} \times 21 \text{ ft.}$$

∴ lateral surface of pyramid

$$= 4 \times \frac{21 \times 29}{2 \times 2} \text{ sq. ft.} = 21 \times 29 \text{ sq. ft.} = 21 \times 29 \times 144 \text{ sq. in.}$$

And area of 1 slate =  $12 \times 9$  sq. in. ∴ no. of slates required for pyramidal roof =  $(21 \times 29 \times 144) \div (12 \times 9) = 28 \times 29$ . And their cost

$$= 28 \times 29 \times \frac{1}{100} \times 18s. 9d. = \frac{28 \times 29}{100} \times £\frac{15}{16} = £\frac{609}{80} = £7. 12s. 3d.$$

80. Let  $a$  and  $b$  represent corresponding dimensions of the solids, and  $3c$  and  $c$  the surfaces of these respectively. Then  $3c : c :: a^3 : b^3$ ;

$$\therefore a^3 = 3b^3; \therefore a = b\sqrt[3]{3}.$$

Again, vol. of 1<sup>st</sup> solid : vol. of 2<sup>nd</sup> solid ::  $a^3 : b^3$ ;

$$\text{i.e. as } b^3 \times 3\sqrt[3]{3} : b^3; \text{ i.e. as } 3\sqrt[3]{3} : 1.$$

81. Vol. of parallelepiped =  $36 \times 75 \times 80$  cub. in. =  $4^3 \times 5^3 \times 3^3$  cub. in.

$$= 60^3 \text{ cub. in.} = (\text{by hyp.}) \text{ vol. of cube.}$$

$$\therefore \text{edge of cube} = \sqrt[3]{(60^3 \text{ cub. in.})} = 60 \text{ in.} = 5 \text{ ft.}$$

82. Vol. of water flowing per minute

$$= \frac{1}{8} \times (30 \text{ ft.} \times 200 \text{ yds.} \times 4 \text{ miles}) = 3600 \times 1760 \text{ cub. ft.};$$

and 1 cub. foot of water weighs 1000 oz. Av.; ∴ weight of water required

$$= 3600 \times 1760 \times 1000 \text{ oz.} = \frac{3600 \times 1760 \times 1000}{16 \times 28 \times 4 \times 20} \text{ tons} = \frac{1237500}{7} \text{ tons}$$

$$= 176785\frac{1}{7} \text{ tons} = 176786 \text{ tons nearly.}$$

83. Side of cube =  $\sqrt[3]{(5 \cdot 359375 \text{ cub. ft.})} = \sqrt[3]{(9261 \text{ cub. in.})} = 21 \text{ in.}$

84. Vol. of cistern =  $(10^3 + 2^3)$  cub. in. = 1008 cub. in.; and area of its base =  $\{(1\frac{1}{2})^2 - (1\frac{1}{2})^2\}$  sq. ft. =  $\frac{1}{4}$  sq. ft. =  $1\frac{1}{2} \times 1\frac{1}{2}$  sq. in. Now vol. of cistern likewise = area of its base  $\times$  its depth =  $1\frac{1}{2} \times 1\frac{1}{2} \times \text{depth}$ ; ∴  $1\frac{1}{2} \times 1\frac{1}{2} \times \text{depth}$  of cistern = 1008 cub. in.; ∴ depth of cistern =  $1008 \times \frac{1}{1\frac{1}{2} \times 1\frac{1}{2}} \text{ in.} = 27 \text{ in.}$

85. Vol. of cube in cub. in. =  $\frac{£1206. 4s. 4d.}{£3. 10s. 4d.} = \frac{72373}{60} \times \frac{60}{211} = 343$ ; ∴ edge of cube =  $\sqrt[3]{(343 \text{ cub. in.})} = 7 \text{ in.}$ ; ∴ surface of cube =  $6 \times 7^2 \text{ sq. in.}$  And cost of gilding =  $6 \times 49 \times \frac{1}{4}d. = 4\frac{1}{2}s. = 12s. 3d.$

86. Thickness of 1 sheet of paper =  $\frac{1}{4 \times 10^6}$  of an inch; ∴ vol. of the required no. of leaves =  $56 \times \frac{1}{4 \times 10^6} \text{ cub. in.} = \frac{1}{5 \times 10^6}$  of a cub. in.; ∴ their weight =  $\frac{1}{5 \times 10^6} \times 17 \times 18 \times 1215 \text{ lbs. Av.} = \frac{6}{5 \times 10^6} \text{ lbs. Av.} = \frac{6}{5 \times 10^6} \times 7000 \text{ grs. Tr.} = \frac{11 \times 10^3}{10^6} \text{ grs. Tr.}$ ; but, by hyp., 1 leaf weighs 1 gr. Tr.; ∴ no. of leaves required =  $\frac{11 \times 10^3}{10^6} = 689$  very nearly.

87. Vol. of basin =  $\frac{1}{3} \times \frac{3}{4} \times 3.1416 \times (\frac{1}{4})^2 \times 15$  cub. ft. =  $3.1416 \times \frac{1125}{128}$  cub. ft.;  
 $\therefore$  vol. of tub =  $3.1416 \times \frac{1125}{128} \times \frac{1}{12}$  cub. ft. =  $3.1416 \times \frac{1}{4}$  cub. ft. But vol. of  
 latter likewise =  $3.1416 \times \text{radius}^2 \times \frac{1}{2}$  ft.;

$$\therefore \text{radius}^2 \text{ of tub} = (3.1416 \times \frac{1}{4}) \div (3.1416 \times \frac{1}{2}) \text{ sq. ft.} = \frac{1}{2} \text{ sq. ft.}$$

$$\therefore \text{radius of tub} = \frac{1}{2} \text{ ft.}; \therefore \text{its diam.} = \frac{1}{2} \text{ ft.} = 2 \text{ ft. 6 in.}$$

88. (Vide diag. to Chap. XXV. 30), here  $AD=4$  in., and  $BD=3$  in.;  
 $\therefore$  vol. of cone  $ABC = \frac{1}{3} \times 3.1416 \times 3^2 \times 4$  cub. in. =  $37.6992$  cub. in. Now (by  
 Euc. I. 47)  $AB = \sqrt{(AD^2 + DB^2)} = \sqrt{\{(4^2 + 3^2) \text{ sq. in.}\}} = \sqrt{\{25 \text{ sq. in.}\}} = 5$  in.  
 $\therefore$  whole surface of cone  $ABC = (3.1416 \times 3^2 + 3.1416 \times 3 \times 5) \text{ sq. in.}$   
 $= 3.1416 \times 3 (3 + 5) \text{ sq. in.} = 75.3984 \text{ sq. in.}$

89. Here  $ML=4$  in.,  $FG=9$  in.,  $FK=12$  in.,  
 $BE=8$  in.

Join  $BD$ , and  $FH$ . Then  $\triangle BED$  and  $FKH$  are  
 similar. Hence (by Euc. VI. 4),  $DE : EB :: HK : KF$ ;

$$\therefore DE = \frac{8 \times 9}{12} \text{ in.} = 6 \text{ in.}$$

Now, let  $AL$  be the height of the pyramid, of which  
 $CBFGHKED$  is the given frustum. Now  $BM = \frac{1}{2} BD$

$$= \frac{1}{2} \sqrt{(BE^2 + ED^2)} = \frac{1}{2} \sqrt{\{(8^2 + 6^2) \text{ sq. in.}\}} = 5 \text{ in.}$$

And  $FL = \frac{1}{2} FH = \frac{1}{2} \sqrt{(FK^2 + KH^2)}$

$$= \frac{1}{2} \sqrt{\{(12^2 + 9^2) \text{ sq. in.}\}} = \frac{15}{2} \text{ in.}$$

Now from similar  $\triangle ABM$  and  $\triangle AFL$ ,

$$AM : BM : AL : FL; \therefore AM = \frac{5 \text{ in.} (AM + 4 \text{ in.})}{\frac{15}{2} \text{ in.}};$$

$$\therefore 15AM = 10AM + 40 \text{ in.}; \therefore AM = 8 \text{ in., and } AL = (8 + 4) \text{ in.} = 12 \text{ in.};$$

$\therefore$  vol. of the given frustum = vol. of pyramid  $AFGHK$  - vol. of pyramid  
 $ABCDE = (\frac{1}{3} \times 9 \times 12 \times 12 - \frac{1}{3} \times 6 \times 8 \times 8) \text{ cub. in.}$

$$= \frac{1}{3} \times 3 \times 16 (27 - 8) \text{ cub. in.} = 304 \text{ cub. in.}$$

With reference to this solution, see the note prefixed to Chapter XXVI.  
 In this solution, as in those of the similar problems, the axis  $AML$  of the  
 pyramid is assumed to be perpendicular to the ends of the frustum. If we  
 apply the rule of Art. 268 of the text, we have

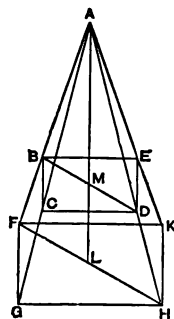
$$\begin{aligned} \text{Vol. of frustum} &= \frac{1}{3} \times 4 \times (48 + \sqrt{\{48 \times 108\}} + 108) \\ &= \frac{1}{3} \times 4 \times (48 + 72 + 108) \\ &= 304 \text{ cub. in.} \end{aligned}$$

90. Surface of sphere =  $3.1416 \times \text{diam.} \times \text{diam.} = 3.1416 \times \text{diam.}^2$  (by  
 hyp.)  $25 \text{ sq. in.}; \therefore \text{diam.}^2 = (25 \div 3.1416) \text{ sq. in.} = 7.957728 \text{ sq. in.};$

$$\therefore \text{diam.} = \sqrt{(7.957728 \text{ sq. in.})} = 2.821 \text{ in.};$$

$\therefore$  vol. of sphere

$$= \frac{1}{6} \times 3.1416 \times \text{diam.}^3 = \frac{1}{6} \times 3.1416 \times \frac{25}{3.1416} \times 2.821 \text{ cub. in.} = 11.754 \text{ cub. in.}$$



91. Vol. of log =  $20\frac{1}{2} \times 1\frac{1}{8} \times 1\frac{1}{2}$  cub. ft. =  $\frac{61 \times 11}{12}$  cub. ft.; and vol. of all the planks = area of their surface  $\times \frac{1}{2}$  ft.;  $\therefore$  area of their surface

$$= \frac{61 \times 11}{12} \times 12 \text{ sq. ft.} = 671 \text{ sq. ft.}$$

92. Vol. of water =  $6 \times 3 \times 1\frac{3}{4}$  cub. ft. =  $\frac{9}{2}$  cub. ft.; and 1 cub. foot of water weighs 1000 oz. Av.;  $\therefore$  weight of water in bath =  $\frac{9}{2} \times 1000$  oz.

$$= \frac{9}{2} \times 1\frac{3}{4} \text{ lbs.} = \frac{13\frac{3}{4}}{2} \text{ lbs.} = 1968\cdot75 \text{ lbs.} = 1969 \text{ lbs. nearly.}$$

93. Is worked like 83.

94. Vol. of box =  $2\frac{1}{2} \times 3\frac{1}{2} \times 4\frac{1}{2}$  cub. ft. =  $13^3 \times 24$  cub. in.; and vol. of package =  $(\frac{13}{27} \text{ in.})^3 = \frac{13^3}{27}$  cub. in.  $\therefore$  no. of packages required

$$= (13^3 \times 24) \div \frac{13^3}{27} = 24 \times 27 = 648.$$

95. Area of floor =  $14\frac{1}{2} \times 13\frac{1}{2}$  sq. ft. = 190 sq. ft.; and area of 1 plank

$$= \frac{3}{4} \times 10 \text{ sq. ft.} = \frac{3}{2} \text{ sq. ft.};$$

$\therefore$  no. of planks required =  $190 \times \frac{2}{3} = \frac{380}{3} = 126\frac{2}{3}$ ;  $\therefore$  vol. of all the planks

$$= \frac{380}{3} \times \frac{3}{2} \text{ sq. ft.} \times \frac{1}{2} \text{ in.} = 57 \times 240 \text{ cub. in.};$$

$\therefore$  their weight =  $57 \times 240 \times \frac{1}{2}$  oz. Av. =  $\frac{6840}{2}$  lbs. = 427 $\frac{1}{2}$  lbs.

96. Vol. of room = length<sup>3</sup> = (by hyp.) 46656 cub. ft. =  $\frac{36^3}{27}$  cub. yds.;  $\therefore$  length of room =  $\frac{36}{3}$  yds. = 12 yds.  $\therefore$  area of floor =  $12^2$  sq. yds.; and length of carpet required =  $144 \times \frac{12 \times 3}{27}$  yds. = 192 yds.;  $\therefore$  cost of carpet

$$= 192 \times 4s. 6d. = 192 \times \frac{54}{100} = \frac{10368}{100} = \frac{2592}{25} = \frac{10368}{100} = 103\frac{68}{100} = 103\frac{17}{25} = 103\frac{1}{2} \text{ } 4s.$$

97. (Vide diag. to Chap. XXV. 25), here  $AB = 16$  ft.,  $DE = 24$  ft.,  $\therefore AC$  (by Enc. I. 47) =  $(AB^2 + BC^2) = \sqrt{\left\{AB^2 + \left(\frac{DF}{2}\right)^2\right\}} = \sqrt{\{(16^2 + 12^2) \text{ sq. ft.}\}}$

$$= \sqrt{400 \text{ sq. ft.}} = 20 \text{ ft.}$$

$\therefore$  area of the 4 triangular faces of pyramid

$$= 4 \times \frac{24 \times 20}{2} \text{ sq. ft.} = 960 \text{ sq. ft.} = 960 \times 144 \text{ sq. in.};$$

$\therefore$  quantity of lead required =  $960 \times 144 \times \frac{1}{12}$  cub. in. = 960  $\times$  9 cub. in.; and weight of lead =  $960 \times 9 \times 7$  oz. = 3780 lbs. Again, vol. of one bullet

$$= \left\{8\cdot1416 \times \left(\frac{1}{11}\right)^3 \times \frac{1}{2} + \frac{1}{2} \times 3\cdot1416 \times \left(\frac{1}{11}\right)^3 \times \frac{8}{3}\right\} \text{ cub. in.}$$

$$= 8\cdot1416 \times \frac{1}{11^3} \left(\frac{1}{2} + \frac{1}{2}\right) \text{ cub. in.} = \frac{45 \times 3927}{121} \text{ cub. in.};$$

$\therefore$  no. of bullets that can be cast out of the lead

$$= (960 \times 9) \div \frac{45 \times 3927}{121} = \frac{7744}{1309} = 59159 \text{ nearly.}$$

98. Vol. of ball =  $\frac{3}{4} \times 3.1416 \times (\frac{3}{4} \text{ in.})^2 \times 9 \text{ in.} = 3.1416 \times 2\frac{3}{4} \text{ cub. in.}$  = (by hyp.) vol. of cone =  $\frac{1}{3} \times 3.1416 \times 9^2 \text{ sq. in.} \times \text{height} = 3.1416 \times 27 \text{ sq. in.} \times \text{height of cone}$ ;  $\therefore \text{height of cone} = (3.1416 \times 2\frac{3}{4}) \div (3.1416 \times 27) \text{ in.} = \frac{2}{9} \text{ in.} = 4\frac{1}{2} \text{ in.}$

99. Whole surface =  $(3.1416 \times 4 \times 24 \text{ sq. ft.} + 2 \times \frac{1}{2} \times 3.1416 \times 4 \times 4) \text{ sq. ft.}$   
 $= 3.1416 \times 16 (6 + 1) \text{ sq. ft.} = 3.1416 \times 112 \text{ sq. ft.} = 351.8592 \text{ sq. ft.}$

100. Using the Rule of Art. 341 of the text for the conical part of the funnel, its surface

$$= \frac{1}{2} (20 + 1\frac{1}{2}) 6 = 2\frac{1}{2} \text{ sq. in.}$$

And curved surface of cylindrical part of funnel =  $\frac{1}{2} \times 8 \text{ sq. in.} = 10 \text{ sq. in.}$ ;  
 $\therefore \text{no. of sq. in. of tin required} = 2\frac{1}{2} + 10 = 73\frac{1}{2}$ .

101. Vol. of large cube =  $22^3 \text{ cub. in.}$ ; and vol. of one of the small cubes =  $(\frac{1}{2})^3 \text{ cub. in.}$ ;  $\therefore \text{no. of latter that can be cut out of former}$

$$= 22^3 \times \frac{4^3}{11^3} = 2^3 \times 4^3 = 512.$$

102. Let  $a$  = breadth of room; then  $\frac{2}{3}a$  = length, and  $\frac{3}{4}a$  = height;  $\therefore$  content of room =  $a \times \frac{2}{3}a \times \frac{3}{4}a = a^3$  = (by hyp.) 5832 cub. ft. =  $18^3 \text{ cub. ft.}$ ;

$\therefore a = 18 \text{ ft.}$ ; and length =  $\frac{2}{3} \times 18 \text{ ft.} = 27 \text{ ft.}$ , and height =  $\frac{3}{4} \times 18 \text{ ft.} = 12 \text{ ft.}$

103. Is worked like 83.

104. Multiple required =  $\frac{4\frac{1}{2} \times 27 \text{ cub. ft.}}{30\frac{3}{8} \text{ cub. ft.}} = \frac{9 \times 27}{2} \times \frac{8}{243} = 4.$

105. Vol. of water that passes under bridge in 10 min.

$$= \frac{1}{2} \times 17\frac{1}{2} \text{ ft.} \times 10\frac{1}{2} \text{ ft.} \times 4 \text{ miles} = 53 \times 181 \times 880 \times 192 \text{ cub. in.}$$

$$= 1173089280 \text{ cub. in.};$$

and 1 gallon = 277.274 cub. in.;  $\therefore \text{no. of gallons of water required}$

$$= \frac{1173089280}{277.274} = 4230794 \text{ nearly.}$$

106. Side of unexpanded metal =  $\sqrt{(1 \text{ sq. in.})} = 1 \text{ in.}$ ; and side of expanded metal =  $\sqrt{(1 + .0201) \text{ sq. in.}} = 1.01 \text{ in.}$ ;

$\therefore \text{vol. of latter} = (1.01 \text{ in.})^3 = 1.030301 \text{ cub. in.}$

$\therefore \text{increase of vol.} = (1.030301 - 1) \text{ cub. in.} = .030301 \text{ cub. in.}$

107. Let  $x$  = length of edge of cube; surface of cube =  $6x^2$  = (by hyp.) surface of sphere =  $3.1416 \times \text{diam.}^2$ ;  $\therefore x^2 = .5236 \times \text{diam.}^2$ ;

$$\therefore x = \text{diam.} \sqrt{.5236} = \text{diam.} \times .7236.$$

Now vol. of sphere : vol. of cube ::  $\frac{1}{6} \times 3.1416 \times \text{diam.}^3 : x^3$ ;

which is as  $.5236 \times \text{diam.}^3 : .5236 \times \text{diam.}^3 \times .7236$ ;

$\therefore \text{vol. of sphere} = \frac{1}{.7236} \text{ of vol. of cube} = 1.382 \text{ of vol. of cube.}$

In this and the three following examples the method of solution adopted has been somewhat Algebraical. The student who understands Algebra will have seen that he may often use it advantageously in the more difficult examples. Let  $r$  be the radius of the sphere,  $x$  the edge of the cube,  $\pi$  the number 3.1416.

Then as the sphere and the cube have the same surface  $4\pi r^2 = 6x^2$ .

$$\therefore 4r^2 = \frac{6}{\pi} x^2 \text{ and } 2r = x \sqrt{\frac{6}{\pi}}.$$

$$\text{Also } \frac{\text{vol. of sphere}}{\text{vol. of cube}} = \frac{4\pi r^3}{6x^3} = \frac{4\pi r^2}{6x^2} \cdot \frac{2r}{x} = \frac{2r}{x} = \sqrt{\frac{6}{\pi}} = 1.382.$$

108. Let  $x$ =radius of base of cylinder; then  $2x$ =height of cylinder.  
 $\therefore$  whole surface of cylinder

$$= 2 \times 3.1416 \times x^2 + 3.1416 \times 2x \times 2x = 6 \times 3.1416 \times x^2 \\ = (\text{by hyp.}) \text{ surface of sphere} = 3.1416 \times \text{diam.}^2 \text{ of sphere;}$$

$$\therefore x^2 = \frac{1}{3} \text{ diam.}^2 \text{ of sphere; } \therefore x = \text{diam.} \times \sqrt{\frac{1}{3}} = \text{diam.} \times .40825.$$

Now vol. of sphere : vol. of cylinder ::  $\frac{1}{3} \times 3.1416 \times \text{diam.}^3$  :  $3.1416 \times x^2 \times 2x$ ;  
 which is as  $\frac{1}{3} \text{ diam.}^3$  :  $2 \times \frac{1}{3} \times .40825 \times \text{diam.}^3$ , or as 1 : .8165;

$$\therefore \text{vol. of sphere} = \frac{1}{.8165} \text{ of vol. of cylinder} = 1.2247 \text{ of vol. of cylinder.}$$

*Otherwise.* Let  $r$  be the radius of the sphere,  $x$  that of the cylinder of which the height is therefore  $2x$ . Then, their surfaces being the same,

$$4\pi r^2 = 2\pi x^2 + 4\pi x^2 = 6\pi x^2.$$

$$\therefore 2r^2 = 3x^2 \text{ and } r = x \sqrt{\frac{3}{2}}.$$

$$\text{Also } \frac{\text{vol. of sphere}}{\text{vol. of cylinder}} = \frac{\frac{4}{3}\pi r^3}{2\pi x^3} = \frac{2r^3}{3x^3} = \frac{2r^2}{3x^2} \times \frac{r}{x} = \frac{r}{x} = \sqrt{\frac{3}{2}} = 1.2247.$$

109. Let  $x$ =length of edge of cube;  $\therefore$  vol. of cube= $x^3$ =(by hyp.) vol. of sphere= $\frac{1}{3} \times 3.1416 \times \text{diam.}^3$   $\therefore x^3 = \text{diam.}^3 \times .5236$ ;

$$\therefore x = \text{diam.} \sqrt[3]{.5236} = \text{diam.} \times .806;$$

Now surface of cube : surface of sphere ::  $6x^2$  :  $3.1416 \times \text{diam.}^2$ ;  
 which is as  $6 \times (.806)^2 \times \text{diam.}^2$  :  $3.1416 \times \text{diam.}^2$ ; or as  $6 \times .649636$  :  $3.1416$ ;

$$\therefore \text{surface of cube} = \frac{.649636}{.31416} \text{ of surface of sphere} = 1.2407 \text{ of surface of sphere.}$$

*Otherwise.* Let  $r$  be the radius of the sphere,  $x$  the edge of the cube. Then, as the volumes are the same,

$$\frac{4}{3}\pi r^3 = x^3; \therefore x^3 = \left(\frac{4\pi}{3}\right)^{\frac{1}{3}} \cdot r^3.$$

$$\text{Also } \left(\frac{\text{surface of cube}}{\text{surface of sphere}}\right)^3 = \left(\frac{6x^2}{4\pi r^2}\right)^3 = \left(\frac{3}{2\pi}\right)^3 \times \frac{x^3}{r^3} = \left(\frac{3}{2\pi}\right)^3 \times \left(\frac{4\pi}{3}\right)^{\frac{1}{3}} = \frac{6}{\pi}.$$

$$\therefore \frac{\text{surface of cube}}{\text{surface of sphere}} = \sqrt[3]{\frac{6}{\pi}} = 1.2407.$$

110. Let  $x$  = radius of base of cylinder; then  $2x$  = height of cylinder.  
 $\therefore$  vol. of cylinder =  $8 \cdot 1416 \times x^2 \times 2x = 2 \times 8 \cdot 1416 \times x^3$  = (by hyp.) vol. of  
 sphere =  $\frac{1}{2} \times 8 \cdot 1416 \times \text{diam.}^3$ ;  $\therefore x^3 = \frac{1}{4} \text{diam.}^3$  of sphere.  $\therefore x$  = diam. of  
 sphere  $\times \sqrt[3]{(\cdot 083)} = \text{diam.} \times \cdot 4368$ . Now surface of cylinder (as in 108)

$$= 6 \times 8 \cdot 1416 \times x^2 = 6 \times 8 \cdot 1416 \times (\cdot 4368)^2 \times \text{diam.}^2 \text{ of sphere} \\ = 6 \times 8 \cdot 1416 \times \cdot 19079424 \times \text{diam.}^2 = 8 \cdot 1416 \times \text{diam.}^2 \times 1 \cdot 1447;$$

$\therefore$  surface of cylinder : surface of sphere

$$:: 8 \cdot 1416 \times \text{diam.}^2 \times 1 \cdot 1447 : 8 \cdot 1416 \times \text{diam.}^2; \text{ or as } 1 \cdot 1447 : 1;$$

$\therefore$  surface of cylinder =  $1 \cdot 1447$  of surface of sphere.

*Otherwise.* Let  $r$  be the radius of the sphere,  $x$  that of the cylinder of which the height is therefore  $2x$ . Then, the volumes being the same,

$$\frac{4}{3} \pi r^3 = 2 \pi x^3; \therefore x^6 = \left(\frac{2}{3}\right)^3 r^6.$$

$$\text{Also } \left( \frac{\text{surface of cylinder}}{\text{surface of sphere}} \right)^3 = \left( \frac{6 \pi x^2}{4 \pi r^2} \right)^3 = \left( \frac{3}{2} \right)^3 \cdot \frac{x^6}{r^6} = \left( \frac{3}{2} \right)^3 \times \left( \frac{2}{3} \right)^3 = \frac{8}{27}.$$

$$\therefore \frac{\text{surface of cylinder}}{\text{surface of sphere}} = \sqrt[3]{\frac{8}{27}} = 1 \cdot 1447.$$

111. Vol. of water in cistern

$$= 5 \frac{1}{2} \times 3 \frac{3}{4} \times 1 \frac{1}{2} \text{ cub. ft.} = 1 \frac{1}{2} \times 1 \frac{1}{2} \times \frac{5}{2} \text{ cub. ft.} = \frac{25}{8} \text{ cub. ft.};$$

and 1 cub. foot of water weighs 1000 oz. Av. or  $1 \frac{3}{4}$  lbs.;  $\therefore$  weight of  
 water in cistern =  $\frac{25}{8} \times 1 \frac{3}{4}$  lbs. =  $10 \frac{5}{8} \times 1 \frac{3}{4}$  lbs. = 1611 lbs. nearly.

112. Vol. of gravel

$$= 3 \frac{3}{4} \text{ ac.} \times 4 \frac{1}{2} \text{ ft.} = 1 \frac{1}{2} \times 4840 \times \frac{9}{8} \times \frac{1}{2} \text{ cub. yds.} = 45 \times 605 \text{ cub. yds.}$$

$$\therefore \text{cost} = 45 \times 605 \times 6d. = 45 \times 605 \times \frac{6}{4} = \frac{15975}{2} = \text{£} 7987 \frac{1}{2} = \text{£} 680. 12s. 6d.$$

113. Edge of cube =  $\sqrt[3]{(5 \text{ cub. ft. } 621 \text{ in.})} = \sqrt[3]{(9261 \text{ cub. in.})} = 21 \text{ in.}$  Now  
 in the diag. to Chap. XXV. 39 join  $GC$ ; and  $EG$ . Then  $GC$  is the diagonal  
 of the cube.  $EG$  (by Euc. I. 47)

$$= \sqrt{\{(21^2 + 21^2) \text{ sq. in.}\}} = \sqrt{(21^2 \times 2 \text{ sq. in.})};$$

$$\therefore GC = \sqrt{(EG^2 + EC^2)} = \sqrt{\{(21^2 \times 2 + 21^2) \text{ sq. in.}\}} = \sqrt{(21^2 \times 3 \text{ sq. in.})} = 21 \sqrt{3} \text{ in.}$$

114. Vol. of brick =  $\frac{2}{3} \times \frac{3}{4} \times \frac{1}{2} \text{ cub. ft.} = \frac{1}{4} \text{ cub. ft.}$ ; and vol. of stack  
 =  $10 \times 6 \times 3 \text{ cub. ft.} = 180 \text{ cub. ft.};$

$$\therefore \text{weight required} = 180 \times 1 \frac{3}{4} \times 5 \text{ lbs.} = 12800 \text{ lbs.}$$

115. Use the diag. of Chap. XXV. 21.

$ADE$  is an equilateral triangle, each side being 20 ft.  $\therefore$  the height

$$AC = \sqrt{(20^2 - 10^2)} = 10 \sqrt{3}.$$

And  $CB = \frac{1}{2} DF = 10$ .  $\therefore AB$ , the height of the pyramid

$$= \sqrt{(AC^2 - CB^2)} = \sqrt{(300 - 100)} = \sqrt{200} = 10 \sqrt{2}.$$

Thus vol. of pyramid =  $\frac{1}{3} \times 20^2 \times 10 \sqrt{2} = \frac{4000}{3} \sqrt{2} = 1885 \cdot 6 \text{ cub. ft.}$

116. Vol. of cylinder =  $3 \cdot 1416 \times 3^2 \times 20$  cub. ft. =  $565 \cdot 488$  cub. ft.

117. The ditch is in the form of a prism, the four faces of which are rectangles or parallelograms, and the two ends trapezoids. The problem is the same as that of dividing one of the trapezoidal ends into two trapezoids of equal area, as in Chapter XIX. 19. In the diagram to that example let  $ABCD$  be the trapezoid,  $AB$  the bottom of the ditch = 12 ft.,  $CD$  = 16 ft., and  $EI$  the depth of the ditch = 4 ft. It is required to find  $IL$  so that the trapezoid  $FGCD$  may equal the trapezoid  $ABFG$ .

From similar triangles  $KAB$ ,  $KCD$ ,

$$IK : AB :: KE : CD,$$

that is

$$IK : 12 :: IK + 4 : 16.$$

$$\therefore 16 \times IK = 12 (IK + 4),$$

therefore

$$IK = 12 \text{ ft. and } KE = 16 \text{ ft.}$$

Thus the area of  $KAB$  is 72 sq. ft. and that of  $KCD$  = 128 sq. ft.; and the triangle  $KFG$  is equal to one-half the sum of  $KAB$  and  $KCD$ .  $\therefore$  area of  $KFG$  = 100 sq. ft.

Now by the rule in Art. 205

$$KL^2 : KE^2 :: 100 : 128;$$

$$\therefore KL^2 = \frac{100 \times 256}{128} = 200 \text{ and } KL = 10\sqrt{2},$$

and

$$IL = 10\sqrt{2} - 12 = 2 \cdot 142 \text{ ft.}$$

118. Use the rule in Art. 268. We have area of one end

$$= 3 \cdot 1416 \times 18^2 \text{ sq. in.};$$

and area of the other end =  $3 \cdot 1416 \times 12^2$  sq. in.; also the square root of the product of these =  $3 \cdot 1416 \times 18 \times 12$  sq. in.;  $\therefore$  vol. of frustum

$$= \frac{1}{3} \times 15 (18^2 + 18 \times 12 + 12^2) \times 3 \cdot 1416 \\ = 10744 \cdot 272 \text{ cub. in.}$$

119. Vol. of iron ball =  $\frac{1}{6} \times 3 \cdot 1416 \times 4^3$  cub. in. and vol. of shell

$$= \frac{1}{6} \times 3 \cdot 1416 \times (7^3 - 4^3) \text{ cub. in.}$$

$$\therefore \text{weight of shell} = \frac{7^3 - 4^3}{4^3} \times \text{weight of ball}$$

$$= \frac{316}{64} \times 9 \text{ lbs.}$$

$$= 44 \text{ lbs. 7 oz.}$$

120. Whole surface of cone =  $(3 \cdot 1416 \times \text{radius}^2 + 3 \cdot 1416 \times \text{radius} \times 3 \text{ radius})$   
 $= 4 \times 3 \cdot 1416 \times \text{radius}^2 = (\text{by hyp.}) 32 \text{ sq. ft.};$

$$\therefore \text{radius}^2 = \frac{8}{3 \cdot 1416} \text{ sq. ft. Now height}^2 \text{ of cone (by Euc. I. 47)}$$

$$= \text{slant height}^2 - \text{radius}^2 = 9 \text{ radius}^2 - \text{radius}^2 = 8 \text{ radius}^2$$

$$= \frac{64}{3 \cdot 1416} \text{ sq. ft.} = 20 \cdot 37178507 \text{ sq. ft.}$$

$$\therefore \text{height} = \sqrt{(20 \cdot 37178507 \text{ sq. ft.})} = 4 \cdot 5135 \text{ ft.}$$

$$\therefore \text{vol. of cone} = \frac{1}{3} \times 3 \cdot 1416 \times \text{radius}^2 \times \text{height}$$

$$= \frac{1}{3} \times 3 \cdot 1416 \times \frac{8}{3 \cdot 1416} \times 4 \cdot 5135 \text{ cub. ft.} = 12 \cdot 036 \text{ cub. ft.}$$

$$121. \text{ Vol.} = 9 \times 2\frac{1}{2} \text{ cub. ft.} = 4\frac{1}{2} \text{ cub. ft.} = 22\frac{1}{2} \text{ cub. ft.}$$

$$122. \text{ Vol. of coal}$$

$$= 120 \text{ ac.} \times 5 \text{ ft.} = 120 \times 4840 \times \frac{1}{3} \text{ cub. yds.} = 968000 \text{ cub. yds.};$$

$$\therefore \text{its weight} = 968000 \text{ tons. And value of coal} = 968000 \times 12s. = \pounds 580300.$$

$$123. \text{ Vol. of glass}$$

$$= 5 \text{ ft.} \times 3\frac{1}{2} \text{ ft.} \times \frac{1}{2} \text{ in.} = 5 \times 1\frac{1}{2} \times 144 \times \frac{1}{8} \text{ cub. in.} = 900 \text{ cub. in.}$$

$$124. \text{ Vol. of coal in field}$$

$$= 1000 \text{ sq. miles} \times 60 \text{ ft.} = 1000 \times 1760^2 \times 20 \text{ cub. yds.};$$

$$\therefore \text{its weight} = 1000 \times 1760^2 \times 20 \text{ tons}; \therefore \text{no. of years required}$$

$$= 1000 \times 1760^2 \times 20 \times \frac{1}{7000000} = \frac{30976}{85} = 885 \text{ very nearly.}$$

125. Let  $h$  = height of whole pyramid;  $\therefore$  vol. of smaller pyramid : vol. of pyramid  $:: \left(\frac{h}{2}\right)^3 : h^3$ ; i.e. as  $\frac{1}{8} : 1$ ;  $\therefore$  vol. of smaller pyramid =  $\frac{1}{8}$  of vol. of whole pyramid;  $\therefore$  vol. of frustum =  $\frac{7}{8}$  of vol. of whole pyramid = 7 times vol. of smaller pyramid.

$$126. \text{ Vol. of cylinder}$$

$$= 3 \cdot 1416 \times \left(\frac{1}{2} \text{ ft.}\right)^2 \times \text{height} = 7854 \times 25 \text{ sq. ft.} \times \text{height} = (\text{by hyp.}) 785 \cdot 4 \text{ cub. ft.}$$

$$\therefore \text{height} = \frac{785 \cdot 4}{7854 \times 25} \text{ ft.} = \frac{1000}{25} \text{ ft.} = 40 \text{ ft.}$$

127. Use Art. 283. Area of one rectangular end =  $8 \times 7 = 56 \text{ sq. ft.}$ ; and area of the other rectangular end =  $10 \times 6 = 60 \text{ sq. ft.}$  The middle section is a rectangle of which the dimensions are  $\frac{1}{2}(8+10)$ ,  $\frac{1}{2}(7+6)$ , that is 9 ft. and  $1\frac{1}{2} \text{ ft.}$ , and the area =  $1\frac{1}{2} \times 9 \text{ sq. ft.}$  Thus vol. of prismoid

$$= \frac{1}{6} \times 4 \times (56 + 4 \times 1\frac{1}{2} \times 9 + 60) \\ = 233\frac{1}{2} \text{ cub. ft.}$$

128. (Vide diag. to Chap. XXVI. 16.) Here  $DH = 4 \text{ ft.}$ ,  $BL = 5 \text{ ft.}$ ,  $HL = 7 \text{ ft.}$ , and  $HK = 3 \cdot 884 \text{ ft.}$   $AH$  (found as in Chap. XXVI. 16) =  $28 \text{ ft.}$ , and  $AL = 35 \text{ ft.}$  Now we shall suppose that  $FG$  bisects the frustum, and we shall hence shew that  $HK = 3 \cdot 884 \text{ ft.}$  Frustum  $DFGE$  = cone  $AFG$  - cone  $ADE$ ; and frustum  $FBCG$  = cone  $ABC$  - cone  $AFG$ ; but frustum  $FBCG$  = (by hyp.) frustum  $DFGE$ ;  $\therefore$  cone  $AFG$  - cone  $ADE$  = cone  $ABC$  - cone  $AFG$ ;

$$\therefore \text{cone } AFG = \frac{1}{2} (\text{cone } ABC + \text{cone } ADE).$$

$$\text{Now cone } ABC : \text{cone } ADE :: AL^3 : AH^3;$$

$$\frac{\text{cone } ABC + \text{cone } ADE}{2} : \frac{\text{cone } ADE}{2} :: AL^3 + AH^3 : AH^3;$$

i. e. cone  $AFG : \frac{1}{2}$  cone  $ADE :: AL^3 + AH^3 : AH^3$ ;

but cone  $AFG : \frac{1}{2}$  cone  $ADE :: AK^3 : \frac{1}{2} AH^3$ ;

$$\therefore AK^3 : \frac{1}{2} AH^3 :: AL^3 + AH^3 : AH^3;$$

$$\therefore AK^3 = \frac{1}{2} (AL^3 + AH^3) = \frac{1}{2} (35^3 + 28^3) \text{ cub. ft.} = 32413.5 \text{ cub. ft.};$$

$$\therefore AK = \sqrt[3]{(32413.5 \text{ cub. ft.})} = 31.884 \text{ ft.};$$

$$\therefore HK = AK - AH = (31.884 - 28) \text{ ft.} = 3.884 \text{ ft.}$$

$$129. \text{ Vol. of solid ball} = \frac{4}{3} \times 3.1416 \times 4^3; \text{ and vol. of shell}$$

$$= \frac{4}{3} \times 3.1416 \times (5^3 - 4^3).$$

$$\therefore \text{ weight of shell} = \frac{5^3 - 4^3}{4^3} \times \text{weight of ball}$$

$$= \frac{61}{64} \times 8 \text{ lbs.}$$

$$= 7\frac{7}{8} \text{ lbs.}$$

$$130. \text{ Diam. of ball} = \frac{15.708}{3.1416} \text{ ft.} = 5 \text{ ft.}; \therefore \text{ surface of ball}$$

$$= 15.708 \times 5 \text{ sq. ft.} = 78.54 \text{ sq. ft.}$$

$$131. \text{ Vol. of excavation}$$

$$= 30 \times 28 \times 18 \text{ cub. ft.} = \text{vol. of wall} = 10\frac{1}{2} \times 2\frac{1}{2} \text{ sq. ft.} \times \text{length of wall};$$

$$\therefore \text{ length of wall} = (30 \times 28 \times 18) \div (2\frac{1}{2} \times 10\frac{1}{2}) \text{ ft.} = 640 \text{ ft.}$$

$$132. \text{ Vol. of cistern} = 5 \times 4 \times 3 \text{ cub. ft.} = 60 \text{ cub. ft.} = \text{quantity of water discharged through pipe in } 1\frac{1}{2} \text{ hours, i. e. in 90 min.}; \text{ quantity discharged per minute} = \frac{3}{2} \text{ cub. ft.} = \frac{3}{2} \text{ cub. ft.} = 1152 \text{ cub. in.}; \text{ and 1 gallon of water contains } 277.274 \text{ cub. in.}; \therefore \text{ no. of gallons required} = \frac{1152}{277.274} = 4.15.$$

$$133. \text{ No. of cub. feet required} = 200 \times 15 \times \frac{5}{8} \times \frac{1}{2} = 1875 = 312\frac{1}{2}.$$

$$134. \text{ Vol. of ice} = 4 \text{ ac.} \times 6 \text{ in.} = 4 \times 4840 \times 9 \times \frac{1}{2} \text{ cub. ft.} = 18 \times 4840 \text{ cub. ft.};$$

$$\therefore \text{ weight of ice} = 18 \times 4840 \times 896 \text{ oz.} = \frac{18 \times 4840 \times 896}{16 \times 28 \times 4 \times 20} \text{ tons} = 2178 \text{ tons.}$$

$$135. \text{ Vol. of coal consumed in 1 month} = \frac{70000000}{12} \text{ cub. yds. (vide 124)} \\ = \frac{70000000 \times 9}{4} \text{ cub. ft., and vol. of pyramid} = \frac{1}{3} \times 764^2 \text{ sq. ft.} \times \text{height (vide$$

$$\text{Chap. XXV. 41)}; \therefore \text{ height of pyramid} = \frac{70000000 \times 27}{764^2 \times 4} \text{ ft.} = \frac{29531250}{36481} \text{ ft.} \\ = 809 \text{ ft. nearly.}$$

$$136. \text{ A cross section of the trench will be a trapezoid. Hence its vol.}$$

$$= \frac{10+14}{2} \times 8 \text{ sq. ft.} \times \text{length} = 96 \text{ sq. ft.} \times \text{length.}$$

And a cross section of the bank will be a triangle, the height of which =  $\frac{3}{4}$  of the base. The length of the bank is the same as that of the trench. Let  $b$  and  $h$  represent the base and height of the triangle.  $\therefore$  vol. of bank

$$= \frac{b \times h}{2} \times \text{length} = \frac{1}{2} \times \frac{1}{2} h \times h \times \text{length} = \frac{1}{2} h^2 \times \text{length} = (\text{by hyp.}) 96 \text{ sq. ft.} \times \text{length};$$

$$\therefore h^2 = 144 \text{ sq. ft.}; \therefore h = 12 \text{ ft.}$$

137. Vol. of cylinder

$$= 3 \cdot 1416 \times 6^2 \times 40 \text{ cub. ft.} = 8042 \cdot 496 \text{ cub. ft.} = 8042 \cdot 5 \text{ cub. ft. nearly.}$$

138. Area of one end of frustum  $= 3 \cdot 1416 \times (\frac{1}{2})^2 \text{ sq. ft.}$ ; and area of the other end  $= 3 \cdot 1416 \times (\frac{3}{2})^2 \text{ sq. ft.}$  Also the square root of their product

$$= 3 \cdot 1416 \times \frac{1}{2} \times \frac{3}{2};$$

$\therefore$  vol. of frustum, or bucket

$$= \frac{1}{2} \times \frac{3}{2} \times 3 \cdot 1416 \left\{ \frac{1}{4} + \frac{9}{4} + \frac{3}{2} \right\} = \frac{1}{2} \times 3 \cdot 1416 \times \frac{3}{2} \times \frac{7}{2} \text{ cub. ft.};$$

and 1 cub. foot of water weighs 1000 oz. or  $1\frac{3}{4}$  lbs.;  $\therefore$  weight of water in

$$\text{bucket} = \frac{1}{2} \times 3 \cdot 1416 \times \frac{3}{2} \times \frac{7}{2} \times 1\frac{3}{4} \text{ lbs.} = \frac{2994 \cdot 3875}{32} \text{ lbs.} = 94 \text{ lbs. nearly.}$$

$\therefore$  when full of water, the bucket weighs 94 lbs. more than when empty.

139. Interior vol. of required sphere

$$= 11 \times 30 \text{ cub. in.} = 330 \text{ cub. in.} = \frac{1}{6} \times 3 \cdot 1416 \times \text{diam.}^3;$$

$$\therefore \text{diam.}^3 = \frac{330 \times 6}{3 \cdot 1416} \text{ cub. in.} = 630 \cdot 25210084 \text{ cub. in.};$$

$$\therefore \text{diam.} = \sqrt[3]{(630 \cdot 25210084 \text{ cub. in.})} = 8 \cdot 574 \text{ in.}$$

140. Curved surface of zone

$$= 3 \cdot 1416 \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \text{ sq. ft.} = 7854 \times 65 \text{ sq. ft.} = 51 \cdot 051 \text{ sq. ft.}$$

141. Quantity of earth required

$$= 1\frac{1}{2} \text{ ac.} \times 18 \text{ in.} = \frac{3}{8} \times 4840 \times \frac{1}{2} \text{ cub. yds.} = 3872 \text{ cub. yds.}$$

$$142. \text{ Thickness required} = 1 \text{ cub. in.} \div 7 \text{ sq. ft.} = \frac{1}{7 \times 144} \text{ in.} = \frac{1}{1008} \text{ in.}$$

143. Vol. of air required

$$= 24\frac{3}{4} \times 18\frac{1}{2} \times 10\frac{3}{8} \text{ cub. ft.} = \frac{9}{2} \times \frac{37}{2} \times \frac{83}{8} \text{ cub. ft.} = 4840 \text{ cub. ft.}$$

144. Vol. of excavation  $= 40 \times 30 \times 6 \text{ cub. ft.}$ ;  $\therefore$  vol. of earth when removed  $= \frac{1}{2} \times 40 \times 30 \times 6 \text{ cub. ft.} = 66 \times 120 \text{ cub. ft.}$ ;  $\therefore$  height of earth

$$\text{spread over field} = \frac{66 \times 120 \text{ cub. ft.}}{\frac{1}{2} \times 4840 \times 9 \text{ sq. ft.}} = \frac{4}{11} \text{ ft.}$$

145. Vol. of pyramid

$$= \frac{1}{3} \times \frac{10^2}{4} \sqrt{3} \times 12 \text{ cub. in.} = 100 \times 1 \cdot 73205 \text{ cub. in.} = 173 \cdot 205 \text{ cub. in.}$$

$$146. \text{ Vol. of wire} = 3 \cdot 1416 \times (\cdot 04)^2 \times \text{length} = (\text{by hyp.}) 1 \text{ cub. in.}; \therefore \text{length}$$

$$\text{of wire} = \frac{1}{3 \cdot 1416 \times \cdot 0016} \text{ in.} = \frac{1}{\cdot 00502656} \text{ in.} = 199 \text{ in. nearly.}$$

147. The gutter is in the form of a triangular prism. The base of each triangular end is 8 in., and each of the other sides of these ends is 5 in. The length of the prism is 144 in.;  $\therefore$  vol. of gutter = area of triangular end  $\times$  length of prism

$$= \frac{1}{2} \sqrt{\{(8+5)^2 - 5^2\} \{5^2 - (8-5)^2\}} \times 144 \text{ cub. in.} = 12 \times 144 \text{ cub. in.} = 1728 \text{ cub. in.}$$

148. (Vide diag. to Chap. XXVI. 5.) Here  $DF=4$  in.,  $BG=5$  in.,  $FG=3$  in.,  $\therefore$  (as in Chap. XXVI. 5),  $AF=\frac{4 \times 3}{5-4}$  in. = 12 in., and  $AG=15$  in.  
 $\therefore$  vol. of cone  $ABC = \frac{1}{3} \times 3 \cdot 1416 \times 5^2 \times 15 \text{ cub. in.} = 392 \cdot 7 \text{ cub. in.}$

149. Vol. of 1<sup>st</sup> shot =  $\frac{1}{3} \times 3 \cdot 1416 \times (4 \cdot 4)^3 \text{ cub. in.}$ ; and vol. of 2<sup>nd</sup> shot =  $\frac{1}{3} \times 3 \cdot 1416 \times (3 \cdot 96)^3 \text{ cub. in.}$ ;  $\therefore$  weight of latter

$$= \frac{(3 \cdot 96)^3}{(4 \cdot 4)^3} \times 12 \text{ lbs.} = \frac{\cdot 09^3}{\cdot 1^3} \times 12 \text{ lbs.} = \frac{\cdot 008748 \text{ lbs.}}{\cdot 001} = 8 \cdot 748 \text{ lbs.}$$

150. Curved surface of cylinder

$$= 2 \times 3 \cdot 1416 \times \text{radius} \times 25 \text{ in.} = (\text{by hyp.}) 600 \text{ sq. in.};$$

$$\therefore \text{radius} = \frac{600}{50 \times 3 \cdot 1416} \text{ in.} = \frac{1}{\cdot 2618} \text{ in.} = 3 \cdot 82 \text{ in. nearly.}$$

151. Area of field =  $440 \times 154 \text{ sq. yds.} = \frac{440 \times 154}{4840} \text{ ac.} = 14 \text{ ac.}$   $\therefore$  area of smaller portion =  $\frac{220 \times 154}{2} \text{ sq. yds.} = \frac{220 \times 154}{2 \times 4840} \text{ ac.} = \frac{7}{2} \text{ ac.} = 3 \frac{1}{2} \text{ ac.}$ ; and area of larger portion =  $(14 - 3 \frac{1}{2}) \text{ ac.} = 10 \frac{1}{2} \text{ ac.}$

152. Area of 4 walls =  $2 \times 11 \frac{1}{2} (21 + 15 \frac{1}{2}) \text{ sq. ft.} = \frac{35 \times 49}{2} \text{ sq. ft.}$ ; and cost of painting them =  $\pounds 9 \frac{3}{8} = \pounds 9 \frac{3}{8}$ ;  $\therefore$  cost of painting per sq. foot

$$= \pounds \frac{77}{8} \times \frac{2}{35 \times 49} = \pounds \frac{11}{980};$$

$\therefore$  cost of painting ceiling =  $21 \times 15 \frac{1}{2} \times \pounds \frac{11}{980} = \pounds 3 \frac{14}{10} = \pounds 3 \cdot 14 \text{ s. } 3 \text{ d.}$

153.  $(8 \frac{1}{2} \text{ ft.})^2 + (7 \frac{1}{2} \text{ ft.})^2 = (\frac{17}{2})^2 + (\frac{15}{2})^2 \text{ sq. ft.} = \frac{187}{4} \text{ sq. ft.}$  And

$$(11 \frac{1}{2} \text{ ft.})^2 = \frac{161}{4} \text{ sq. ft.};$$

$\therefore$  (by Euc. I. 48), the parallelogram is *not* a rectangle.

154. Hypotenuse<sup>2</sup> =  $(13^2 + 17^2) \text{ sq. in.} = (169 + 289) \text{ sq. in.} = 458 \text{ sq. in.}$  Now (by Euc. III. 31), this hypotenuse is the diam. of the circle.  $\therefore$  area of circle =  $3 \cdot 1416 \times (\frac{\text{diam.}}{2})^2 = 3 \cdot 1416 \times \frac{458}{4} \text{ sq. in.} = 359 \cdot 7132 \text{ sq. in.}$

155. Is solved as in 124.

156. Vol. of coal consumed in 1 year = 70000000 cub. yds. (vide 124) = vol. of stack = 10 ac.  $\times$  height of stack = 10  $\times$  4840 sq. yds.  $\times$  height of stack;

$$\therefore \text{height of stack} = \frac{70000000}{10 \times 4840} \text{ yds.} = \frac{175000}{121} \text{ yds.} = 1446 \text{ yds. nearly.}$$

157. Thickness required

$$= \frac{1728 \text{ cub. in.}}{6 \times 4840 \times 9 \times 144 \text{ sq. in.}} = \frac{1}{21780} \text{ in.} = .000046 \text{ in. nearly.}$$

158. 1 cub. foot. = 1728 cub. in.; and 1 gallon contains 277.274 cub. in.;  
 $\therefore$  no. of gallons in 1 cub. foot =  $\frac{1728}{277.274} = 6.232$ .

159. Is solved like 146.

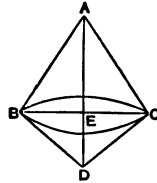
160. Here  $AB = AC = 4$  in.,  $BD = DC = 3$  in., and (by Euc. I. 47),  $AD = \sqrt{(AB^2 + BD^2)} = \sqrt{(25 \text{ sq. in.})} = 5$  in.  
 Now (by Euc. VI. 8, Cor.),  $AE : AB :: AB : AD$ ;

$$\therefore AE = \frac{AB^2}{AD} = \frac{16}{5} \text{ in.} \quad \therefore \text{also } ED = \frac{4}{5} \text{ in.};$$

again,  $AE : EB :: EB : ED$  (by the same proposition);

$$\therefore EB^2 = AE \times ED = \frac{16}{5} \times \frac{4}{5} \text{ sq. in.} \quad \therefore \text{area of double cone } ABDC = \frac{1}{3} \times 3.1416 \times EB^2 \times AE + \frac{1}{3} \times 3.1416 \times EB^2 \times ED \\ = \frac{1}{3} \times 3.1416 \times EB^3 (AE + ED) = \frac{1}{3} \times 3.1416 \times \frac{16}{5} \times \frac{4}{5} \times 5 \text{ cub. ft.}$$

$$= .62832 \times 48 \text{ cub. ft.} = 30.15936 \text{ cub. ft.} = 30.16 \text{ cub. ft. nearly.}$$



161. Let  $x$  = no. of yards in side of square base; then  $\frac{x}{2}$  = height.

$$\therefore \text{surface of aquarium} = x^2 + 4x \times \frac{x}{2} = 3x^2; \quad \therefore \text{cost of glass}$$

$$= 3x^2 \times 15s. = 45x^2 \times x^2 = (\text{by hyp.}) \text{ £} 1 \frac{9}{16} = \text{£} 1 \frac{3}{4}.$$

$\therefore x^2 = \frac{3}{4} \times \frac{4}{3} \text{ sq. yds.} = \frac{3}{4} \text{ sq. yds.}; \quad \therefore x = \frac{3}{2} \text{ of a yard.} \quad \therefore \text{vol. of aquarium}$

$$= (\frac{3}{2})^3 \times \frac{1}{12} \text{ cub. yds.} = \frac{125}{36 \times 12} \text{ cub. yds.} = 13500 \text{ cub. in.};$$

and 1 gallon contains 277.274 cub. in.

$$\therefore \text{no. of gallons in aquarium} = \frac{13500}{277.274} = 48.7 \text{ nearly.}$$

162. Vol. of wire =  $3.1416 \times (\frac{1}{16} \text{ in.})^2 \times 1 \text{ yd.} = \frac{3.1416}{240 \times 720} \text{ cub. ft.}; \quad \therefore \text{weight}$   
 of wire =  $\frac{3.1416}{240 \times 720} \times 8500 \text{ oz.} = \frac{11.1265}{72} \text{ oz.} = 1.545 \text{ oz. nearly.}$

163. (Vide diag. to 160.) Here  $AB=AC=12$  in.,  $BD=DC=5$  in., and (by Euc. I. 47),  $AD=\sqrt{\{(12^2+5^2)\text{ sq. in.}\}}=\sqrt{\{169\text{ sq. in.}\}}=13$  in.  $AE$  and  $ED$  (found as in 160) = respectively,  $2\frac{14}{13}$  in. and  $1\frac{1}{13}$  in.; and

$$BE^2 = \frac{144 \times 25}{169} \text{ sq. in.} \therefore BE = \frac{60}{13} \text{ in.}$$

Now vol. of double cone  $ABDC = \frac{1}{3} \times 3 \cdot 1416 \times \frac{144 \times 25}{169} \times \left( \frac{144}{13} + \frac{25}{13} \right) \text{ cub. in.}$   
 $= 3 \cdot 1416 \times \frac{48 \times 25}{13} \text{ cub. in.} = \frac{3769 \cdot 92}{13} \text{ cub. in.} = 289 \cdot 99 \text{ cub. in.} = 290 \text{ cub. in.}$   
 nearly. And curved surface of the double cone  $= 3 \cdot 1416 \times BE (AB + BD)$   
 $= 3 \cdot 1416 \times \frac{60}{13} \times 17 \text{ sq. in.} = \frac{3204 \cdot 432}{13} \text{ sq. in.} = 246 \cdot 49 \text{ sq. in.} = 246 \text{ sq. in. nearly.}$

164. Let  $W$  and  $w$  represent the respective weights, and  $D$  and  $d$  the respective diameters of the two globes. Then  $W : w :: 9 : 25$ ;  $\therefore W = \frac{9}{5} w$ . Now weight of 1 cub. inch in each globe = the weight of the globe divided by its volume in cubic inches, and this is, in the two globes,

$$\frac{W}{\frac{1}{6} \times 3 \cdot 1416 \times D^3}, \text{ and } \frac{w}{\frac{1}{6} \times 3 \cdot 1416 \times d^3} \text{ respectively;}$$

$\therefore$  weights of 1 cub. inch in each globe are to each other as  $\frac{W}{D^3} : \frac{w}{d^3}$ ;

hence (by hyp.),  $\frac{W}{D^3} : \frac{w}{d^3} :: 15 : 9$ ; i.e. as  $5 : 3$ ;  $\therefore 5 \times w \times D^3 = 3 \times W \times d^3$ .

And we have shewn above that  $W = \frac{9}{5} w$ ;  $\therefore 5 \times w \times D^3 = 3 \times \frac{9}{5} w \times d^3$ ;

$$\therefore 125D^3 = 27d^3; 5D = 3d; \therefore D : d :: 3 : 5.$$

165. Length of ring

$$= 3 \cdot 1416 \cdot (0 \cdot 1 + 1) \text{ in. (vide Chap. XXIV. 2)} = 3 \cdot 1416 \times 1 \cdot 01 \text{ in.};$$

$$\therefore \text{vol. of ring} = 3 \cdot 1416 \cdot (0 \cdot 005)^2 \times 3 \cdot 1416 \times 1 \cdot 01 \text{ cub. in. (Art. 261.)}$$

$$= (3 \cdot 1416)^2 \times (0 \cdot 005)^2 \times 1 \cdot 01 \text{ cub. in.} = \text{vol. of gilding.}$$

Now surface enclosed by the ring is a circle whose diameter is 1 inch.

$\therefore$  vol. of gilding  $= 3 \cdot 1416 \times 5^2 \text{ sq. in.} \times \text{thickness};$

$$\therefore 3 \cdot 1416 \times 5^2 \text{ sq. in.} \times \text{thickness of gilding} = (3 \cdot 1416)^2 \times (0 \cdot 005)^2 \times 1 \cdot 01 \text{ cub. in.}$$

$$\therefore \text{thickness of gilding} = 3 \cdot 1416 \times 0 \cdot 000101 \text{ in.} = 0 \cdot 0003173016 \text{ in.} = 0 \cdot 00032 \text{ in. nearly.}$$

166. When the height of a cone equals the radius of the base, the vertical angle is a right angle, and conversely by Euc. I. 32.

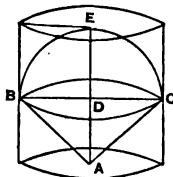
$$\therefore DA = DB = DC = \frac{1}{2} BC = 1 \text{ ft.}$$

$$\therefore \text{vol. of solid} = \frac{1}{3} \times 3 \cdot 1416 \times DB^2 \times DA$$

$$+ \frac{1}{2} \times \frac{1}{2} \times 3 \cdot 1416 \times DB^2 \times 2BD = 3 \cdot 1416 \times DB^3$$

$$= 3 \cdot 1416 \text{ cub. ft.}$$

Now height of cylinder being the same as that of



the solid  $ABEC = AE = 2DE = 2DB = 2$  ft. ;  $\therefore$  vol. of water remaining in cylinder = vol. of the cylinder - vol. of the solid  $ABEC$

$$= (3 \cdot 1416 \times 1^2 \times 2 - 3 \cdot 1416) \text{ cub. ft.} = 3 \cdot 1416 \text{ cub. ft.}$$

167. Let  $ABCD$  represent the covered portion;  $EFD$  the uncovered portion,  $G$  the centre. Then  $EF = \frac{1}{2} EG = 1$  ft. Join  $AG$ . Then

$$AF^2 = AG^2 - FG^2 = (3^2 - 2^2) \text{ sq. ft.} = 5 \text{ sq. ft.}$$

$\therefore$  vol. of buried portion = vol. of hemisphere - vol. of spherical segment  $AED$  (see Art. 301,)

$$= \left\{ \frac{1}{6} \times 3 \cdot 1416 \times \frac{6^3}{2} - \frac{1}{6} \times 3 \cdot 1416 \times 1 \times (3 \times 5 + 1) \right\} \text{ cub. ft.}$$

$$= \frac{1}{6} \times 3 \cdot 1416 (108 - 16) \text{ cub. ft.} = 3 \cdot 1416 \times \frac{4}{3} \text{ cub. ft.}$$

Let  $HBCK$  represent the cylinder of earth to be left.  $\therefore$  vol. of earth to be dug out = vol. of this cylinder - vol. of  $ABCD$

$$= (3 \cdot 1416 \times 3^2 \times 2 - 3 \cdot 1416 \times \frac{4}{3}) \text{ cub. ft.}$$

$$= 3 \cdot 1416 (18 - \frac{4}{3}) \text{ cub. ft.} = 3 \cdot 1416 \times \frac{50}{3} \text{ cub. ft.} = 8 \cdot 3776 \text{ cub. ft.}$$

168. Use the diagram in Chapter XXVI. 16. Let  $DECB$  be the given frustum divided into two equal parts by the plane  $FG$ . Then the vol. of the cone  $AFG$  is equal to half the sum of the cones  $ADE$ ,  $ABC$ .

Now by similar triangles  $AL : AH :: BL : DH$ ;  
that is  $AH + 6 \text{ ft.} : AH :: 3 : 2$ .

$$\therefore AH = 12 \text{ ft. and } AL = 18 \text{ ft.}$$

Thus vol. of cone  $ABC = \frac{1}{3} \times 3 \cdot 1416 \times 3^2 \times 18 \text{ cub. ft.}$ ,  
and that of  $ADE = \frac{1}{3} \times 3 \cdot 1416 \times 2^2 \times 12 \text{ cub. ft.}$

$$\therefore \text{ vol. of cone } AFG = \frac{1}{2} \times 3 \cdot 1416 \times 105 \text{ cub. ft.}$$

But by similar triangles  $AHD$ ,  $AFK$

$$AK : KF :: AH : HD,$$

that is  $:: 12 : 2$ .

$$\therefore AK = 6 \text{ times } KF.$$

But the vol. of  $AFG = \frac{1}{3} \times 3 \cdot 1416 \times KF^3 \times AK$

$$= \frac{1}{3} \times 3 \cdot 1416 \left( \frac{AK}{6} \right)^3 \times AK.$$

$\therefore$  comparing these two values for vol. of cone  $AFG$  we see that

$$(AK)^3 = 86 \times 105.$$

$$\therefore AK = \sqrt[3]{86 \times 105} \\ = 15 \cdot 5775 \text{ ft.}$$

and

$$HK = 8 \cdot 577 \text{ ft.}$$

Also the vol. of the given frustum, using Art. 268,

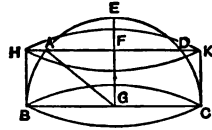
$$= \frac{1}{3} \times 6 \times 3 \cdot 1416 (3^2 + 2 \times 3 + 2^2)$$

$$= 3 \cdot 1416 \times 38$$

$$= 119 \cdot 38 \text{ cub. ft.}$$

and each half of the frustum = 59 \cdot 69 cub. ft.

T. M. K.



169. Surface of sphere =  $23 \text{ in.} \times \frac{28}{8 \cdot 1416} \text{ in.} = \frac{529}{8 \cdot 1416} \text{ sq. in.} = 168 \text{ sq. in.}$   
 nearly = quantity of leather required.

170. Using the rule of Art. 381 we have

$$39 \times (28 \cdot 6)^3 + 25 \times (26 \cdot 5)^3 + 26 \times 28 \cdot 6 \times 26 \cdot 5 \\ = 31900 \cdot 44 + 17556 \cdot 25 + 19705 \cdot 4 = 69162 \cdot 09;$$

$$\therefore \text{vol. of cask} = 69162 \cdot 09 \times 47 \cdot 5 \times \cdot 000031478 \text{ gallons} \\ = 108 \cdot 4 \text{ nearly.}$$

171. Let  $x$  = no. of feet in side of equil. triangle.  $\therefore$  cost of paving

$$= \frac{x^3}{4} \sqrt{3} \times 9d. = x^3 \times \frac{9}{4} \sqrt{3}d.;$$

and cost of fencing =  $8x \times 5s. = x \times 180d.;$

$$\therefore \text{cost of fencing : cost of paving} :: x \times 180d. : x^3 \times \frac{9}{4} \sqrt{3}d.;$$

$$\text{i.e. as } 80 : x \times \sqrt{3}; \text{ i.e. as } 80 \sqrt{3} : 8x.$$

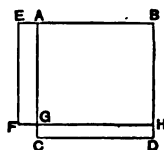
172. Let  $x$  ft. = side of equilateral triangle.  $\therefore$  cost of paving =  $\frac{1}{4} x^3 \sqrt{3}d.$   
 (vide preceding), and cost of fencing =  $x \times 180d.$   $\therefore \frac{1}{4} x^3 \sqrt{3} = x \times 180;$

$$\therefore x = 180 \times \frac{4}{9 \sqrt{3}} \text{ ft.} = \frac{80}{\sqrt{3}} \text{ ft.} = \frac{80}{8} \sqrt{3} \text{ ft.} = \frac{80}{8} \times 1 \cdot 73205 \text{ ft.} = 46 \cdot 188 \text{ ft.}$$

173. Let  $x$  in. = side of given square  $ACDB$ . Let  $EFHB$  be the given rectangle, so that  $AE = \cdot 202$  in., and  $GC = \cdot 2$  in. Now since  $ACDB = EFHB$ ;  $\therefore$  area of rectangle  $EG$  = area of rectangle  $GD$  (by Euc. I. Ax. 3), i.e.  $(x - \cdot 2) \times \cdot 202 \text{ sq. in.} = x \times \cdot 2 \text{ sq. in.};$

$$\therefore x \times \cdot 202 \text{ in.} - \cdot 2 \times \cdot 202 \text{ sq. in.} = x \times \cdot 2 \text{ in.};$$

$$\therefore x (\cdot 202 - \cdot 2) \text{ in.} = \cdot 2 \times \cdot 202 \text{ sq. in.}; \therefore x = \frac{\cdot 2 \times \cdot 202}{\cdot 202 - \cdot 2} \text{ in.}$$



174. Let  $x$  in. = length of edge of cube. Then  $x^3 \times x$  = true volume of cube. And calculated volume =  $(x + \cdot 202)(x - \cdot 2) \times x$  cub. in. Now since the height in both cases is the same, and the vols. are equal, the areas of the bases must likewise be equal.  $\therefore (x + \cdot 202)(x - \cdot 2) \text{ sq. in.} = x^2 \text{ sq. in.};$

$$\therefore (\text{as in preceding}), x = \frac{\cdot 2 \times \cdot 202}{\cdot 202 - \cdot 2} \text{ in.} = \frac{\cdot 0404}{\cdot 002} \text{ in.} = 20 \cdot 2 \text{ in.}$$

$\therefore$  true vol. of cube =  $(20 \cdot 2 \text{ in.})^3 = 8242 \cdot 408 \text{ cub. in.}$

175. Here  $AD = \sqrt{3} \text{ in.}$ ,  $DE = 1 \text{ in.}$ , and segment  $ABC$  = segment  $FGH$ . Now (by Euc. I. 47),

$$AE = \sqrt{(AD^2 - DE^2)} = \sqrt{\{(3 - 1) \text{ sq. in.}\}} = 1 \text{ in.}$$

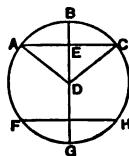
$\therefore$  (by Euc. I. 5, and 82),  $\angle EAD$  and  $\angle EDA$  are half a right angle, as also are  $\angle ECD$  and  $\angle EDC$ ;

$$\therefore \text{whole } \angle ADC = 90^\circ.$$

Now area of sector  $ADC$  : area of circle ::  $90^\circ : 360^\circ$ ;

$\therefore$  area of sector  $ADC = \frac{1}{4}$  area of circle.  $\therefore$  area of segment  $ABC = \frac{1}{4}$  area of circle - area of  $\triangle ADC$

$$= \frac{1}{4} \text{ area of circle} - \frac{\sqrt{2} \times \sqrt{2}}{2} \text{ sq. in.} = \frac{1}{4} \text{ area of circle} - 1 \text{ sq. in.}$$





178. (Note. The most advantageous shape for a parallelepiped with a lid, is that in which the parallelepiped is a cube (vide Art. 317); and that for a cylinder with a lid, is that in which height = 2 radius of base (vide Art. 325).) Take  $a$  and  $b$  as in preceding. Then vol. of cube =  $a^3$ , and vol. of cylinder =  $3.1416 \times b^2 \times 2b = 2 \times 3.1416 \times b^3$ ;

$$\therefore \text{(as in preceding)} b = \sqrt[3]{\frac{1}{2 \times 3.1416} \times a};$$

$\therefore$  quantity of material in cube =  $6a^3$ , and quantity of material in cylinder =  $2 \times 3.1416 \times b^3 + 3.1416 \times 2b \times 2b = 6 \times 3.1416 \times b^3 = (\text{as in preceding}) 6 \times .92 \times a^3$ ;  
 $\therefore$  material in cylinder : material in cube ::  $6 \times .92 \times a^3 : 6a^3$ ; i. e. as .92 : 1;  
 $\therefore$  material in cylinder = .92 of material in cube.

179. (Vide diag. to Chap. XXV. 21)

$$FE = \sqrt{(FD^2 + DE^2)} = \sqrt{\{(100^2 + 100^2) \text{ sq. ft.}\}} = 100\sqrt{2} \text{ ft.} \quad \therefore BE = 50\sqrt{2} \text{ ft.}$$

$$\therefore AB = \sqrt{(AE^2 - BE^2)} = \sqrt{\{(100^2 - 50^2 \times 2) \text{ sq. ft.}\}} = 50\sqrt{2} \text{ ft.}$$

$\therefore$  vol. of pyramid  $ADFE$

$$= \frac{1}{3} \times 100^2 \times 50\sqrt{2} \text{ cub. ft.} = 235702.2604 \text{ cub. ft.} = \text{vol. of cube.}$$

$\therefore$  edge of cube =  $\sqrt[3]{(235702.2604 \text{ cub. ft.})} = 61.77 \text{ ft. nearly.}$

180. Let  $AF = a$ ; then  $GF = \frac{1}{3}a$ , and  $AG = \frac{2}{3}a$ .  
 Let  $BF = b$ . Now from similar  $\Delta AGD$  and  $AFB$ ,  
 (by Euc. VI. 4),  $DG : GA :: BF : FA$ ;

$$\therefore DG = \frac{\frac{1}{3}a \times b}{a} = \frac{1}{3}b.$$

$\therefore$  vol. of cylinder  $DHKE$

$$= 3.1416 \times \left(\frac{1}{3}b\right)^2 \times \frac{1}{3}a = \frac{1}{3} \times \frac{1}{3} \times 3.1416 \times ab^2.$$

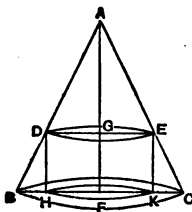
And vol. of cone  $ABC = \frac{1}{3} \times 3.1416 \times b^2 a$ ;  $\therefore$  vol. of cylinder  $DHKE = \frac{1}{3}$  of vol. of cone  $ABC$ . And this cylinder  $DHKE$  is the greatest possible. For, suppose  $k \times a = GF$ , where  $k$  is a proper fraction,  $\therefore AG = a - ka = a(1 - k)$ ;  
 $\therefore$  (as above),  $DG = b(1 - k)$ .  $\therefore$  vol. of the cylinder, the radius of the base of which =  $b(1 - k)$ , and the height of which =  $ka = 3.1416 \times b^2(1 - k)^2 \times ka$ ;  
 and this vol. will be less than vol. of former cylinder  $DHKE$  if

$$3.1416 \times b^2(1 - k)^2 \times ka \text{ is less than } 3.1416 \times b^2 a \times \frac{1}{3};$$

that is, if  $(1 - k)^2 k$  is less than  $\frac{1}{3}$ , or  $\frac{1}{3}$  greater than  $(1 - k)^2 k$ . Now

$$\frac{1}{3} - (1 - k)^2 k = \frac{1}{3} - k + 2k^2 - k^3 \\ = \left(\frac{1}{3} - k\right)^2 \left(\frac{1}{3} + k\right).$$

Thus when  $k = \frac{1}{3}$  this expression = 0 and at all other times is greater than 0; for  $\left(\frac{1}{3} - k\right)^2$  is always a positive quantity, and  $\frac{1}{3} - k$  is also positive as  $k$  is a proper fraction. Thus the trimmed frustum is the greatest when  $k = \frac{1}{3}$ .



THE END.





